

CRITICAL CONCEPTS FOR EXAM SUCCESS

ETHICAL AND PROFESSIONAL STANDARDS

I Professionalism

- I (A) Knowledge of the Law
- I (B) Independence and Objectivity
- I (C) Misrepresentation
- I (D) Misconduct

II Integrity of Capital Markets

- II (A) Material Nonpublic Information
- II (B) Market Manipulation

III Duties to Clients

- III (A) Loyalty, Prudence, and Care
- III (B) Fair Dealing
- III (C) Suitability
- III (D) Performance Presentation
- III (E) Preservation of Confidentiality

IV Duties to Employers

- IV (A) Loyalty
- IV (B) Additional Compensation Arrangements
- IV (C) Responsibilities of Supervisors

V Investment Analysis, Recommendations, and Action

- V (A) Diligence and Reasonable Basis
- V (B) Communication with Clients and Prospective Clients

VI Conflicts of Interest

- VI (A) Disclosure of Conflicts
- VI (B) Priority of Transactions
- VI (C) Referral Fees

VII Responsibilities as a CFA Institute Member or CFA Candidate

- VII (A) Conduct as Members and Candidates in the CFA Program
- VII (B) Reference to CFA Institute, the CFA designation, and the CFA Program

Global Investment Performance Standards (GIPS®)

- **Compliance Statement:** "[Insert name of firm] has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS)." Compliance must be applied on a firm wide basis.
- **Eight sections:** fundamentals of compliance; input data; calculation methodology; composite construction; disclosures; presentation and reporting; real estate; and private equity.

$$\text{var}(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B \sigma(R_A) \sigma(R_B) \rho(R_A, R_B)$$

Normal Distributions

Normal distribution is completely described by its mean and variance.

68% of observations fall within $\pm 1\sigma$.

90% fall within $\pm 1.65\sigma$.

95% fall within $\pm 1.96\sigma$.

99% fall within $\pm 2.58\sigma$.

Computing Z-Scores

Z-score: "standardizes" observation from normal distribution; represents # of standard deviations a given observation is from population mean.

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

Binomial Models

Binomial distribution: assumes a variable can have one of two values (success/failure) or, in the case of a stock, movement (up/down). Binomial distribution can describe direction of change in value of asset or portfolio; can be used to compute its expected value over several periods.

Sampling Distribution

Sampling distribution: probability distribution of all possible sample statistics computed from set of equal size samples randomly drawn from same population. The *sampling distribution of the mean* is the distribution of estimates of the mean.

Central Limit Theorem

Central limit theorem: when selecting simple random samples of size n from population with mean μ and finite variance σ^2 , the sampling distribution of sample mean approaches normal probability distribution with mean μ and variance equal to σ^2/n as the sample size becomes large.

Standard Error

Standard error of the sample mean is the standard deviation of distribution of the sample means.

known population variance: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

unknown population variance: $s_{\bar{x}} = \frac{s}{\sqrt{n}}$

Confidence Intervals

Confidence interval: gives range of values the mean value will be between, with a certain probability (say 90% or 95%). With known variance, formula for a confidence interval is:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$z_{\alpha/2} = 1.645$ for 90% confidence intervals (significance level 10%, 5% in each tail)

$z_{\alpha/2} = 1.960$ for 95% confidence intervals (significance level 5%, 2.5% in each tail)

$z_{\alpha/2} = 2.575$ for 99% confidence intervals (significance level 1%, 0.5% in each tail)

Null and Alternative Hypotheses

Null hypothesis (H_0): hypothesis the researcher wants to reject; the hypothesis that is actually tested; the basis for selection of the test statistics.

QUANTITATIVE METHODS

Time Value of Money Basics

- **Future value (FV):** amount to which investment grows after one or more compounding periods.
- **Future value:** $FV = PV(1 + I/Y)^N$.
- **Present value (PV):** current value of some future cash flow.
- **Present value:** $PV = FV/(1 + I/Y)^N$.
- **Annuities:** series of equal cash flows that occur at evenly spaced intervals over time.
- **Ordinary annuity:** cash flow at *end-of-time* period.
- **Annuity due:** cash flow at *beginning-of-time* period.
- **Perpetuities:** annuities with an infinite life.

$$PV_{\text{perpetuity}} = PMT/(I/Y)$$

Means

Arithmetic mean: sum of all observation values in sample/population, divided by # of observations.

Geometric mean: used when calculating investment returns over multiple periods or to measure compound growth rates.

Geometric mean return:

$$\bar{R}_G = [(1 + R_1) \times \dots \times (1 + R_N)]^{1/N} - 1$$

$$\text{harmonic mean} = \frac{N}{\sum_{i=1}^N \left(\frac{1}{X_i} \right)}$$

Variance and Standard Deviation

Variance: average of squared deviations from mean.

$$\text{population variance} = \sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N}$$

$$\text{sample variance} = s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}$$

Standard deviation: square root of variance.

Holding Period Return (HPR)

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} \text{ or } \frac{P_t + D_t}{P_{t-1}} - 1$$

Coefficient of Variation

Coefficient of variation (CV): expresses how much dispersion exists relative to mean of a distribution; allows for direct comparison of dispersion across different data sets. CV is calculated by dividing standard deviation of a distribution by the mean of the distribution:

$$CV = \frac{s}{\bar{X}}$$

Sharpe Ratio

Sharpe ratio: measures *excess* return per unit of risk.

$$\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{\sigma_p}$$

$$\text{Roy's safety - first ratio} = \frac{\bar{r}_p - r_{\text{target}}}{\sigma_p}$$

For both ratios larger is better.

