Willi Semmler

# Asset Prices, Booms and Recessions

Second Edition

Springer

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## Financial Economics from a Dynamic Perspective

Second Edition

With 45 Figures
and 27 Tables

Springer

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# Preface to the Second Edition

Since the publication of the first edition of this book, the links between economic activity and global financial markets have grown only stronger and more important. Thus, in this new edition, we continue and expand our exploration of a dynamic framework in which to study Financial Economics. By financial markets, we mean those activities, institutions, agents and strategies that typically play significant roles in the markets for bonds, equity, credit, and currencies. Economic activity encompasses those actions of firms, banks, households, and governments insofar as they are concerned with the production of goods and services, savings, investment, consumption, etc. Of course, the financial marketplace is but a subset of the larger economy. However, it is an increasingly important subset and its boundary with the rest of the economy has become progressively more blurry over time. In this new edition, we will more extensively study those mechanisms by which the performance, volatility, and instability of financial markets influence, reinforce, and counteract economic activity. Additionally, we examine the reverse processes wherein actual or expected economic activity acts to sway asset prices, foreign exchange rates, and financial markets in general.

The focus of the book is on theories, dynamic models and empirical evidence as they serve to enhance our understanding of the interrelationships between financial markets and economic activity. We illustrate certain real-world situations wherein the interactions of financial markets and economic activity have shown themselves in the United States, Latin America, Asia, and Europe. Additionally, we consider various episodes of instability and crisis and how economic theory can be of explanatory value.

In this edition, we have substantially revised several chapters and updated the literature references. Chapter 13 is completely new and deals with issues of choice in the management of international portfolios. In a new section, Part VI, we present three new chapters, 14–16, concerning recent advances in asset pricing and dynamic portfolio decision-making. As a pedagogical aid, we have added an extensive collection of exercises collected at the end of the book.

Originally, the book was based on lectures delivered at the University of Bielefeld in Germany and at The New School for Social Research in New York City. I am very grateful to my colleagues at those institutions as well as the several generations of students who took my classes in Financial Economics and listened to these lectures in their formative stages. Individually, many of the chapters of the book have been presented at conferences, workshops, and seminars throughout the United States, Europe, and Japan. Specifically, in Italy, Spain, and Portugal, several of the chapters have been presented in the context of the Euro-wide Quantitative Doctoral Program in Economics.

I want to thank Gaby Windhorst for typing many versions of the manuscript. Lucas Bernard, Jens Rubart, and Leanne Ussher provided valuable editorial assistance and Uwe Köller and Mark Meyer prepared the figures. I also want to thank Sabine Guschwa for providing the data set used for the estimation presented in section 4.4 and my various co-authors who have allowed me to draw on our joint work. In particular, I want to thank Toichiro Asada, Carl Chiarella, Peter Flaschel, Reiner Franke, Gang Gong, Lars Grüne, Chih-ying Hsiao, Levent Kockesen, Martin Lettau, Christian Proano, Malte Sieveking, and Peter Wöhrmann.

Finally, I want to note that although linear and nonlinear econometric methods are used throughout the book, a more extensive treatment of those methods for the estimation of dynamic relationships is given in Semmler and Wöhrmann (2002). The text of this can be found at http://www.wiwi.uni-bielefeld.de/∼cem.

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# Introduction

*"Those who want to be rich in a day,*
*will be hanged in a year".*

(Leonardo da Vinci, 1452-1519)

Financial markets perform the essential role of channeling funds to firms that have potentially productive investment opportunities. They also permit households to borrow against future income and allow countries to access foreign funds and, thus, accelerate growth. As financial markets have expanded, they have significantly impacted not only on economic growth, but employment and policy as well. Financial liberalization has actively been advocated by such organizations as the International Monetary Fund (IMF) and the World Bank (WB) and has been pursued by many governments since 1980s. Financial deepening is also the result of financial innovations and recently developed financial instruments such as financial derivatives. Since the number of innovative financial products, e.g., credit derivatives and mortgage-backed securities, has expanded exponentially, so too the markets for them have correspondingly greatly enlarged.

It is not surprising, therefore, that the rapid enlargement of the financial market has led to more financial instability which, in turn, can be devastating. For example, the Mexican (1994), Asian (1997/8) and Russian (1998) financial crises demonstrated the degree to which a too-rapid market liberalization could lead to a currency crisis wherein a sudden reversal of capital flows was followed by financial instability and a consequent decline in economic activity. Again, during the period from 2001 through 2002, the United States and Europe experienced a significant decline in asset prices, commonly referred to as the bursting of the Information Technology (IT) Stock Market Bubble. Here, the combination of a decade of dubious accounting practices, shortsighted investment, and outright fraud led to a situation in which the public-at-large became suspicious of equity markets with consequent high volatility and negative pressure on asset prices became the not so surprising result. It is interesting to note that this very volatility and lack of trust, especially when combined with the increasing globalization of the markets, have led to new products and new excitement in these same markets. The post-crash phenomena were seen as opportunities by clever traders and globally operated investment firms.

Our book deals with financial markets and their relationship to economic activity. At the outset, let us first enlarge upon what we mean by financial markets and by economic activity. An important part of the financial market is represented by the money and bond markets. This is where, to a great extent, short and long-term interest rates are determined. An important component is the credit market, where commercial paper is traded and where households and firms obtain bank loans. In

fact, as we will see, bank credit is still the dominant source of financing for real activity (firms and households), yet credit may also depend on the equity market and asset prices. As the two are frequently observed to move together, they will both be important objects of our study. Also important is the international capital market where borrowing and lending across boarders and foreign exchange all affect each other. By economic activity, we mean the actions of households, firms, banks, governments and countries. Thus, as this is a book on Financial Economics, we will pursue a broad set of questions such as:

- What are the specifics of the major financial markets and do they differ in importance as to how they impact economic activity? Does the deepening and liberalization of the financial marketplace stimulate or retard economic growth? Will developed financial markets lead to a more efficient use of resources?
- Has the deepening and liberalization of the financial marketplace decreased or increased the volatility of macroeconomic variables, e.g., output, employment, balance of trade, long-term interest rates, exchange rates, money wages, the price level, and stock prices? Has financial risk increased and will financial liberalization lead to booms and crashes?
- What theories explain the relationship between economic activity, asset prices, and returns? What economic factors, macroeconomic factors in particular, are important for asset prices and returns? How do asset prices and returns behave over business cycles? Do the equity premium and Sharpe-ratio, a measure of the risk-return trade-off, move with the business cycle and are they driven by the varying risk-aversion of the economic agents?
- Are asset price inflation, deflation, and volatility harmful to economic activity? How do asset prices, alone or through credit channels, affect business cycles? Can an asset price boom also lead to an economic boom? Do asset price booms have a persistent effect on economic growth?
- Do monetary and fiscal policies influence the financial market and how do financial markets influence government policies? How effective are these policies in open economies with free capital flows and volatile exchange rate? Can and how should financial markets be regulated? Should governments or monetary authorities intervene to stabilize asset prices?

Both theoretical and empirical work on the relationship of financial and real activities has been undertaken by different schools of economic thought. One currently prominent school refers to the theory of perfect capital markets. Perfect capital markets are mostly assumed in intertemporal general equilibrium theory (stochastic growth and Real Business Cycle (RBC) theory). Yet they include no explicit modeling for the interaction of credit, asset prices, and real activity. In contrast to this, many theoretical and empirical studies have applied the theory of imperfect capital markets. Moreover, there are other traditions, e.g., the Keynesian tradition as revived by Minsky (1975) and Tobin (1980) that have been very influential in studying the interaction between financial markets and economic activity. There is, currently, also another important view on this interaction and this is represented by Shiller's (1991, 2001) overreaction

hypothesis. The research that will be presented in this volume is heavily influenced by Keynesian tradition, yet we also draw upon recent developments in information economics, as developed by Stiglitz and others wherein systematic attempts have been made to describe how actual financial markets operate.

Many studies of financial markets claim that a crucial impediment to the functioning of the financial system is asymmetric information. In this situation, one party to a financial contract has much less information than the other. Borrowers, for example, usually have much better information about the potential returns of their investment projects and the associated risks than do the potential lenders. Asymmetric information leads to two other basic problems: adverse selection and moral hazard.

Adverse selection occurs when those borrowers with the greatest potential for default actively seek out loans. Since they are not likely to repay the loan anyway, they may offer a high interest rate. Thus, those borrowers who lenders should most avoid are most likely to obtain loans. If the percentage of potentially "bad" borrowers is perceived as too high by the lender, he/she may simply decide to ration loans or to make no loans at all.

Moral hazard takes place after a transaction has taken place. Here, lenders are subject to hazards since the borrower has incentives to engage in activities that are undesirable from the lenders point of view. Moral hazard occurs if the borrower does well when the project succeeds, but the lender bears most of the cost when the project fails. Borrowers may also use loans inefficiently, e.g., personal expenses. Lenders may impose restrictions, face screening and enforcement costs, and this may lead, in turn, to credit rationing for the entire population of borrowers.

The existence of asymmetric information, adverse selection, and moral hazard also explains why there is an important role for the government to play in the regulation and supervision of the financial marketplace. To be useful, regulation and supervision mechanisms must be aim towards the maximization of access to information, while minimizing adverse selection and moral hazard. This requires the production of information through screening and monitoring. Firms and banks need to be required to adhere to standards of accounting and to publicly state information about their sales, assets, and earnings. Additionally, safety nets for institutions as well as for individuals are necessary to avoid the risks of a rapid liberalization of financial markets.

Mishkin (1998), for example, has posited an explanation of the Asian financial crises of 1997/8 using the above information-theoretic ideas. A similar theory by Krugman (1999a, b) laid the blame on banks' and firms' deteriorating balance sheets. Miller and Stiglitz (1999) employ a multiple-equilibria model to explain financial crises in general. Now, whereas these theories point to the perils of too fast a liberalization of financial markets and to the role of government bank supervision and guarantees, Burnside, Eichenbaum, and Rebelo (2001) view government guarantees as actual causes of financial crises. These authors argue that the lack of private hedging of exchange rate risk by firms and banks led to financial crises in Asia. Other authors, following the bank run model of Diamond and Dybvig (1983) argue that

financial crises occur if there is a lack of short-term liquidity. Further modeling of financial crises triggered by exchange rate shocks can be found in Edwards (1999) and Rogoff (1999) who discuss the role of the IMF as the lender of last resort. Recent work on the roles of currency in financial crises can be found in Aghion, et. al. (2000), Corsetti, et. al. (1998), Proano, et. al., (2005), Kato and Semmler (2005) and Roethig, et. al. (2005). The latter authors pursue a macroeconomic approach to model currency and financial crises and consider also the role of currency hedging in mitigating financial crises.