Expected Return/Standard Deviation

Expected return: $E(X) = \sum P(x_i) x_i$

$$E(X) = P(x_1)x_1 + P(x_2)x_2 + \dots + P(x_n)x_n$$

Probabilistic variance:

$$\begin{aligned} \sigma^2(X) &= \sum P(x_i) [x_i - E(X)]^2 \\ &= P(x_1) [x_1 - E(X)]^2 + P(x_2) [x_2 - E(X)]^2 + \dots + P(x_n) [x_n - E(X)]^2 \end{aligned}$$

Standard deviation: (take square root of variance).

Correlation and Covariance

Correlation = covariance divided by product of the two standard deviations.

$$\text{corr}(R_i, R_j) = \frac{\text{COV}(R_i, R_j)}{\sigma(R_i) \sigma(R_j)}$$

Expected return, variance of 2-stock portfolio:

$$E(R_p) = \sum_{i=1}^N w_i E(R_i)$$

Alternative hypothesis (H_a): concluded if there is sufficient evidence to reject the null hypothesis.

Difference Between One & Two-Tailed Tests

One-tailed test: tests whether value is greater than or less than zero.

Two-tailed test: tests whether value is different from zero.

One-tailed test: $H_0: \mu \leq 0$ versus $H_a: \mu > 0$

Two-tailed test: $H_0: \mu = 0$ versus $H_a: \mu \neq 0$

Type I And Type II Errors

When hypothesis testing, two possible errors:

- **Type I error:** rejection of null hypothesis when it is actually true.
- **Type II error:** failure to reject null hypothesis when it is actually false.

Regression Equation

General form of linear regression model:

$$Y_i = b_0 + b_1 X_i + \varepsilon_i$$

- Y_i and X_i are the i^{th} observation of the dependent and independent variables, respectively.
- b_0 = intercept term (represents the value of Y if X is zero).
- b_1 = slope coefficient (measures the change in Y for a one unit change in X).
- ε_i = residual error of the i^{th} observation.

Regression Coefficients: Testing Significance

Appropriate test structure for null and alternative hypotheses:

$$H_0: b_1 = 0 \text{ versus } H_a: b_1 \neq 0$$

The decision rule for tests of significance for regression coefficients:

$$\text{Reject } H_0 \text{ if: } t > t_{\text{critical}} \text{ or } -t < -t_{\text{critical}}$$

Rejection of the null means the slope coefficient is statistically different from hypothesized value of b_1 .

MICROECONOMICS

Elasticity of Demand

$$\text{Price elasticity of demand} = \frac{\% \Delta \text{ quantity demanded}}{\% \Delta \text{ price}}$$

If absolute value > 1 , demand is elastic; if absolute value < 1 , demand is inelastic.

Price elasticity has two main determinants:

- Availability of substitutes.
- Share of budget spent on product.

Accounting Costs vs. Economic Costs

Accounting costs include firm's **explicit costs**;

economic costs include **both explicit/implicit costs** (opportunity cost of equity capital).

Short run vs. long run: in **short run**, size of plant/equipment cannot be changed. In **long run**, all resources (costs) are variable.

Law of diminishing returns: as more resources are devoted to production process, they increase output but at ever-decreasing rate.

Competitive Models

Price taker accepts market price to sell product.

Price searcher seeks price that maximizes profit.

Purely competitive markets:

- Large number of independent firms.
- All firms produce a homogeneous product.
- Each seller is small relative to the market.
- No barriers to entry.

Competitive price searcher markets:

- Large number of independent firms.
- Each firm produces a **differentiated** product.
- Low barriers to entry.
- Demand is highly elastic.

Monopoly is a market where **one** firm sells a well-defined product that has no good substitutes and high entry barriers. **Oligopoly** is a similar structure, but has a small number of firms.

Any firm will maximize profits by expanding output until marginal revenue = marginal cost.

Marginal Revenue Product

If a firm uses an additional unit of an input, **increase in output** is **marginal product (MP)** of last unit of resource employed. **Increase in revenue** from producing/selling marginal product is the **marginal revenue product (MRP)**. Profit-maximizing firms will increase use of each resource until $MRP = \text{price of the last resource unit}$.

In **equilibrium**, the price of a resource will equal the resource's marginal revenue product.

INTERNATIONAL ECONOMICS

Foreign Exchange

Direct quote: domestic currency (DC) per foreign currency (DC/FC).

Bid-ask spread stated as percent of asking price:

$$\% \text{ spread} = \frac{\text{ask price} - \text{bid price}}{\text{ask price}} (100)$$

Foreign currency is at forward **discount (premium)** if forward rate is below (above) spot rate, using **direct** quotes.

$$\left(\begin{array}{c} \text{forward} \\ \text{prem or disc} \end{array} \right) = \left(\frac{F - S}{S} \right) \left(\frac{360}{\# \text{ of forward contract days}} \right)$$

Currency appreciates/depreciates due to:

- Relative income growth rates.
- Relative rates of inflation.
- Changes in real interest rates.

Interest Rate Parity (IRP)

If you can borrow in low interest rate country, and lend those borrowed funds at a higher rate in another country, the forward exchange rate must adjust to offset interest difference.

$$\text{IRP: } \frac{F}{S} = \frac{1 + i_d}{1 + i_f} \quad (\text{Note: } S = \text{DC/FC})$$

IRP must hold due to arbitrage opportunities.

Purchasing Power Parity (PPP)

The **law of one price**: identical goods should have same price in all locations. **Absolute PPP** asks if law of one price is correct **on average**. **Relative PPP** requires that exchange rate changes be proportional to price level changes (i.e. differences in inflation).

$$\text{Formula for relative PPP: } c_t = c_0 \times \frac{(1 + i_{DC})^t}{(1 + i_{FC})^t}$$

MACROECONOMICS

Inflation

Inflation rate: rate of change in price index over given period of time:

$$i = \frac{\text{current price index} - \text{last period price index}}{\text{last period price index}}$$

Unemployment

- **Frictional:** economic changes prevent matching **qualified** workers with job openings.
- **Structural:** workers lack needed skills.
- **Cyclical:** change in economic activity level.

Demand Side Fiscal Policy Models

- **Keynesian:** deficit spending can increase aggregate demand.
- **Crowding out:** benefits of deficit spending are offset by increases in interest rates.
- **New classical:** deficit spending has no effect.
- **Supply side:** lower tax rates enhance aggregate supply (leads to more efficient allocation of capital).

Timing of Fiscal Policy

Time Lags:

(1) Recognition (2) Implementation (3) Impact

Automatic stabilizers:

- (1) Unemployment compensation.
- (2) Corporate profit taxes.
- (3) Progressive personal income taxes.

Monetary Policy

$$\text{Deposit Expansion Multiplier} = \frac{1}{\text{Reserve Requirement}}$$

Fed controls money supply three ways:

- Reserve requirements.
- Open market operations—most used
- Discount rate.

FINANCIAL STATEMENT ANALYSIS

Revenue Recognition

Two requirements: (1) completion of earnings process and (2) assurance of payment.

Revenue recognition methods:

- Sales basis method.
- Percentage-of-completion method.
- Completed contract method.
- Installment sales.
- Cost recovery method.

Unusual or Infrequent Items

- Gains/losses from disposal of a business segment.
- Gains/losses from sale of assets or investments in subsidiaries.
- Provisions for environmental remediation.
- Impairments, write-offs, write-downs, and restructuring costs.

- Integration expenses associated with businesses recently acquired.

Extraordinary Items

Similar to unusual/infrequent items, except extraordinary items are **both unusual and infrequent** in occurrence, and material in nature (e.g., losses from expropriation of assets).

Discontinued Operations

To be accounted for as a discontinued operation, a business—assets, operations, investing, financing activities—must be physically/operationally distinct from rest of firm. Income/losses are reported net of tax after net income from continuing operations.

Compute Cash Flows From Operations (CFO)

Direct Method—Start with cash collections (cash

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→ equivalent of sales); cash inputs (cash equivalent of cost of goods sold); cash operating expenses; cash interest expense; cash taxes.

Indirect Method—Start with net income, subtracting back gains and adding back losses resulting from financing or investment cash flows, adding back all noncash charges, and adding and subtracting asset and liability accounts that result from operations.

Free Cash Flow

Free cash flow (FCF) measures cash available for discretionary purposes. It is equal to operating cash flow less net capital expenditures.

Critical Ratios

Common-size balance sheets/income statements.

Common-size balance sheet expresses all balance sheet accounts as a percentage of total assets.

Common-sized income statement expresses all income statement items as a percentage of sales.

Current ratio and quick ratio:

$$\text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{quick ratio} = \frac{\text{cash} + \text{marketable securities} + \text{receivables}}{\text{current liabilities}}$$

Receivables, inventory, payables turnover, and days supply ratios—all of which are used in the cash conversion cycle:

$$\text{receivables turnover} = \frac{\text{net annual sales}}{\text{average receivables}}$$

$$\text{inventory turnover} = \frac{\text{cost of goods sold}}{\text{average inventory}}$$

$$\text{payables turnover ratio} = \frac{\text{cost of goods sold}}{\text{average trade payables}}$$

$$\text{average receivables collection period} = \frac{365}{\text{receivables turnover}}$$

$$\text{average inventory processing period} = \frac{365}{\text{inventory turnover}}$$

$$\text{payables payment period} = \frac{365}{\text{payables turnover ratio}}$$

$$\text{cash conversion cycle} = \left(\frac{\text{average receivables}}{\text{collection period}} \right) + \left(\frac{\text{average inventory}}{\text{processing period}} \right) - \left(\frac{\text{payables payment}}{\text{period}} \right)$$

Total asset, fixed-asset, and equity turnover ratios:

$$\text{total asset turnover} = \frac{\text{net sales}}{\text{average total net assets}}$$

$$\text{fixed asset turnover} = \frac{\text{net sales}}{\text{average net fixed assets}}$$

$$\text{equity turnover} = \frac{\text{net sales}}{\text{average equity}}$$

Gross, operating, and net profit margins:

$$\text{gross profit margin} = \frac{\text{gross profit}}{\text{net sales}}$$

$$\text{operating profit margin} = \frac{\text{operating profit}}{\text{net sales}} = \frac{\text{EBIT}}{\text{net sales}}$$

$$\text{net profit margin} = \frac{\text{net income}}{\text{net sales}}$$

Return on assets: (return on total capital: "ROTC")

$$\text{return on assets} = \frac{\text{net income} + \text{interest exp.}}{\text{average total capital}}$$

Debt to equity ratio and total debt ratio:

$$\text{debt to equity ratio} = \frac{\text{total long term debt}}{\text{total equity}}$$

$$\text{total debt ratio} = \frac{\text{current liabilities} + \text{total long term debt}}{\text{total debt} + \text{total equity}}$$

Interest coverage and the CF to LTD ratio:

$$\text{interest coverage} = \frac{\text{EBIT}}{\text{Interest}}$$

cash flow to CFO

$$\text{LTD} = \frac{\text{BV of LTD} + \text{PV of operating leases}}$$

Growth rate (g): $g = \text{RR} \times \text{ROE}$

The calculation of the *retention rate* is:

$$\text{retention rate} = 1 - \frac{\text{dividends declared}}{\text{operating income after taxes}}$$

Liquidity ratios indicate company's ability to pay its short-term liabilities.