As shown above, many observers of the financial crises in emerging markets during the period 1997 - 1999 were very quick to blame loose standards of accounting, the lack of safety nets, etc. as being root causes. Yet, the years 2001 - 2002 have shown that even advanced countries e.g. the United States, Europe, and Japan cannot escape excessive asset price volatility and financial instability. As things have turned out, however, the same loose accounting practices, the lack of supervision by executive boards and regulatory institutions, and the role of big banks in helping to disguise huge corporate debt has led to a general distrust by shareholders and the general public with respect to the “fair" asset pricing of markets.

The content of this book is as follows: Part I deals with money, bonds, and economic activity. In Chapter 1, we consider the basics of the money and bond markets and the role of monetary policy in determining interest rates. Chapter 2 focuses on interest rates, which play an important role in economic activity as well as in asset and derivative pricing. We will study the determination of short and long-term interest rates and the term structure of interest rates both from theoretical and empirical points of view.

Part II treats the credit market and economic activity. In Chapters 3 and 4, we will present theories and empirical evidence relating to credit markets, i.e., borrowing, lending, and the causes and consequences of credit risk. We focus on the theory of perfect and imperfect capital markets and the role of the banking system for the relationship of credit and economic activity by positing that firms and households finance their activity largely through credit market instruments, e.g., bank loans or commercial paper. We also show that asset prices play an important role in credit markets.

Part III takes up the topic of the stock market and its relationship to economic activity. Chapters 5 - 7 examine the equity market as a significant part of the securities market and explore approaches that focus on the interaction between asset pricing and economic activity. Here we also show exactly how asset-price booms may go hand-in-hand with a rapid implementation of new technology.

Part IV, Chapter 8 - 10, elaborate on asset pricing theories such as the Capital Asset Pricing Model (CAPM), the Present Value (PV) approach, and the consumption and production-based intertemporal asset pricing theory. An important issue from Part III, one that we take up again, is the relationship between stock market volatility, excess asset returns, credit booms, and economic activity. Further, we show to what extent stylized facts can be explained by macroeconomic models, intertemporal asset pricing models, stochastic growth models, and some non-conventional approaches,

e.g., Shiller's overreaction theory and evolutionary as well as heterogeneous agent-based models.

Part V focuses on the foreign exchange market, financial instability, and economic activity. In Chapter 11, by using a macroeconomic portfolio approach, we first present an integrated view of the money, credit, bond, and equity markets in a unified framework. We will here refer to the portfolio approach developed by Tobin and study the relationship of the financial sector, as it appears in portfolio theory, to economic activity. The main tool in this section will be the balance sheets of economic agents. This will help us explain financial instabilities, financial crises, and declining economic activity that occasionally occurs in certain countries and regions. While in Chapter 11 the role of balance sheets is explored in the analysis of financial instability, in Chapter 12 we attempt to include foreign exchange, international borrowing, and international lending. Here, we will focus of the volatility of exchange rates, credit market asset prices, and the domestic spillover effects into real activity. Lastly, Chapter 13 extends the static portfolio choice model of Chapter 8 into an international portfolio.

Part VI of the book treats some more advanced topics in financial economics. Chapter 14 surveys and discusses agent-based and evolutionary methods in the modeling of asset markets. Chapter 15 considers non-expected utility-maximizing models, namely habit formation and loss aversion models to study asset price dynamics and the equity premium. Chapter 16 treats dynamic portfolio choice models where agents can choose both a consumption path as well as an asset allocation in the context of an intertemporal decision model.

Finally, Chapter 17 draws some policy conclusions. Useful econometric toolkits for studying linear and nonlinear dynamic relationships in financial economics are summarized in Wöhrmann and Semmler (2002).

# Part I

# Money, Bonds and Economic Activity

CHAPTER 1

# Money, Bonds and Interest Rates

## 1.1 Introduction

We start this book on financial economics with money, bonds and interest rates. Interest rates are major determining factors for asset markets. Interest rate processes are important for credit markets, equity markets, commercial paper markets, foreign exchange markets and security pricing such as stocks, bonds and options. Interest rates are important for real activity, consumption and investment spending. Interest rate spreads and the term structure of interest rates affect asset markets as well as real activity. In this chapter we study some major issues in the theory and empirics of interest rates. We will give here only some elementary expositions.[1]

We will first define what money is and how monetary theories help us to determine the interest rate. We will refer to the loanable fund theory and the Keynesian liquidity preference theory. If there are only two assets, money and bonds, either of them can be used to explain interest rates. We will define the different types of bonds and different types of monetary policy aimed at stabilizing inflation and output. In the next chapter we discuss short- and long-term interest rates and the term structure of interest rates.

## 1.2 Some Basics

In modern monetary economies money serves as the medium of exchange, unit of account and store of value. On the international level it also can serve as the medium of international reserve. In the latter case usually only a few currencies have been selected, for example the U.S. Dollar, the Euro or the Yen. Historically, money has developed from metallic money (gold or silver) to fiat money (paper currency) backed by the monetary authority of the country. Monetary aggregates are usually referred to as $M_1$, $M_2$ and $M_3$ money. The subsequent scheme defines those aggregates:

**Monetary aggregates:**

$$M_1 = \text{currency} \quad + \quad \begin{cases} \text{traveller checks} \\ \text{demand deposit} \\ \text{other checkable deposits} \end{cases}$$

[1] A more detailed treatment of bonds and interest rates can be found in Mishkin, 1995 (Chaps. 1-7).

$$
\begin{aligned}
M_2 &= M_1 \quad + \quad \left\{ \begin{array}{l} \text{time deposits} \\ \text{saving deposits} \end{array} \right. \\
M_3 &= M_2 \quad + \quad \left\{ \begin{array}{l} \text{large time deposits} \\ \text{money market mutual funds} \end{array} \right. \\
L &= M_3 \quad + \quad \left\{ \begin{array}{l} \text{short-term Treasury securities} \\ \text{commercial papers} \end{array} \right.
\end{aligned}
$$

Hereby $L$ represents liquidity.Monetary policy when aiming at controlling monetary aggregates usually selects one of these aggregates to stabilize inflation or output.

## 1.3 Macroeconomic Theories of the Interest Rate

Traditionally, in monetary economics, there have been two basic theories of interest rate determination. These are the loanable fund theory and the liquidity preference theory. The first theory originates in classical monetary theory of David Hume and David Ricardo. The second is based on Keynes' work. Both give us a theory of interest rate determination. We give a brief introduction to both theories.[2]

### 1. Loanable Funds Theory

Before we define the theory of loanable fund we want to define some simple principles of bond pricing. Bonds are simple loans that are traded on the bond market. They comprise principle and interest payments. A one period coupon bond is a bond with a face value $F$, of say 1000 that pays a fixed amount of income, say $100$, so the interest rate is $i = \frac{100}{1000}$. A one period discount bond (zero coupon bond) can be obtained at a price below the face value so that the interest rate is $i = \frac{1000-900}{900}$. The value of a console (permanent coupon payment) is given by the present value of multi period income stream from a bond, which is given for $t \Rightarrow \infty$ as a $\frac{100}{0.1} = 1000$. The present value of a bond is thus the solution to the following discounting problem:

$$P_b = \frac{C_1}{1+i} + \frac{C_2}{(1+i)^2} \cdots \frac{C_n}{(1+i)^n}$$

where $C_t$ is an income stream of the payments, some of which can be zero. A yield of a bond, $y$ for example for a one period bond relates the income stream to the (present) value of the bond,

$$P_b = \frac{C}{1+y}$$

with $C$ the payment and $P_b$ the price of the bond. A return on a bond is defined as

$$R_{t+1} = \frac{(C_t + P_{t+1} - P_t)}{P_t}$$

[2] For more details of the subsequent basic description of the money and bond markets, see Mishkin 1995, Chaps. 2-7).

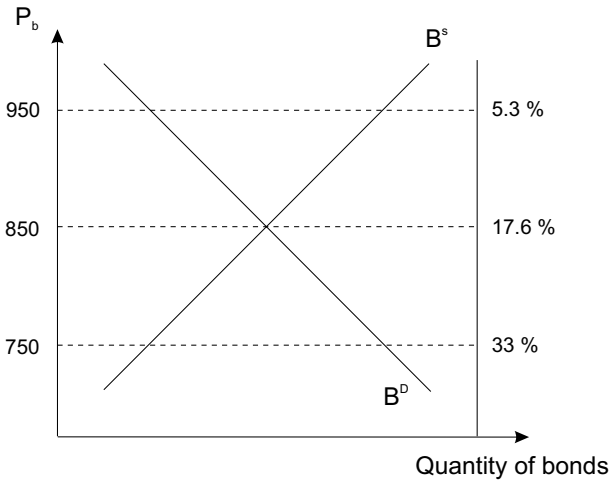



**Fig. 1.1.** Demand and Supply of Bonds

whereby $P_t$ is the price of the bond at period t. For our figure 1.1 assume

$$i = \frac{F - P_d}{P_d}$$

whereby $P_d$ is the purchase price of the discount bond and $F$ the face value of the bond.

The above figure shows the demand and supply of bonds. The purchase price of the bond is on the left axis and the corresponding interest rate on the right axis. So the purchase price of the bond is inversely related to the interest rate. The interest rate — or the price of the bond— at which demand and supply of bonds are equal define equilibrium interest rates or bond prices. On the other hand, there are long-run influences on the demand and supply of bonds which are not shown in the figure. Forces that shift the demand for bonds are defined next (whereby the arrow indicates in what direction the demand is shifting)

$$\text{Shift in the demand for bonds:} \begin{cases} \text{1. wealth } (\text{B}^{\text{d}} \rightarrow) \\ \text{2. expected interest rate rise } (\text{B}^{\text{d}} \leftarrow) \\ \text{3. inflation rate } (\text{B}^{\text{d}} \leftarrow) \\ \text{4. risk } (\text{B}^{\text{d}} \leftarrow) \\ \text{5. liquidity } (\text{B}^{\text{d}} \rightarrow) \end{cases}$$

The main force to affect the shift of the supply of bonds are government deficits. These can be written as $\dot{B} = iB + G - T$ whereby G is government expenditure, T government taxes (revenues) and $\dot{B}$ the change of government bonds. Assuming that government expenditures are not financed by money creation the deficit is then solely increasing the supply of government bonds.

## 2. Liquidity Preference Theory

The liquidity preference theory originates in Keynes (1936) and can, in a simplified version, be considered the logical counterpart of the loanable fund theory if we assume an asset market with two assets only. So we might suppose that there is supply and demand of money and bonds

$$B^s + M^s = B^d + M^d$$

So we get

$$B^s - B^d = M^d - M^s$$

Whenever the bond market is in equilibrium the money market will be in equilibrium, too. The liquidity theory can be shown to determine the interest rate as follows.

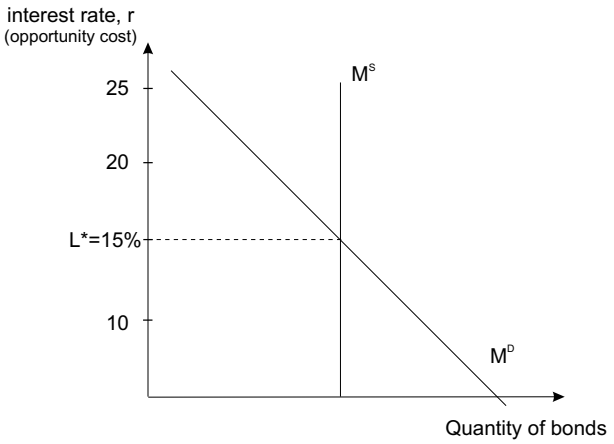



**Fig. 1.2.** Liquidity Preference Theory

Here again we might think about the forces that shift the demand for money. These are:

$$\text{Shift in the demand for money: } \begin{cases} \text{1. income } (M^d \rightarrow) \\ \text{2. price level } (M^d \rightarrow) \end{cases}$$

Shift in supply of money is solely at the discretion of the monetary authority. Making use of the standard LM-equation of the macroeconomic textbook we can write:

$$M^s = PYe^{-\alpha_1 i}$$

where $Y$ is income and $P$ the price level.

Taking logs with $m = \log M$, $p = \log P$, we get

$$m - p = y - \alpha_1 i$$

It follows that

$$i = \frac{y - (m - p)}{\alpha_1}$$

or

$$i = \delta y - \delta(m - p); \delta = 1/\alpha_1$$

Thus, any change in the money supply will shift the money supply curve to the right in the LM schedule and decrease the interest rate. Details are discussed in Chap. 6. More specifically we want to discuss two important policies that affect the interest rate.

## 1.4 Monetary Policy and Interest Rates

In fact there are two monetary policy rules that have recently been discussed. The first policy rule, originating in the monetarist view of the working of a monetary economy, can be formulated as follows.

(1) Control of the monetary aggregates:
This view prevailed during a short period in the 1980s in the US and, until recently at the German Bundesbank. It can formally be written by using the following equations.

$$MV = PY$$

Then, with V a constant, and taking logs we can write for growth rates,

$$\hat{m} = \hat{p} + \hat{y}$$

From this we get the p*-concept:

$$\hat{m} = \hat{p}^* + \hat{y}^*$$

with $\hat{m}$ = constant.

Hereby, the growth rate of money supply, $\hat{m}$, has to be set such that it equals $\hat{p}^*$, the target rate of inflation, plus $\hat{y}^*$, the potential output growth. As can be noted, the above inflation rate, although there is a target for it, is only indirectly targeted through the growth rate of money supply. A further disadvantage is that given an unstable money demand function — which is usually found in the data — this concept is not a very robust one, i.e., shifts in the money demand will create problems for the monetary authority in stabilizing the inflation rate.

(2) Control of the short-term interest rates:
To describe this type of monetary policy the following equation can be used.

$$r_{t+1} = r_0 + \underset{\text{(interest gap)}}{\beta_r(r_t - r_0)} + \underset{\text{(inflation gap)}}{\beta_p(\hat{p_t} - \pi_t)} + \underset{\text{(output gap)}}{\beta_u(y - y^*)}$$

Here, $\pi_t$ is the inflation rate targeted by the central bank, $r_t$ the short-term interest rate, $y$ actual and $y^*$ the potential output. The $\beta_i$ are reaction coefficients that determine how strongly the monetary authority stresses interest rate smoothing, inflation stabilization and output stabilization.

This concept originates in Taylor (1999). Svensson (1997) has demonstrated its application to OECD countries and it has become the dominant paradigm in central banks' monetary policy. It has the advantage that the inflation rate is directly targeted and is, therefore, called inflation targeting by the central bank. The central bank is made accountable for its targets and efforts and the decision making process is rendered more transparent. The European Central Bank (ECB) originally followed the first concept stabilizing inflation through controlling monetary aggregates. It had been argued that the German Bundesbank had achieved a solid reputation in keeping the inflation rate down with monetary targeting. However, since the second concept, of direct inflation targeting is more realistic by not relying on the (unstable) money demand function, it has been more emphasized by the ECB. The stabilizing properties of these two monetary policy rules are studied in a macroeconometric framework in Flaschel, Semmler and Gong (2001). There it is found that, by and large the second rule, since it is a direct feedback rule, has better stabilizing properties. Usually, the above interest rate reaction function, the Taylor rule, is studied for closed economies. A notable exception is the work by Ball (1999) who studies monetary policy rules for an open economy.

Note, however, that in either of the above cases the monetary authority can only directly affect the short-term interest rate. The long-term interest rates and the term structure of interest rates is affected by the financial market. In Chap. 2 we deal with the term structure of interest rates.

## 1.5 Monetary Policy and Asset Prices

It is the task of central banks not only to care about inflation rates and unemployment but also about the stability of the financial sector and possibly about asset prices. In most countries the central bank is also the lender of last resort.

An interesting feature of the monetary and financial environment in industrial countries over the past decade has been that inflation rates remained relatively stable and low, while the prices of equities, bonds, and foreign exchanges experienced a strong volatility with the liberalization of the financial markets. Central banks, therefore have become concerned with such volatility. The question has been raised whether such volatility is justifiable on the basis of economic fundamentals. A question that has become important is whether a monetary policy should be pursued that

takes financial markets and asset price stabilization into account. In order to answer this question, it is necessary to model the relationship between asset prices and the real economy. Extended models, going beyond the one underlying the above interest rate reaction of the central bank, are needed to take into account the central bank's task to stabilize asset prices. An early study of such type can be found in Blanchard (1981) who has analyzed the relationship between stock value, output and the interest rate under different scenarios. Recent examples of models incorporating the central bank's task of stabilizing asset prices include Bernanke and Gertler (2000), Smets (1997), Kent and Lowe (1997), Dupor (2001), Cecchetti et al. (2002), Semmler and Zhang (2002), and Kato and Semmler (2005). The latter consider also an open economy.

The difference of the approach by Semmler and Zhang (2002) from others lies in the fact that they employ a different framework. Bernanke and Gertler (2000), for example, by using a representative agent model, analyze how output and inflation will be affected by different monetary policy rules, which may or may not take into account asset price bubbles. The work by Semmler and Zhang (2002) aims at deriving optimal policy rules under the assumption that asset price bubbles do affect output and even inflation (asset prices may also affect the real economy through other channels e.g., credit channel, see Ch. 12 for example). Semmler et al. (2005, Ch. 8) analyze the effects of policy rules on output and inflation both with and without asset prices considered and show that welfare improving results are obtained if the central bank directly targets asset prices.

We note that there are, of course, other means of decreasing asset price volatility and preventing its adverse impact on the macroeconomy. As remarked above, the improvement of the stability of financial institutions and financial market supervision and regulation undertaken by the central bank appear to be the most important means toward this end. Yet, given financial institutions and financial market regulations an important contribution of the central bank might be in stabilizing output, inflation and asset prices when asset prices are volatile.

## 1.6 Conclusions

In this Chap. we have summarized some basic theories on money, bonds and interest rates. The reader might want to also look at the actual empirical trends in monetary variables for some economies. For the U.S., for example, those trends can be found in Mishkin (1995, Chaps. 1-7). There, one can find trends in money supply and the price level, the correlation of the different monetary aggregates, trends in real interest rates, the business cycle and money growth rates, trends in bond rates (public and private bonds) and an example of the term structure of interest rates. Those empirical trends and stylized facts are important for a study of the financial market and the macroeconomy, since theoretical models should be able to explain such empirical trends and facts.

CHAPTER 2

# Term Structure of Interest Rates

## 2.1 Introduction

We will introduce some definitions of the various terms used in the study of the term structure of interest rates and provide some economic theories that attempt to explain the term structure. After that we will summarize some empirical work on the term structure of the interest rates and show how one can model the interest rate process as a stochastic process. As we will show stochastic processes are very useful tools for interest rate and, more generally, financial market analysis. Basic stochastic processes are summarized in appendix 1.

## 2.2 Definitions and Theories

We will first give some formal definitions of the terms used in the theory of the term structure of interest rates, also called yield curve[3].

For a zero coupon bond and a full spectrum of maturities u$\in [t, T]$ and a price of the bond B(u, t) the spectrum of yields $\{R_t^u,\ \text{u} \in [t, T]\}$ is called term structure of interest rates, where

$$B(u,t) = 100\, e^{-R_t^u (u-t)},\ t < u \tag{2.1}$$

For example, take $R_t^u = r$ then one can compute the present value of an income stream with r the discount rate. If the income occurs at period $u > t$ and is 100 then we can write the present value of the income

$$B(u,t) = \underbrace{100}_{\text{paid at period u}} \times \underbrace{e^{-r(u-t)}}_{\text{discount factor}} \tag{2.2}$$

For example, for a given information set $I_t$ the price of a bond that pays 100 after three periods gives us the discrete time formula

$$B(3,1) = E\left[\underbrace{\frac{100}{(1+r_1)(1+r_2)(1+r_3)}}_{\text{discounting by the expected short term interest rates}} \mid I_t\right] \tag{2.3}$$

[3] For a more detailed technical description of the following, see Neftci (1996, Chap. 16).

If we have a time varying discount factor $r_s$ we get the following modification

$$B(u,t) = 100\ E\ \left[e^{-\int_t^u r_s\ ds}\mid I_t\right] \tag{2.4}$$

If we have the price of a bond, determined by (2.4), we can calculate the yield (and the spectrum of yields)

$$R_t^u = \frac{\log B(u,t) - \log(100)}{t-u} \tag{2.5}$$

The above relates the bond prices (the spectrum of bond prices) to the yield (spectrum of yields). One can obtain the yield curve from the future short rates. Equating (2.1) and (2.4) and applying logs on both sides gives

$$R_t^u = log\ E\ \frac{\left[e^{-\int_t^u r_s\ ds}\mid I_t\right]}{u-t} \tag{2.6}$$

Note that in continuous time the slope of the yield curve is $dR_t^u/du$.