Operating performance ratios indicate how well management operates the business.

DuPont Analysis

Traditional DuPont equation:

$$\text{return on equity} = \left(\frac{\text{net income}}{\text{sales}} \right) \left(\frac{\text{sales}}{\text{assets}} \right) \left(\frac{\text{assets}}{\text{equity}} \right)$$

You may also see it presented as

$$\text{return on equity} = \left(\frac{\text{net profit}}{\text{margin}} \right) \left(\frac{\text{asset}}{\text{turnover}} \right) \left(\frac{\text{equity}}{\text{multiplier}} \right)$$

Extended DuPont equation further breaks down net profit margin:

$$\text{ROE} = \left(\frac{\text{EBIT}}{\text{sales}} \right) \left(\frac{\text{sales}}{\text{assets}} \right) \left(\frac{\text{interests}}{\text{assets}} \right) \left(\frac{\text{assets}}{\text{equity}} \right) (1 - r)$$

You may also see it presented as

$$\text{ROE} = \left(\frac{\text{operating profit}}{\text{margin}} \right) \left(\frac{\text{total asset}}{\text{turnover}} \right) \left(\frac{\text{int. exp. rate}}{\text{rate}} \right) \left(\frac{\text{financial leverage}}{\text{multiplier}} \right) \left(\frac{\text{tax retention rate}}{\text{rate}} \right)$$

Inventory Accounting

In periods of rising prices and stable or increasing inventory quantities:

LIFO results in:

Higher COGS
Lower taxes
Lower net income (EBT & EAT)
Lower inventory balances
Lower working capital
Higher cash flows (↓ taxes paid out)

FIFO results in:

Lower COGS
Higher taxes
Higher net income (EBT & EAT)
Higher inventory balances
Higher working capital
Lower cash flows (↑ taxes paid out)

Basic and Diluted EPS

Basic EPS calculation *does not* consider effects of any dilutive securities in computation of EPS

$$\text{basic EPS} = \frac{\text{net income} - \text{preferred dividends}}{\text{wtd. avg. no. of common shs. outstanding}}$$

$$\text{diluted EPS} = \frac{\text{adj income avail for common shares}}{\text{wtd avg common shares plus potential common shares outstanding}}$$

Therefore, diluted EPS is:

$$\left[\frac{\text{net income}}{\text{div}} + \frac{\text{convertible preferred dividends}}{\text{dividends}} + \left(\frac{\text{convertible debt interest}}{\text{interest}} \right) (1 - r) \right] + \left(\frac{\text{wtd avg sh's}}{\text{sh's}} \right) + \left(\frac{\text{shares from conversion of conv. pfd. sh's}}{\text{conv. pfd. sh's}} \right) + \left(\frac{\text{sh's from conversion conv. debt}}{\text{conv. debt}} \right) + \left(\frac{\text{shares issuable from stock options}}{\text{stock options}} \right)$$

Financial Statement Impacts:

Capitalizing vs. Expensing

Capitalizing: lowers income variability and increases near-term profits. Increase TA, Equity. **Expensing:** opposite effect.

Depreciation and Impairment

Calculation: Straight line and SYD methods subtract salvage value, the double-declining balance method does not.

Assuming continued investment in new assets, relative to a firm that uses accelerated depreciation, a company that uses straight line depreciation will have:

- **Lower:** Depreciation expense, turnover ratios.
- **Higher:** Net income, assets, equity, ROA, ROE.
- **Same:** Cash flows.

Impairment

- **Recognized** when the carrying value of an asset is higher than the sum of the future cash flows (undiscounted) from their use and disposal. This makes impairments subject to management manipulation.
- **Loss** measured as the carrying value of the asset minus the fair value of the asset.
- **Financial Statement impact.** Write-down will immediately cause income, asset value, deferred taxes, and equity to decline. It will cause future depreciation to decline, and as a result, future net income to rise and future asset turnover ratios to increase.

Deferred Taxes

- Created when **taxable income** (on tax return) differs from pretax income (financial statement).
- **Deferred tax liabilities (DTL)** are created by the use of accelerated depreciation methods for tax purposes (resulting in lower taxable income) and straight line for financial reporting (lower depreciation expense results in higher net income).
- If DTL are not expected to reverse, treat as equity for analysis.

Financing Liabilities

- **Premium bond:** coupon rate > market rate at issuance.
- **Discount bond:** coupon rate < market rate at issuance.
- **Interest expense** equals book value at the beginning of the year multiplied by the market rate of interest at the time the bonds were issued.
- **Analytic cash flow perspective:** For premium bonds, CFF is overstated and CFO is understated relative to par and discount bonds.

Leases

Financial statement/ratio impact of lease accounting from the lessee perspective: Capital leases result in:

- **Higher:** Assets, liabilities, CFO, Debt/Equity.
- **Lower:** Net income (early years), CFF, current ratio, working capital, asset turnover, ROA, ROE.
- **Same:** Total cash flow.

Weighted Average Cost of Capital

$$WACC = (w_d)[k_d(1-t)] + (w_{ps})(k_{ps}) + (w_{ce})(k_s)$$

Cost of retained earnings using CAPM:

$$k_s = RFR + \beta(R_{mkt} - RFR)$$

Cost of Newly Issued Equity

Cost of new common equity (k_e) is higher than cost of retained earnings because of flotation costs.

$$k_e = \left[\frac{D_1}{P_0(1-F)} \right] + g$$

Capital Budgeting

$$NPV = CF_0 + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \dots + \frac{CF_n}{(1+k)^n}$$

IRR: discount rate that sets NPV equal to zero.

Operating Leverage

Operating leverage: variable/fixed cost tradeoff.

$$DOL = \frac{\% \Delta EBIT}{\% \Delta Sales}$$

$$DOL_{Qty} = \frac{Q(P-V)}{Q(P-V)-F}$$

$$DOL_{Sales} = \frac{S-VC}{S-VC-F}$$

Financial Leverage

Refers to use of fixed-income securities (debt and preferred stock).

$$DFL = \frac{\% \Delta EPS}{\% \Delta EBIT} = \frac{EBIT}{EBIT - INTEREST}$$

$$\% \Delta EPS = (DFL)(\% \Delta EBIT)$$

Total Leverage

Degree of total leverage (DTL) combines degree of operating leverage and financial leverage. Measures effect on EPS of given change in sales.

$$DTL = (DOL)(DFL)$$

Dividend Irrelevance Theory:

(Modigliani and Miller) A firm's dividend policy

has no effect on stock price or cost of capital and is irrelevant (shareholders effectively create their own dividends). Holds only in perfect world with no taxes, no brokerage costs, infinitely divisible shares.

Bird-in-the-Hand Theory:

Cost of equity declines as dividend payout increases (investors are less certain of receiving future capital gains from reinvested retained earnings than they are of receiving current certain dividend payments).

Tax Preference Theory:

Income/capital gains taxes influence investor preferences.

Residual Dividend Model

Four steps to determine target payout ratio:

Step 1: Identify optimal capital budget.

Step 2: Determine equity needed to finance capital budget for capital structure.

Step 3: Meet equity requirements to maximum extent possible with retained earnings.

Step 4: Pay dividends with "residual" earnings available after needs of optimal capital budget are supported (i.e., dividends are paid out of leftover earnings).

PORTFOLIO MANAGEMENT

Investment Policy Statement

Investment objectives:

- Return objectives.
- Risk tolerance.

Constraints:

- Liquidity needs.
- Time horizon.
- Tax concerns.
- Legal and regulatory factors.
- Unique needs and preferences.

Required Rate of Return

Components:

1. Real risk-free rate.
2. Expected inflation rate premium (IP).
3. Risk premium.

Formula for nominal RFR:

$$RFR_{nominal} = (1 + RFR_{real})(1 + IP) - 1$$

Risk Premium

Investors demand risk premium for the uncertainty associated with investment; addresses factors such as business, financial, liquidity, exchange rate, and country risk.

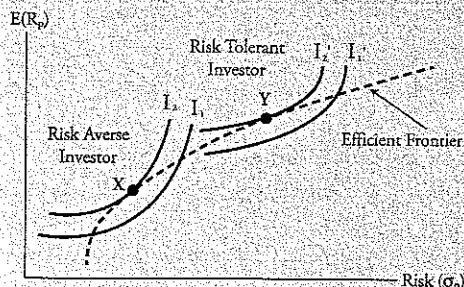
$$E(R) = (1 + RFR_{real})(1 + IP)(1 + RP) - 1$$

Approximation formula for nominal required rate:

$$E(R) \cong RFR + IP + RP$$

Combining Preferences with the Optimal Set of Portfolios

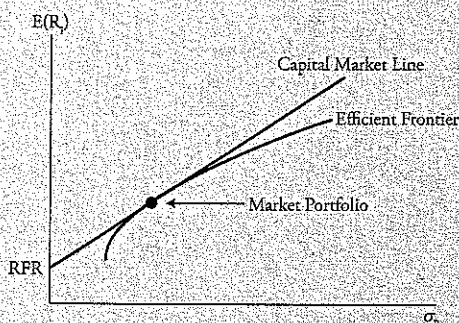
Markowitz efficient frontier is the set of portfolios that have highest return for given level of risk.



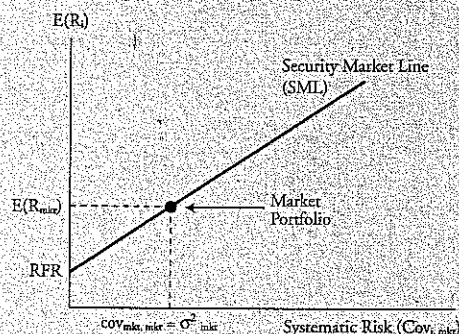
Security Market Line (SML)

Investors should only be compensated for risk relative to market. Unsystematic risk is diversified away; investors are compensated for systematic risk. The equation of the SML is the CAPM, which is a return/systematic risk equilibrium relationship.

total risk = systematic + unsystematic risk



$$CAPM: E(R_i) = RFR + \beta_i [E(R_{mkt}) - RFR]$$



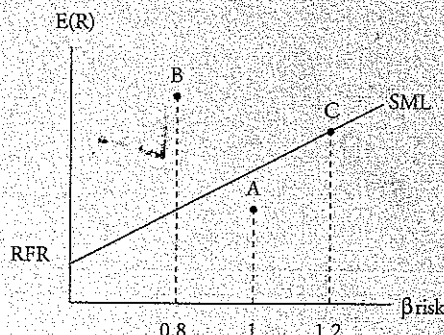
The SML and Equilibrium

Identifying Mispriced stocks

Consider three stocks (A, B, C) and SML:

Estimated stock returns should plot on SML!