Moreover, we can define forward rates (on a loan that begins at time u and matures at T) as:

$$F(t,u,T) = \frac{\log\ B(u,t) - \log\ B(T,t)}{T-u};\ t<u<T$$

The instantaneous forward rate is:

$$f(t,u) = \lim_{T\to u}\ F(t,u,T);\ f(t,t) = r_t$$

The spot rate can be defined as the interest rate paid on a dollar borrowed at time s, where $t<s<T$ and held an infinitesimal period of time.

Empirically, first the short- and long-term interest rates usually move together. Second, the yield curve is mostly upward sloping, but sometimes it is flat or downward sloping. Third, there is some mean reverting process: if the short term interest rate is low one expects some high interest rates in the future and the reverse holds, if the current interest rate is high. There is some economic theory that gives us some guidance in the study of the empirical behavior of the term structure of nominal the interest rates.[4] In economic theory the yield curve is seen to be determined by

1. *expectations about the future path of* $r_t$ :
   In the standard approach, bonds with different maturities are perfect substitutes and, given rational expectations, the expected interest rate on long term bonds is given by the expected future short term interest rates. If one thinks, for example, about the short-term interest rate following some mean reverting process and the current $r_t$ is low the expected $r_t$ would be high. Thus, expected future interest rates would tend to rise. This theory cannot sufficiently explain why the yield curve is mostly upward sloping.

[4] For details, see Mishkin (1995, Chap. 7).

2. *segmented markets:*
   Here it is assumed that bonds with different maturities are determined in different markets. Interest rates of bonds with different maturities are determined by supply and demand of bonds with those maturities. This theory can explain why the yield curve has an upward slope, but it cannot explain why interest rates of bonds with different maturities usually move together.
3. *liquidity premium:*
   This theory posits that a positive term (liquidity) premium must be offered to buyers of long term bonds to compensate them for the higher risk. If one thinks of a liquidity premium as a compensation for risk then the future interest rate should include a risk premium and the term structure should always be upward sloping. Although it can explain the upward slope it needs to assume substantial fluctuations in the term premiums for long term bonds.

On the other hand, as aforementioned, it is useful for financial analysis to model the expected interest rate process— the expected short term interest rates —as a stochastic process. Take $r$ as the short term interest rate. Then a stochastic process might be defined such as

$$dr = a(r_t, t)dt + \sigma(r_t, t)dW_t \tag{2.7}$$

where the first term on the right hand side is the drift term and the second the diffusion term with $dW_t$ the increment of a Brownian motion. Then (2.7) can be used for (2.6).[5] Details of such processes as (2.7) are discussed in appendix 1. Next we will employ a specific stochastic process to model the movement of the short term interest rate.

## 2.3 Empirical Tests on the Term Structure

As already mentioned[6] above, a standard view on the term structure of interest rates is that the term structure can be inferred from expected future short term interest rates. Accordingly, the term structure of interest rates is given by the expected future short rates. As aforementioned, modeling and estimating expected short rates is essential for credit markets, equity and derivative markets and foreign exchange markets as well as real activity such as consumption and investment spending.

One usually attempts to capture the process of the short-term interest rate in a stochastic equation which describes the future path of the short term interest rate. The process, describing the interest rate path, is particularly useful for derivative contracts for example on stocks, bonds or foreign exchange. Often the value of the underlying asset is formulated in reference to a stochastic process of the short term interest rate. The appendix 1 describes several of such stochastic processes which might be employed to model and estimate interest rate processes and the movements

[5] For details using a stochastic process such as (2.7) to solve for bond prices and yields, see Cochrane (2001, Chap. 9) and Chap. 19 of this book.

[6] Details of this section can be found in Hsiao and Semmler (1999).

of other asset prices. Recently the mean reverting process has been used by a number of researchers for formulating and estimating the process of short term interest rates. For a detailed survey of recent empirical studies, see Chan et al. (1992). This is called a one factor approach to modelling interest rates.

On the other hand, recent models have extended this approach to a two factor model. Thus, econometric regression studies on the process of short-term interest rates have also used information on longer-term rates to forecast future short-term rates. Long rates are the second factor. Examples of this approach can be found in Fama (1984), Fama and Bliss (1987), Mankiw (1996) and Campbell and Shiller (1992). Following Balduzzi (1997) we in particular assume that longer maturity bond yields incorporate useful information about the central tendency – the mean – of the short term rates. We propose a simplified version of the more complex model by Balduzzi (1997) who allows for an additional stochastic process to determine the central tendency. In our case the mean reversion process is simply determined by the spread between two long rates. We show that the spread between two longer maturity bond rates gives, for periods of stronger changes of the central tendency, additional significant information of the mean of the short rate.

Technically, in our estimations we propose the Euler approach of turning a continuous time stochastic process into a discrete time estimable process. As our experiment with a univariate stochastic process has shown the discrete time Euler estimation appears to be a useful estimation method. The Euler procedure is then applied to a stochastic interest rate process with mean reversion. This discrete time method is employed to estimate the dynamic process of the monthly U.S.- T-bill rate with mean reversion where, however, the mean is allowed to undergo changes depending on long term interest rates. The time series data employed are from 1960.1 to 1995.1. In addition sub-periods are studied in order to find differences in the mean reverting behavior of the interest rate. As has been shown in Hsiao and Semmler (1999) although one can undertake continuous time estimations for such a process they are not always superior to the discrete time estimations using the Euler approximation. This encourages us to directly use the Euler approach in estimating the parameters of an interest rate process with mean reversion.

Recently it has become popular to define the short term interest rate process as a mean reverting process[7]. One could think that interest rates are generated from the following discrete time mean reverting process.

$$\Delta_h r_t = r_{t+h} - r_t = (\theta - \kappa r_t)h + \sigma \Delta_h B_t \tag{2.8}$$

where $B_t$ is one-dimensional Brownian motion and $\Delta_h B_t := B_{t+h} - B_t$ with $h$ the time step.

The corresponding continuous time stochastic differential equation to (2.8) is:

$$dr_t = (\theta - \kappa r_t)dt + \sigma dB_t \tag{2.9}$$

[7] Several types of processes are discussed in the appendix and empirical tests are reported in Chan et al. (1992).

Note that both (2.8) and (2.9) represent a mean reverting process. We know that the solution to (2.9) is

$$r_t = e^{-\kappa t}(r_0 - \frac{\theta}{\kappa}(1 - e^{\kappa t}) + \sigma \int_0^t e^{\kappa s} dB_s) \tag{2.10}$$

One can generate data from using (2.8) "quasi-continuously" that means with an iteration time interval that is much finer than the observation intervals and we only take the data from the observation points. For the purpose of testing the usefulness of the Euler procedure this has been undertaken in Hsiao and Semmler (1999). There the Euler procedure has been tested against other alternatives, for example continuous time estimations. In our experiments the Ito integral represents the best approximation for our continuous time integral, yet, the continuous time estimation with primitive sums turns out to be better, for Ito's Lemma, see the appendix 1. Since the discrete time Euler procedure is equivalent to taking primitive sums in the continuous time estimation we can conclude that the Euler method comes out best. Our hypothesis is therefore that the finer discreteness of the data does not add much independent information and thus does not give significantly better estimation results.[8] This seems to justify the use of the Euler procedure for discrete time estimations. Next, by employing the discrete time Euler procedure we undertake an estimation for actual data using a type of model such as represented by equ. (2.8).

In fact a model as represented by equ. (2.8) has often been employed for describing a mean reverting interest rate process, see Cox, Ingersoll and Ross (1985) and Balduzzi (1997).

A general mean reverting interest rate process with changing "central tendency" (Balduzzi 1997) can be written as follows:

$$dr = \kappa(\theta - r)dt + \sqrt{\sigma_0^2 + \sigma_1^2 r} dZ \tag{2.11}$$

$$\hat{\theta} = a_0 + a_1(B(\tau_2)\tau_1 y(\tau_1) - B(\tau_1)\tau_2 y(\tau_2)) \tag{2.12}$$

$$B(\tau) = \frac{2(e^{\delta\tau} - 1)}{(\lambda_1 + \delta + \kappa)(e^{\delta\tau} - 1) + 2\delta} \quad , \quad \delta = \sqrt{(\lambda_1 + \kappa)^2 + \sigma_1^2}$$

Assuming a stochastic process for $\theta$ such as $d\theta = m(\theta)dt + s(\theta)dW$.

Balduzzi (1997) takes $\hat{\theta}$ in (2.12) as an approximation of $\theta$ in (2.11). We use some assumptions to simplify the above model: (1) $\sigma_1^2 = 0$ and (2) $\delta$ is small.

Then

$$\delta = (\lambda_1 + \kappa)$$
$$B(\tau) = \frac{(1 - e^{-\delta\tau})}{(\lambda_1 + \kappa)}$$

[8] Balduzzi (1997) also employs the Euler discretization in his estimation strategy. He does not, however, give a justification for it.

When $\delta$ is small

$$\begin{aligned} e^{-\delta\tau} &\sim 1-\delta\tau \\ &\Rightarrow B(\tau)=\frac{(1-e^{-\delta\tau})}{(\lambda_1+\kappa)} \sim \frac{(\delta\tau)}{(\lambda_1+\kappa)} \\ &\Rightarrow B(\tau_2)\tau_1 \sim B(\tau_1)\tau_2 \end{aligned}$$

then,

$$\hat{\theta} \sim a_0 + \tilde{a_1}(y(\tau_1)-y(\tau_2))$$

and

$$\begin{aligned} dr &= (\kappa a_0 + \kappa\tilde{a_1}(y(\tau_1)-y(\tau_2)) - \kappa r)dt + \sigma_0 dZ \\ &= (b_0 + b_1(y(\tau_1)-y(\tau_2)) + b_2 r)dt + \sigma_0 dZ \end{aligned} \tag{2.13}$$

Model (2.13) is linear in the variables. We use the following data:[9]

$r_t$ : short-term interest rate, U.S. monthly T-bill rate, annualized
$y(\tau_1)$ : long-term interest rate, U.S. T-bonds, constant maturity, 1YR
$y(\tau_2)$ : long-term interest rate, U.S. T-bonds, constant maturity, 3YR

All data employed are monthly data. Estimations are undertaken with non-linear least square estimation (NLLS). We use the AIC (Akaike Information Criterion) for evaluating the estimation results without and with long-term interest rate effect on the expected short-term interest rate. The AIC is computed as:

$$\ln \hat{\sigma^2} + 2\frac{k}{n}$$

where $k =$ number of parameters, $n =$ number of observations.