- a return plot over the line is underpriced.
- a return plot under the line is overpriced.



SECURITIES MARKETS & EQUITY INVESTMENTS

Well-Functioning Security Markets

- Timely, accurate information.
- Liquidity.
- Internal efficiency (lowest possible transactions costs).
- Informational (external) efficiency (prices rapidly adjust to new information).

Margin Purchases

For margin transactions:

- leverage factor = 1/margin percentage.
- levered return = HPR × leverage factor.

Margin Calls

Margin Call Trigger Prices

$$\text{Margin Purchase: } \frac{P_0(1 - \text{initial margin \%})}{1 - \text{maintenance margin \%}}$$

$$\text{Short Sale: } \frac{P_0(1 + \text{initial margin \%})}{1 + \text{maintenance margin \%}}$$

Computing Index Prices

$$\text{Price-weighted Index} = \frac{\sum \text{stock prices}}{\text{adjusted divisor}}$$

Value-weighted Index

$$= \frac{\sum (\text{current prices})(\# \text{ shares})}{\sum (\text{base year prices})(\# \text{ base year shares})} \times \text{base value}$$

Forms of EMH

- **Weak form.** Current stock prices fully reflect available security market info. Volume information/past price do not relate to future direction of security prices. Investor cannot achieve excess returns using tech analysis.
- **Semi-strong form.** Security prices instantly adjust to new public information. Investor cannot achieve excess returns using fundamental analysis.
- **Strong form.** Stock prices fully reflect all information from public and private sources. Assumes perfect markets in which all information is cost free and available to everyone at the same time. Even with inside info, investor cannot achieve excess returns.

One-Period Valuation Model

$$V_0 = \frac{D_1}{(1+k_e)} + \frac{P_1}{(1+k_e)}$$

Be sure to use *expected* dividend D_1 in calculation.

Infinite Period Dividend Discount Models

Supernormal growth model (multi-stage) DDM:

$$V_0 = \frac{D_1}{(1+k_e)} + \dots + \frac{D_n}{(1+k_e)^n} + \frac{P_n}{(1+k_e)^n}$$

where: $P_n = \frac{D_{n+1}}{(k_e - g_c)}$

Constant growth model:

$$V_0 = \frac{D_0(1+g_c)}{k_e - g_c} = \frac{D_1}{k_e - g_c}$$

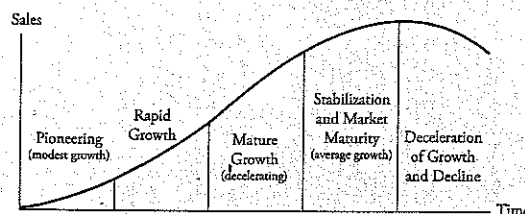
Critical relationship between k_e and g_c :

- As difference between k_e and g_c *widens*, value of stock *falls*.
- As difference *narrows*, value of stock *rises*.
- Small changes in difference between k_e and g_c cause large changes in stock's value.

Critical assumptions of infinite period DDM:

- Stock pays dividends; constant growth rate.
- Constant growth rate, g_c , never changes.
- k_e must be greater than g_c (or math will not work).

Industry Life Cycle



Model breaks down if any assumption is not met.

Required Rate of Return Components

Required rate of return, k_e , is influenced by:

- Real risk-free rate (RFR_{real}).
- Inflation premium (IP).
- Risk premium (RP) to compensate for uncertainty of expected returns.

ROE Decomposition

ROE's component ratios show strengths and weaknesses of the business:

$$\text{ROE} = \left(\frac{\text{Profit margin}}{\text{sales}} \right) \times \left(\frac{\text{Total asset turnover}}{\text{total assets}} \right) \times \left(\frac{\text{Equity multiplier}}{\text{equity}} \right)$$

$$= \frac{\text{net income}}{\text{sales}} \times \frac{\text{sales}}{\text{total assets}} \times \frac{\text{total assets}}{\text{equity}}$$

- If profit margin increases, ROE increases.
- If ROE increases, g_c [(ROE)(RR)], increases.
- If g_c increases, $k - g$ will *decrease*.
- If $k - g$ decreases, price of stock will increase.

Assumptions of Technical Analysis

- Supply/demand determine values (prices).
 - Supply/demand is driven by both rational and irrational behavior.
 - Security prices move in trends that persist for long periods of time.
 - While causes of supply/demand changes are hard to determine, we can see the actual shifts in supply/demand in market price behavior.
- Technicians tend to take one of two views when analyzing general market rules:
- Contrarian.** Thinks majority is wrong, so does the opposite of what investors are doing.
 - Follow the smart money.** Thinks smart investors know best, so wants to "jump on the bandwagon" while there's still time.

DEBT INVESTMENTS

Basic Features of Bond Structures

- Indenture.** Agreement containing the terms under which money is borrowed.
- Term to maturity.** Length of time until loan contract or agreement expires.
- Par value.** Amount borrower promises to pay on or before *maturity* date of the issue.
- Coupon rate.** When multiplied by par value, gives amount of interest to be paid per period.
- Zero-coupon bonds.** No-interest bonds; are sold at a deep discount from their par values.

Repayment/Prepayment Provisions

- Bullet bonds.** Lump sum at maturity pays entire principal.
- Serial bonds.** Pay off principal through series of payments over time.
- Amortizing securities.** Periodic principal and interest payments.
- Sinking fund provisions.** Provide for bond retirement through predefined principal payments over life of the issue.
- Call provisions.** Issuer has right (but not obligation) to *retire all or part of issue prior to maturity*. Issuer owns option to call the bonds away from investor.
- Refunding provisions.** Nonrefundable bonds prohibit premature retirement of an issue from proceeds of a lower coupon bond. Bonds that carry these provisions can be freely callable but nonrefundable.

Basics of Floating Rate Bonds

- These securities pay variable rate of interest.
- Common procedure for setting coupon rates on floating-rate bonds starts with *reference rate*; then adds/subtracts a stated *margin*.

Interest Rate Risk

Key point: there is an *inverse relationship between interest rates and bond prices*.

How bond's features affect interest rate risk:

- Longer maturity bonds. Higher interest rate risk (all else same).
 - Smaller coupon bonds. Higher interest rate risk (all else same).
 - If market interest rates are high, price volatility will be lower than if market interest rates are low.
- Floating rate securities have very low levels of price volatility in relation to interest rate changes.
- If coupon rate > required market yield
⇒ bond price > par value: *premium bond*
- If coupon rate < required market yield
⇒ bond price < par value: *discount bond*
- If coupon rate = required market yield
⇒ bond price = par value: *par bond*

Reinvestment Risk

If interest rates decline, investors are forced to reinvest at lower yields.

Bonds with high coupons have greater risk. Greatest risk is with callable bonds, where all/part of principal can be repaid in low interest rate environment.

Credit Risk

- Default risk.** Issuer might not make payments.
- Credit spread risk.** Difference in bond's yield from yield on risk-free security. All else equal, the riskier the bond, the higher the spread.
- Downgrade risk.** Bond may be reclassified as riskier security by a major rating agency.

Basic Bond Pricing

To value a semi-annual pay bond using a financial calculator:

$$FV = \text{par}, PMT = \frac{\text{coupon}}{2}$$

$$N = (\# \text{ years} \times 2), I/Y = \frac{\text{yield}}{2}$$

There are *two*, equivalent, ways to price a bond:

- Discount at constant rate applied to all cash flows (YTM) to find all future cash flows' PV.
- Treat each cash flow as its own zero-coupon bond and find PV of each "zero" using appropriate spot rates for each cash flow.

Prices *must be the same* to prevent arbitrage.

Accrued Interest and Clean Prices

Bond price without accrued interest is *clean price*.

Full Price includes accrued interest.

Full price = clean price + accrued interest

Yield Calculations

$$\text{current yield} = \frac{\text{annual coupon payment}}{\text{bond price}}$$

Annual Equivalent Yield

Converting a bond-equivalent yield (BEY) to an equivalent annual yield (EAY) or vice versa:

$$\text{BEY of an annual pay bond} = 2 \left[(1 + \text{YTM}_{\text{annual pay bond}})^{1/2} - 1 \right]$$

$$\text{annual equivalent yield} = \left(1 + \frac{\text{BEY}}{2} \right)^2 - 1$$

Spot Rates

To derive a bond's value using spot rates, discount the individual cash flows at appropriate rate for each flow's time horizon; sum PV of the cash flows to get bond's current value. This value is the arbitrage-free value.

Duration and Convexity

- Duration** is the slope of a bond's price-yield function. It is steeper at low interest rates, flatter at high interest rates. So, duration (interest rate sensitivity) is high at low rates and low at higher rates. This holds for non-callable bonds.
- Convexity** is a measure of degree of curvature or convexity in the price/yield relationship. Convexity accounts for amount of error in estimated price (based on duration).

A callable bond is likely to be called as yields fall, so no one will pay a price higher than the call price. The price won't rise significantly as yield falls and you'll see negative convexity at work—as yields fall, prices rise at a decreasing rate. For a positively convex bond, as yields fall, prices rise at an increasing rate.

$$\rightarrow \text{Effective duration (D)} = \frac{V_- - V_+}{2V_0(\Delta y)}$$

Convexity measures curvature of the price-yield function.

$$\% \Delta \text{Price} = [-\text{Duration}(\Delta y) + \text{Convexity}(\Delta y)^2] \times 100$$

Note: Δy is in decimal form

Term Structure Theories

- **Expectations hypothesis.** Yield curve shape reflects investor expectations about future behavior of short

term interest rates. Forward rates computed using today's spot rates are best guess of future interest rates.

- **Liquidity preference theory.** Investors prefer greater liquidity; will demand premium (higher yields to invest in longer-term issues).
- **Market segmentation theory.** Market for debt securities is segmented on basis of investor maturity preferences. Each segment's interest rate level is determined by supply/demand.

ALTERNATIVE INVESTMENTS

Characteristics

Characteristics that generally apply include:

- **Lower liquidity** than traditional investments.
- **Accurate market values** difficult to determine.
- **Limited historical** performance data.