The term $b_1 \neq 0$ stands for the regression with the additional variable $y(\tau_1) - y(\tau_2)$ composed of the two long term rates and $b_1 = 0$ for the regression without the long term rates. The regression with the lower AIC is always the significant result. As can be observed for the periods 1960.1-1993.6, with changes in the central

**Table 2.1.** Parameter Estimates Without and With Long-Term Rates

| | AIC | |
|---|---|---|
| Period | $\mathbf{b_1 \neq 0}$ | $\mathbf{b_1 = 0}$ |
| Jan. 1960 - Jun.1993 | -1.132 | -1.141 |
| Sep. 1971 - Sep. 1978 | -1.671 | -1.660 |
| Oct. 1978 - Sep. 1982 | 0.149 | 0.447 |
| Oct. 1982 - Jun. 1993 | -2.575 | -2.590 |

[9] The data are from Citibase (1998).

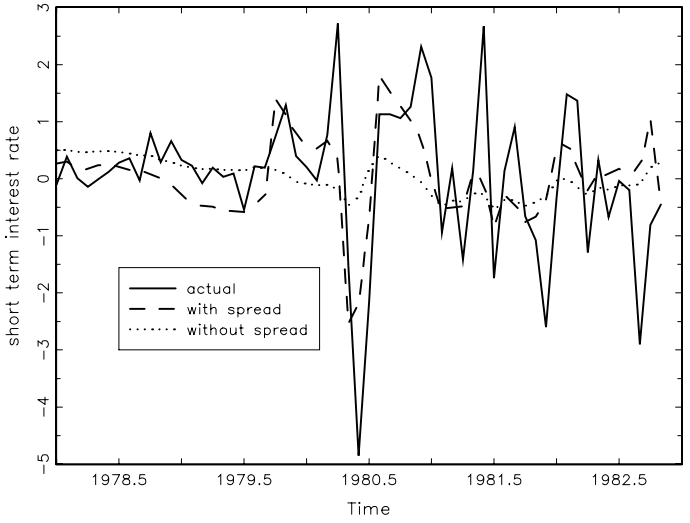



**Fig. 2.1.** Estimated Time Period 1978.1–1982.1

tendency somewhere in the entire time period, the additional variable $y(\tau_1) - y(\tau_2)$ representing the two interest rates has no additional explanatory power. In shorter periods with stronger mean change the term $y(\tau_1) - y(\tau_2)$ has explanatory power. The latter holds for the period 1971-1978 and 1978-1982.

The first period is characterized by the end of the Bretton Woods system and the first oil crisis and the second by a strong change of the interest rate due to monetary policy of the Fed. This confirms that a time varying mean seems to become a relevant explanatory factor when trends in the interest rate change. Information on the changing mean can be extracted from the spread between the two long rates. In figure 2.1 the dotted line represents the regression without the interest rate spread and the dashed line represents the fitted line using interest spread. As the figure 2.1 shows the time period 1978.1-1982.1 is better tracked when the interest rate spread has become the significant additional explanatory variable.

In figure 2.2 it is also shown the 1YR-3YR spread. As the figure 2.2 indicates there is significant information in the 1YR-3YR spread when the mean of the short rate strongly moves during the time period 1978.1-1982.1.

## 2.4 Conclusions

A standard view on the term structure of interest rates is that the term structure can be inferred from expected future short term interest rates. Our experiment has shown that the discrete time Euler estimation appears to be a useful estimation method. The Euler procedure is used for the estimation of the stochastic interest rate process with

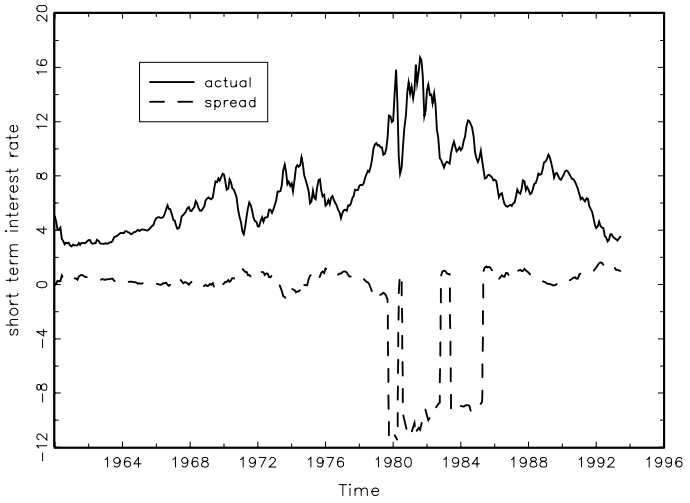



**Fig. 2.2.** Short-Term Interest Rate and 1YR-3YR Spread

mean reversion. Econometric regression studies on the term structure of interest rates have frequently used information on longer term rates to forecast future short term rates. Examples of this approach can be found in Fama (1984), Fama and Bliss (1987), Mankiw (1996) and Campbell and Shiller (1992). Following Balduzzi (1997) we in particular assume that longer maturity bond yields incorporate useful information about the central tendency of the short term rate. We propose, however, a simplified version of the more complex model by Balduzzi (1997) who allows for an additional stochastic process to determine the central tendency. In our case the mean reversion process is simply determined by the spread between two long rates. We show that the spread between two longer maturity bond rates gives, for periods of stronger changes of the central tendency, useful predictions for future short term rate movements and thus for the term structure of interest rates.

CHAPTER 3

# Theories on Credit Market, Credit Risk and Economic Activity

## 3.1 Introduction

The next part deals with the credit market, credit market risk and economic activity. Historically, borrowing and lending have been considered essential for economic activity. The major issues in borrowing and lending theory were already present in the works of the classical economists such as Adam Smith, David Ricardo and Alfred Marshall. The Ricardian equivalence theorem, a modern reformulation of a statement by David Ricardo, has, in the theory of perfect capital markets, become a major issue in modern finance. We will discuss the theory of perfect capital markets and imperfect capital markets. In the latter, asymmetric information, moral hazard and adverse selection as well as asset prices become relevant issues for studying borrowing and lending. Subsequently, this will be applied to the finance of firms, households, governments and countries.

## 3.2 Perfect Capital Markets: Infinite Horizon and Two Period Models

With the extension of perfect competition and general equilibrium theory to the intertemporal decisions of economic agents, studies of borrowing and lending have, thus, often been based on the theory of perfect capital markets (see Modigliani and Miller 1958, Blanchard and Fischer 1989, Chap. 2.).[10] In those multi period models the intertemporal budget constraint of economic agents (households, firms, government and countries) and, often, the so called transversality conditions are employed to make a statement on the solvency of the agents. These mean that the spending of agents can temporarily be greater than their income, and the agents can temporarily borrow against future income with no restriction, but an intertemporal budget constraint has to hold. This sometimes is also called the No-Ponzi condition and represents a statement on the non-explosiveness of the debt of an economic agent. Positing that the agents can borrow against future income, the non-explosiveness

[10] The Modigliani-Miller Theorem means first that corporate leverage, the debt to equity ratio, does not matter for the value of the firm and second that it is irrelevant whether the firm or the share holders do the savings (the firm is a "veil" which acts on behalf of the share holders).

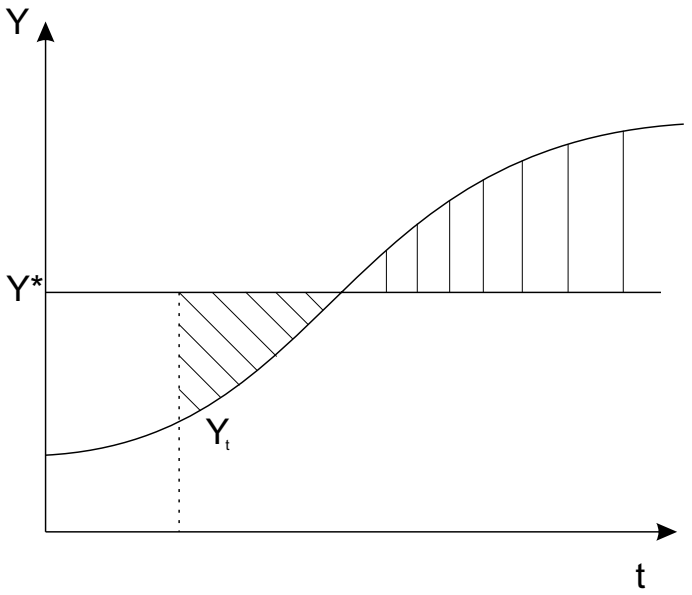



**Fig. 3.1.** Perfect Capital Market

condition is, in fact, equivalent to the requirement that the intertemporal budget constraint holds for the agents. More precisely, this means that agents can borrow against future income but the discounted future income, the wealth of the agents, should be no smaller than the debt that agents have incurred. Indeed, models of this type have been discussed in the literature of households, firms, governments and small open economies (with access to international capital markets). Here, the transversality condition is a statement on the debt capacity of the agents.[11]

Figure 3.1 illustrates the idea of the perfect capital market. The economic agent can borrow when the income, $Y_t$, falls short of the normal spending, $Y^*$. In the long run, however, the segment below the horizontal line should be cancelled out by the segment above the horizontal line. This means that the future (discounted) surplus should be able to pay back the debt incurred.

On the other hand, in practice and as mentioned in the introduction, frequently economists assume an imperfect capital market by positing that borrowing is constrained. Either borrowing ceilings are assumed, agents supposedly preventing from borrowing an unlimited amount, or it is posited that borrowers face an upward sloping supply schedule for debt arising from a risk dependent interest rate. In the first case agents' assets are posited to serve as collateral. A convenient way to define the

[11] For a brief survey of such models for households, firms and governments or countries, see Blanchard and Fischer (1989, Chap. 2) and Turnovsky (1995).

debt ceiling is to assume it is a fraction of the agents' wealth.[12] The risk dependent interest rate, it is frequently assumed is composed of a market interest rate (for example, an international interest rate) and an idiosyncratic component determined by the individual degree of risk of the borrower. Various forms of the agent specific risk premium can be assumed. Frequently, it is posited to be convex in the agents' debt[13] but it may be decreasing with the agents' own capital i.e. that capital which is serving as collateral for the loan.