Exchange Traded Funds (ETF)

Special type of fund that invests in a portfolio of stocks or bonds; designed to mimic performance of a specified index.

Advantages of ETFs

- Efficient method of diversification.
- Trade similar to traditional equity investments.
- Some ETFs patterned after indexes with active futures/option market; better risk management.
- Exact composition is known at all times.
- Typically, very efficient operating expense ratios; no loads to purchase or redeem shares.
- Decreased capital gains tax liability.

Disadvantages of ETFs

- Some countries have fewer indexes than U.S. for ETFs to track (results in mid- or low-cap stocks not being well represented).
- Ability to trade intraday may not be significant to investors with longer time horizons.
- ETFs with low trading volume may have large bid-ask spreads.
- Larger investors may choose to directly invest in an index portfolio, resulting in lower expenses/lower tax consequences.

Valuation of Real Estate Investments

- **Valuation methods:** Cost method, sales comparison method, and income method.
- Income method uses a discounted cash flow model similar to that for a perpetuity:

$$\text{Value} = \frac{\text{NOI}}{\text{Market cap rate}}$$

- **Net operating income (NOI)** = equals gross operating income less estimated vacancy, collections, and other operating expenses, (including property taxes, but excluding income taxes). NOI does not include depreciation or financing costs.

Hedge Funds – Risks

- **Illiquidity.**
- **Potential for mispricing.** Investments in esoteric, infrequently traded securities may lead to difficulty determining true value.
- **Counterparty credit risk.**
- **Settlement errors.** Risk of counterparty failure to deliver security as agreed on settlement day.
- **Short covering.** Risk that managers who short sell as a strategy will have to cover their shorts and repurchase securities at price higher than where they originally sold.
- **Margin calls.** Can result in forced selling of assets, possibly at a loss, on an already highly leveraged position.

DERIVATIVES

Futures vs. Forwards

- **Forward contract.** One party agrees to buy, (counterparty sells) a physical asset/security at specific price on specific *future* date. If asset's future price increases, buyer (at the older, lower price) has a gain and seller has a loss.
- **Futures contract.** Standardized, exchange-traded forward contract. Different from forwards in that futures trade in active secondary market, are regulated, backed by clearinghouse, and require daily settlement of gains/losses.

Arbitrage

- **Law of one price.** Two assets with identical cash flows in the future, regardless of future events, should have the same price. If A and B have identical future payoffs, and A is priced lower than B, buy A and sell B.
- **Second type of arbitrage.** Two assets with uncertain returns can be combined in a portfolio that will have a certain payoff. If a portfolio of A and B has a certain payoff, the portfolio should yield the risk-free rate.

Forward Contracts

- Long must pay a certain amount at specific future date to short, who will deliver the underlying asset.
- A cash settlement forward contract does not require actual delivery of the underlying asset, but a cash payment to the party disadvantaged by the difference between market price of the asset and contract price at settlement date.
- Early termination can be achieved by entering into a new forward contract with the opposite position, at the then-current expected future price. This will fix the amount of payment to be made/received at settlement date.

Forward Rate Agreements (FRA)

Can be viewed as a forward contract to borrow/lend money at a certain rate at some future date.

Formula for payment to the long at settlement is:

$$\left(\frac{\text{notional principal}}{1 + (\text{floating rate}) \left(\frac{\text{days}}{360} \right)} \right) \times \frac{(\text{floating rate} - \text{forward rate}) \left(\frac{\text{days}}{360} \right)}{1 + (\text{floating rate}) \left(\frac{\text{days}}{360} \right)}$$

Futures vs. Forwards

Forwards

Private contracts
Unique contracts
Default risk
Little or no regulation

Futures

Exchange-traded
Standardized contracts
Guaranteed by clearinghouse
Regulated

American vs. European Options

American options let the owner exercise the option any time before or at expiration. *European options* can be exercised *only* at expiration. Value of the American option will equal/exceed value of the European option.

Lower and Upper Bounds for Options

Option	Minimum Value	Maximum Value
European call (c)	$c \geq \text{Max} \left[0, S_t - \frac{X}{(1 + \text{RFR})^t} \right]$	S_t
American call (C)	$C_t \geq \text{Max} \left[0, S_t - \frac{X}{(1 + \text{RFR})^t} \right]$	S_t
European put (p)	$p_t \geq \text{Max} \left[0, \frac{X}{(1 + \text{RFR})^t} - S_t \right]$	$\frac{X}{(1 + \text{RFR})^t}$
American put (P)	$P \geq \text{Max} [0, X - S]$	X

Note: t = time to expiration

Put Call Parity

Put call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. The put-call parity relationship:

$$C + \frac{X}{(1 + \text{RFR})^t} = S + P$$

Each security in the put-call parity relationship can be expressed as:

$$S = C + \frac{X}{(1 + \text{RFR})^t} - P \quad C = S + P - \frac{X}{(1 + \text{RFR})^t}$$

$$P = C + \frac{X}{(1 + \text{RFR})^t} - S \quad \frac{X}{(1 + \text{RFR})^t} = S + P - C$$

Option Payoffs

- Buyer of a call option—long position.
- Writer (seller) of a call option—short position.
- Buyer of a put option—long position.
- Writer (seller) of a put option—short position.
- Intrinsic value of a call option = $\text{Max}[0, S - X]$
- Intrinsic value of a put option = $\text{Max}[0, X - S]$