We will return to borrowing and lending in imperfect capital markets, but, even in the context of the theory of perfect capital markets, one can argue that the non-explosiveness condition may pose some problems. In fact the No-Ponzi condition is state constrained and one has to show the regions where debt is feasible and the borrower remains creditworthy. In Semmler and Sieveking (1998, 1999) and Grüne, Semmler and Sieveking (2004) it is demonstrated that the debt ceiling should not be arbitrarily defined. When studying the debt capacity of the economic agent we can refer to a maximum amount that agents can borrow. Of course, in practice insolvency of the borrower can arise without the borrower moving up to his or her borrowing capacity. One should be interested in the maximum debt capacity up to which creditworthiness is preserved. Insolvency may occur when a borrower faces a loss of his or her "reputational collateral" (Bulow and Rogoff 1989) without having reached the debt capacity. In our view we should be concerned with the "ability to pay" and less with the borrower's "willingness to pay". Recent developments in the latter type of literature, in particular on the problem of incentive compatible contracts is surveyed in Eaton and Fernandez (1995). Recent studies of financial crises appear to pursue the line of ability to pay rather than the willingness to pay.

By undertaking such debt studies, we can often bypass utility theory. Economists have argued that analytical results in models with utility maximizing agents depend on the form of the utility function employed. Moreover, one can argue, economic theory should not necessarily be founded on the notion of utility since such a foundation is not well supported by empirical analysis. Many economists have recently argued that economic theory should refrain from postulating unobservables and employ observable variables as much as possible. We indeed want to argue that a theory of credit risk and creditworthiness, can be formulated without the use of utility theory.[14]

---

[12] The definition of debt ceilings have become standard in models for small open economies; see, Barro, Mankiw and Sala-i-Martin (1995). It has also been pointed out that banks (like the World Bank, see, e.g. Bhandari, Haque and Turnovsky 1990) often define debt ceilings for their borrowers.

[13] The interest rate as function of the default risk of the borrower is posited by Bhandari, Haque and Turnovsky (1990) and Turnovsky (1995).

[14] An analytical treatment why and under what conditions the creditworthiness problem can be separated from the problem of the utility of consumption is given in Semmler and Sieveking (1998).

### 3.2.1 Infinite Horizon Model

Let us make some formal statements in the context of the theory of perfect capital markets. In a contract between a creditor and debtor there are two measurement problems involved. The first pertains to the computation of debt and the second to the computation of the debt ceiling. The first problem is usually answered by employing an equation of the form

$$\dot{B}_t = rB_t - f_t, \tag{3.1}$$

where $B(t)$ is the level of debt[15] at time $t$, $r$ the interest rate and $f(t)$ the net income. The second problem can be settled by defining a debt ceiling such as

$$B_t < B^*, \quad (t > 0)$$

or less restrictively by

$$\sup_{t \geq 0} B_t < \infty$$

or even less restrictively by the aforementioned transversality condition

$$\lim_{t \to \infty} e^{-rt} B_t = 0.$$

The latter condition, which represents the often used transversality condition, means that in the limit the debt should grow no faster than the discount rate which we have taken here as equal to the interest rate, $r$.

The ability of a debtor to service the debt, i.e. the feasibility of a contract, will depend on the debtors source of income, or more simply given the interest rate, $r$, on

$$\dot{B} = r\ B_t - (y_t - y^*)$$

where the transversality condition should hold:

$$\lim_{t \Rightarrow \infty} e^{-rt} B_t = 0.$$

The latter condition means that the debt, $B_o$, incurred by the economic agent will have to be paid off by the discounted future surplus, $S_t$.

$$e^{-rt} B_t = B_o - \int_{t=0}^{\infty} e^{-rt} S_t dt = 0; \text{ where } S_t = y_t - y^*. \tag{3.2}$$

In an economic model with borrowing and lending[16] one can model this source of income as arising from production activity and thus from a stock of capital $k_t$, at time $t$, which changes with investment rate $j_t$ at time $t$ through

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[15] Note that all subsequent state variables are written in terms of efficiency labor along the line of Blanchard (1983).

[16] Prototype models used as basis for our further presentation can be found in Blanchard (1983), Blanchard and Fischer (1989) or Turnovsky (1995).

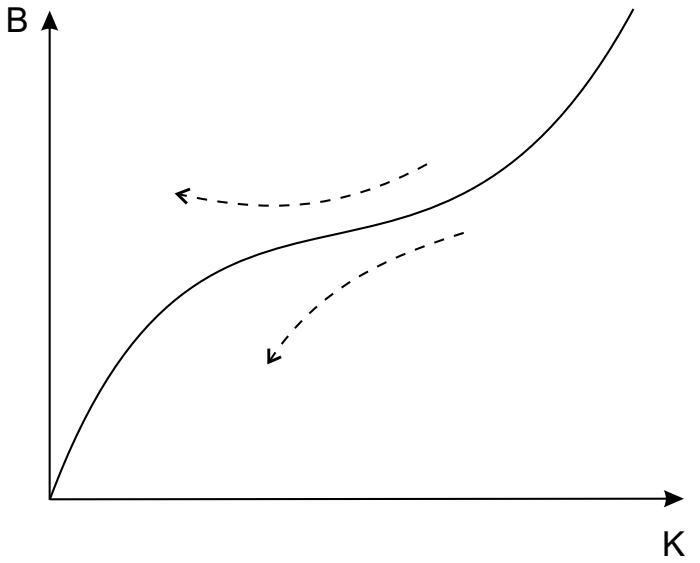



**Fig. 3.2.** Critical Debt Curve

$$\dot{k}_t = j_t - \sigma k_t. \tag{3.3}$$

where $k$, capital stock, $j_t$, investment, and $\sigma k_t$, depreciation. Our theory of credit and credit risk says that the debt capacity of a borrower is limited by a critical curve for each initial unit of capital stock, $k(0) = k_0$. Solvency of the agents and thus the case of no-bankruptcy is established for debt, $B_0$, below that critical debt curve.

This is shown in the figure below; for details see Semmler and Sieveking (1998, 1999).

This is likely to mean that the agent will be cut off from loans if he or she approaches the critical curve and, moreover, loans might be recalled. An empirical study on debt sustainability using the intertemporal budget constraint is given in Chap. 4.4.

For a country such a debt constraint means that once the critical level of debt is reached there will be a sudden reversal of capital flows, possibly triggering an exchange rate devaluation or exchange rate crisis that is possibly followed by a financial crisis and large output loss. Further details of the study of such a process triggered by credit risk and insolvency threat are postponed to Chap. 12.

### 3.2.2 A Two Period Model

A two period model for households, firms, states and countries can be found in Burda and Wyplosz (1997, Chap. 3). We see that even without an initial value of debt, the problem of sustainability of debt already arises in a two period model. This is shown below.

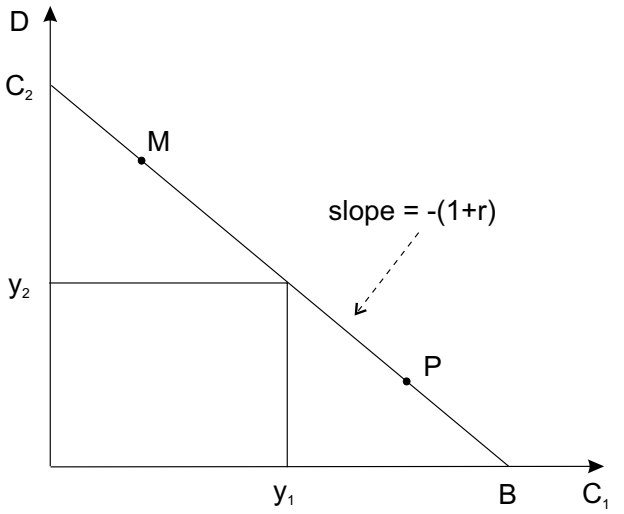



**Fig. 3.3.** Two Period Model

Borrowing and Lending in a two-period model reads as follows. In the first period there are two possibilities

$$y_1 - c_1 \begin{cases} a)\ y_1 > c_1 \Rightarrow \text{lending (see point M)} \\ b)\ y_1 < c_1 \Rightarrow \text{borrowing (see point P)} \end{cases}$$

whereby $c_1$ = first period consumption and $y_1$ = first period income. With $c_2$ = second period consumption and $y_2$ = second period income we have for the second period

$$\text{a) } c_2 = y_2 + (1+r)\underbrace{(y_1 - c_1)}_{>0} \text{ (with lending)}$$
$$\text{b) } c_2 = y_2 + (1+r)\underbrace{(y_1 - c_1)}_{<0} \text{ (with borrowing)}$$

The intertemporal budget constraint (IBC) for a two period model can be derived as follows: From $c_2 = y_2 + (y_1 - c_1)(1+r)$ we obtain in terms of the present value of next period's income and consumption: $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}$ .

The IBC with initial wealth $(V_0)$ reads:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + V_0$$

The IBC with initial debt $(B_0)$ reads:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - B_0$$

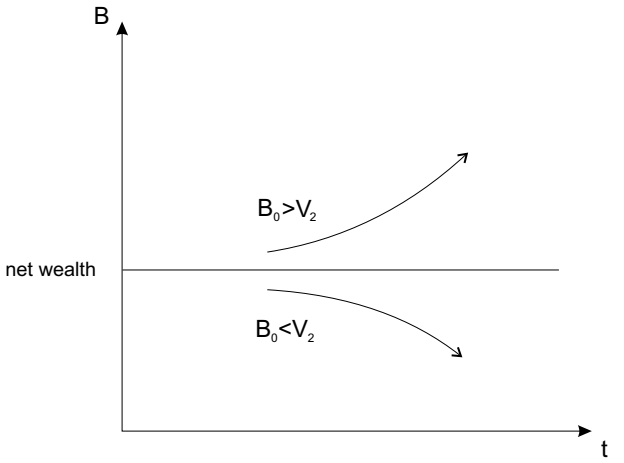



**Fig. 3.4.** Creditworthiness in a Two Period Model

and

$$B_0^* = \underbrace{\underbrace{y_1 + \frac{y_2}{1+r}}_{\text{income}} - \underbrace{(c_1 + \frac{c_2}{1+r})}_{\text{consumption}}}_{\text{net wealth}}$$

or

$$B_0^* - \text{net wealth} = 0$$

Thus, in this latter case the initial value of debt, $B_0$, is not allowed to be greater then the critical debt $B_0^*$ which is equal to the value of net wealth.

Thus in a two period model sustainable debt is

$$B_0^* = V_2 = y_1 + \frac{y_2}{1+r} - \left(c_1 + \frac{c_2}{1+r}\right). \tag{3.4}$$

If $V_2 < B_0$ then the agent has lost creditworthiness and bankruptcy occurs. This is graphically presented in the following figure.

In the infinite horizon case $(t \to \infty)$ we have as the present value:

$$V_t = \int_{t=0}^{\infty} e^{-rt} S_t dt$$

where $S_t = y_t - y^*$ (for one period).

The IBC with initial debt $(B_0)$ reads:

$$B_0^* = \int_{t=0}^{\infty} e^{-rt} S_t dt$$

$$e^{-rt} B_t = B_0 - \int_{t=0}^{\infty} e^{-rt} S_t dt.$$

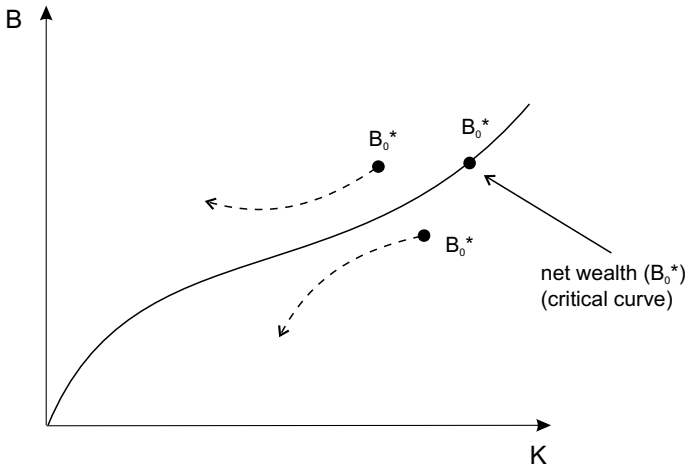



**Fig. 3.5.** Creditworthiness in an Infinite Horizon Model

The right hand side is the remaining debt. The law of motion for debt is:

$$\dot{B} = rB_t - S_t \ .$$

If it is required that the transversality condition should hold, this is equivalent to

$$\lim_{t \Rightarrow \infty} e^{-rt} B_t = 0$$

Thus, in the infinite horizon case the sustainable debt is all the initial debt below the curve in figure 3.5.

If

$$B_0 > \int_0^\infty e^{-rt} S_t dt \tag{3.5}$$

there is a loss of creditworthiness and thus bankruptcy will occur; for details see Semmler and Sieveking (1998), and Grüne et al. (2004).

In Chap. 4.4 we employ a small scale model to demonstrate how an income stream may be generated through a production activity and a process of capital accumulation. There we will also show how debt sustainability can be empirically estimated.

The theory of imperfect capital markets suggests practical rules on how to deal with credit risk and the loss of creditworthiness. Two rules are typically imposed on borrowers. First, there will be credit rationing and debt ceilings. In a two period case there might be a borrowing constraint introduced such that there is credit rationing whereby a debt ceiling, $B_2$, is given by: $B_2 \leq B_0^* = V_2$ (net wealth). In the infinite horizon case credit rationing and debt ceiling might be given by: $B_0 \leq B_0^* = \int_{t=0}^{\infty} e^{-rt} S_t dt$.

Second, there may be endogenous credit costs wherein the interest payment depends on the debts and assets (or net worth) of the economic agents. One could, for example, introduce an equation for the evolution of debt such as

$$\dot{B} = \theta(B,k)B - S_t \tag{3.6}$$

wherein $\theta(B,k)$ is the endogenous credit cost with $S_t$ the net income flow, see Chaps. 4.4 and 12.[17] In the finance literature this credit cost has been treated as default premium caused by both high leverage of the firm as well as high volatility of its asset value, see Merton (1974) and for a more recent study Grüne and Semmler (2005a).

## 3.3 Imperfect Capital Markets: Some Basics

Next we will work out details of the theory of imperfect capital markets, both on the level of agents' actions as well as on the aggregate level. An excellent presentation of the theory of imperfect capital markets is given by Jaffee and Stiglitz (1990). There, the notion of asymmetric information is essential which gives the theory of credit contract a realistic feature. Indeed, credit markets differ from standard markets (e.g. for cars, consumer goods) in some important respects. First, standard markets, which are the focus of classical competitive theory, involve a number of agents who are buying and selling an homogenous commodity. Second, in standard markets, the delivery of a commodity by a seller and payment for the commodity by a buyer occur simultaneously. This is different for credit contracts.

Credit received today by an individual or firm involves a promise of repayment sometime in the future. Yet, one person's promise is different from the promise of another and promises are frequently broken. It is difficult to determine the likelihood that a promise will be kept. Given the little information the lender has about the borrower, moral hazard and adverse selection may indeed affect the likelihood of loan repayment. For most entrepreneurial investment the project is always specific. Credit means allocating resources but those who control existing resources, or have claims on current wealth, are not necessarily those best situated to use these resources. On the other hand, the user of the resource has specific information.

The analysis of credit allocation may go wrong when we apply the standard supply and demand model which is not totally appropriate for the market for promises. If credit markets were like standard markets, then interest rates would be the "prices" that equate the demand and supply for credit. However, an excess demand for credit is common – applications for credit are frequently not granted. As a result, the demand for credit may exceed the supply at the market interest rate. Credit markets deviate from the standard model because the interest rate indicates only what the individual promises to repay, not what he or she will actually repay. This means that credit markets are not necessarily cleared since the interest rate is not the only dimension of a credit contract. Given the above informational and collateral problems

[17] For more recent treatments of this issue from the perspective of information economics, see Semmler and Sieveking (1998), and Grüne et al. (2004) see also Bernanke, Gertler and Gilchrist (1998). A stochastic version can be found in Sieveking and Semmler (1999).

in borrowing and lending in principle there should be a different cost of credit for each economic agent.

As Jaffee and Stiglitz (1990) notice, in most advanced countries complicated, decentralized, and interrelated set of financial markets, institutions, and instruments have evolved to provide credit. We here, focus on loan contracts where the promised repayments are fixed amounts. "At the other extreme, equity securities are promises to repay a given fraction of a firm's income. A spectrum of securities, including convertible bonds and preferred shares, exists between loans and equity. Each of these securities provides for the exchange of a current resource for a future promise. In our discussion we shall uncover a number of 'problems' with the loan market. While some of these problems are addressed by other instruments, these other instruments have their own problems" (Jaffee and Stiglitz 1990: 838).

The problem of the allocation of credit has important implications at both the micro and macro levels. At the micro level, in the absence of a credit market, those with resources would have to invest the resources themselves, possibly receiving a lower return than could be obtained by others. When credit is allocated poorly, inferior investment projects are undertaken, and the nation's resources are misguided. "Credit markets, of course, do exist, but they may not function well – or at least they may not function as would a standard market – in allocating credit. The special nature of credit markets is most evident in the case of credit rationing, where borrowers are denied credit even though they are willing to pay the market interest rate (or more), while apparently similar borrowers do obtain credit" (Jaffee and Stiglitz 1990: 839).

At the macroeconomic level, changes in credit allocations are strongly connected with economic fluctuations and often also with rapid decline in productive activities. For example, the disruption of bank lending during the early 1930s may have created, or at least greatly extended, the Great Depression of the 1930s. Moreover, financial and credit crises have contributed to the Mexican (1994), Asian (1997-1998) and Russian (1998) economic crises. The availability of credit may also strongly be affected by monetary policy. Central banks often provide new liquidity when the financial system is disrupted (e.g. October 1987). Another example is that the Fed often has used credit crunches – enforced credit rationing – to slow down an overheating economy. In fact, as already indicated in Chap. 2, monetary policy effectively works through the credit channel and thus the credit institutions transmit monetary policy shocks; for more details see Chap. 11.

Differences between promised and actual repayments on loans, or even the default of loans, are the result of uncertainty concerning the borrower's ability to make repayments when due. On the other hand, the lender may not be willing to pay and/or deliver the funds to other users. Both the ability or willingness to pay creates the risk of default for the lender. Some aspects of uncertainty may be treated with the standard model, as illustrated by the capital asset pricing model or other models where there is a fixed and known probability of default. The capital asset pricing model will be taken up in Chap. 8.

Given that borrowers and lenders may have different access to information concerning a project's risk, they may evaluate risk differently. In words of Jaffee and

Stiglitz one can refer "to symmetric information as the case in which borrowers and lenders have equal access to all available information. The opposite case – which we will call generically imperfect information – has many possibilities. Asymmetrical information, where the borrower knows the expected return and risk of his project, whereas the lender knows only the expected return and risk of the average project in the economy, is a particularly important case. Uncertainty regarding consumer and (risky) government loans can be described with the same format used for firms, although, of course, the underlying sources of uncertainty are different" (Jaffee and Stiglitz 1990: 840).

Next, let us undertake a more formal presentation. Details of such an exposition can be found in Jaffee and Stiglitz (1990). Most of the subsequent elaborations for the micro as well as macro levels are supposed to hold for one period zero horizon models.

## 3.4 Imperfect Capital Markets: Microtheory

Let us elaborate some elements of the theory of imperfect capital markets. A credit contract involves the relation between a creditor and a borrower.

The first important element in this relation is that of asymmetric information. The borrower knows for what purpose the loan will be used, but the lender is less informed about the use of the loan. The borrower promises to pay back the loan with interest. The lender faces heterogeneous agents and each borrower's promise is different. The risk of not getting the loan back depends on the borrower's willingness to pay ability. In the last section, we discussed the ability to pay. A risk for the lender may, however, also arise if the borrower has some incentives not to pay. This concerns the willingness to pay by the borrower.[18] In recent credit market theories this has been discussed under the topic of incentive compatible debt contracts.[19]

The problem of the ability to pay for the one period zero horizon case can be formalized as follows. Let there be two possible outcomes for the project of the borrower, $x^a$ and $x^b$, whereby $x^a > x^b$ and $x^a =$ good result; $x^b =$ bad result. Let $p^a, p^b$ be the probability of the occurrence of $x^a, x^b$; with $p^a + p^b = 1$. Then we have the expectations: $x^e = p^a x^a + p^b x^b$.

With this notation we describe the second important element in modern debt contracts. This is the limited liability of the borrower which can be described in the following scheme with $B$ the loan and $r$ the interest rate:

$$\begin{aligned} &\text{creditor} \leftarrow (1+r)\,B \text{ from borrower} \\ &(i)\; x^b < (1+r)\,B \text{ (bad result)} \\ &(ii)\; x^b < (1+r)\,B < x^a \text{ (good result).} \end{aligned}$$

[18] Consider for example the case of a sovereign borrower whose value of the debt is B and M is the value of the access to the capital market. Then if sovereign debt B>M the debtor might not be willing to pay.

[19] For this line of research, see Townsend (1979) and Bernanke and Gertler (1989).

In the second case, since $x^a > x^b$, the borrower's gain is $x^a - (1+r)\,B$. Thus, in case there is a good outcome, $x^a$, the borrower has a gain. Note that limited liability refers to the bad outcome where the borrower is not liable for the loss. The creditor would thus be inclined to require a collateral so as to cover the potential loss. A collateral of the borrower promised to be transferred to the creditor in case of a loss, could be of the following type of asset: liquid funds, financial assets or physical capital. Yet, note that in most cases the value of collateral is uncertain.

On the other hand, the creditor may grant credit but charge for different types of borrowers a different interest rate because different borrowers have different risk characteristics (that require different risk rates). So we may have $r_1$ for the risky borrower and $r_2$ for the less risky borrower with $r_1 > r_2$.

A third important element in modern credit markets is rationing of loans. If borrowers have desired loans of $L^*$ and the creditor offers loans of the amount $L$ there are two cases: (1) if $L < L^*$ (desired loans), the interest rate the borrower offers may increase and we get L=L*. In this case no rationing would occur. (2) the borrowers do not receive loans in case even if they offer an interest rate $r^* > r$. Pure credit rationing of credits might occur only for few borrowers, although all potential borrowers are assumed to be equal.

The question is why the creditor is not interested in granting a loan even at a higher interest rate. Why is there usually a disequilibrium in the credit market? Consider a modern banking sector that receives deposits and gives loans to the public (firms or households)

| Assets | Liabilities |
|---|---|
| R (reserves) | D (deposits) |
| L (loans) | r (interest) |

The banking sector could be competitive or there could be, because of a limited number of banks, oligopolistic or monopolistic behavior in the offering of loans. In any case, one can usually observe the following disequilibrium in the credit market.

The reasons for this are as follows. There is the expected rate of return of the bank and there are interest rates offered by borrowers. Yet, with increasing "default" probability, with higher interest rates offered by the borrower, the banks face higher loan losses. In particular, this occurs if there is adverse selection, i.e. that means if the proportion of riskier borrowers increases when $r$ rises.

A profit maximizing bank will, as shown in figure 3.7, restrict loans, since its return will not increase even if borrowers offer a higher interest rate.

Therefore, there is usually excess demand for loans: $\mathrm{Q}^D > \mathrm{Q}^S$ as shown in figure 3.6.

Note that with a default risk of the borrower the profit of the bank is:

$$\pi = \xi\,(1+r)\,B - (1+\delta)\,B \tag{3.7}$$

where $\xi$ is the percent of repaid loans and $\delta$ the interest rate that the bank has to pay.

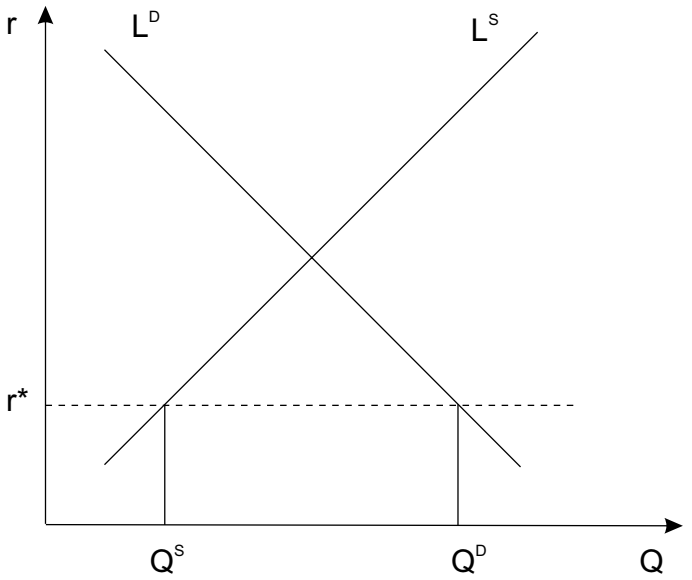



**Fig. 3.6.** Disequilibrium in the Loan Market

This results in the "required" rate of return by banks

$$1 + r^* = \frac{(1+\delta)}{\xi}$$

Here we assume that banks of profit are zero in a competitive banking system. The bank could, however, in order to find out the quality of the borrower suggest the following alternative debt contracts to the borrower: a debt contract with collateral or higher interest rate. The borrower's choice reveals information to the bank about a quality of the borrower.

## 3.5 Imperfect Capital Markets: Macrotheory

The above theory of imperfect capital markets is based on work in the economics of information by Stiglitz and his co-authors. This has greatly influenced the macroeconomic modelling of credit markets and economic activity. We again look at one period zero horizon models. Here asymmetric information, moral hazard and adverse selection as well as asset prices are important. From various studies on credit, asset prices and production activity we can summarize three major results:

1. Asymmetric information induces a wedge between costs of internal finance and external finance (see Townsend 1979, 1984, Myers 1984, Auerbach 1984).

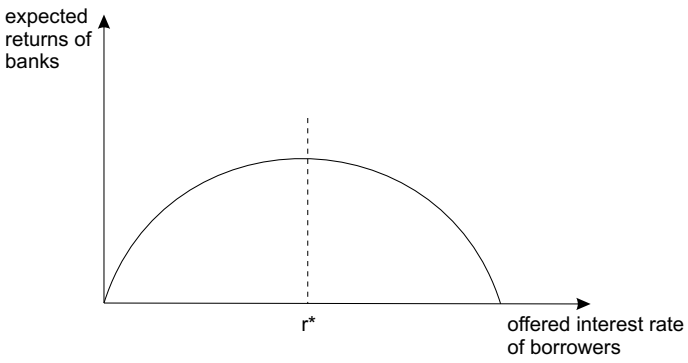



**Fig. 3.7.** Profit Maximizing Bank

2. There is an implied a financial hierarchy where internal finance is the cheapest way of financing investment, debt finance is more expensive and equity finance is the most expensive way to finance investment (see Greenwald and Stiglitz 1993, Bernanke and Gertler 1989), see figure 3.8.
3. The cost of capital depends on the asset price of the firm, i.e. "collaterals" and balance sheets of firms. Investment exhibits an inverse relationship to the cost of capital giving rise to the "financial accelerator". This means that credit and asset prices accelerate the down turn of the economy but also accelerate the upturn. [20]

Figure 3.8 illustrates the financial hierarchy theory. The horizontal line represents the desired investment. When desired investment exceeds a certain amount firms switch from internal to external finance, first using debt finance and then, when further investment is required equity finance. The empirical importance of the credit market for investment is illustrated in Mayer (1991).[21]

As Mayer (1990) has shown a major part of investment is, in the U.S. as well as other countries, financed by credit. Equity financing is, in fact, only a small proportion of the financing options available to firms.

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[20] In earlier literature the procyclical effects of credit were already known. Marshall, for example, states: "As credit by growing makes itself grow, so when distrust has taken the place of confidence, failure and panic breed panic and failure" (Marshall) cited in Boyd and Blatt (1988). A similar statement can be found in Minsky: "Success breeds a disregard of the possibility of failures the absence of serious financial difficulties over a substantial period leads .... to a euphoric economy in which short-term financing of long-term positions become a normal way of life. As a previous financial crisis recedes in time, it is quite natural for central bankers, government officials, bankers, businessmen, and even economists to believe that a new era has arrived" (Minsky 1986: 213).

[21] For further data and figures, see Mayer (1991).

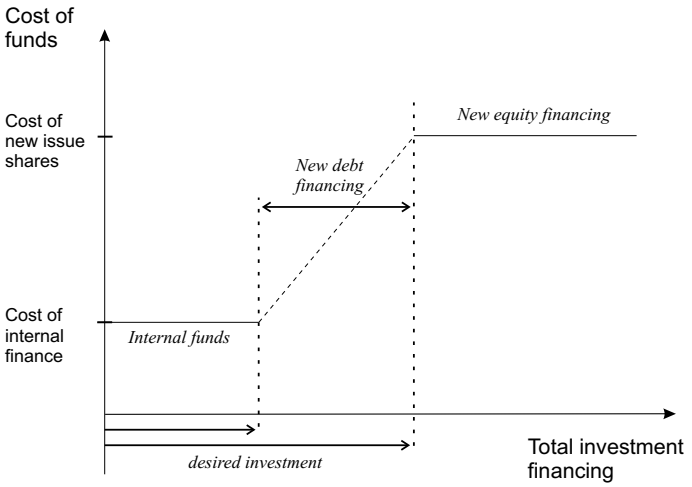



**Fig. 3.8.** Financial Hierarchy

In a simple one period zero horizon model on credit and output some authors have illustrated the above three points where the credit cost depends on the agency cost. This represents the cost of screening and auditing the borrower.[22] Let

$k_i(\pi_i)$: units of capital goods; $q$: their price; $x$: input
$\pi_i$: probability of state $i$ occurring; $r$ : risk-free interest rate;
$S^e$: collateral; $p$: auditing probability.

We assume for a bad state: $i = 1$; and for a good state: $i = 2$. Firms borrow from creditors the amount $x - S^e$. The outcome of the production activity could be

$$qk_i(\pi_i) \lesseqgtr (1 + r)(x - S^e).$$

But note that the risk-free interest rate, $r$, does not represent yet the total borrowing cost. There is an additional agency cost (the cost of screening and auditing) which is

$$\pi_1 p(S^e) q\gamma \text{ where } p_{S^e} < 0, \gamma > 0.$$

Borrowers will thus have to pay an external financing premium which mainly depends on their own equity. The external financing premium will be the lower the higher the internal equity of the borrower. This makes credit cost endogenous and agent specific, for details see Section 12.6.

We thus get the above main three results. First, premium cost for external finance is inversely related to $S^e$, the equity value of the firm. Second, there is a financing hierarchy. Third, investment is inversely related to premium cost (the lower the collateral

[22] See Bernanke and Gertler (1989) and Bernanke, Gertler and Gilchrist (1998) for a multi-period model.

the greater the cost of external finance). This gives rise to the financial accelerator. For the relevance of this theory for macroeconomics and for an empirical test of this theory, see Gertler et al. (1991), Bernanke and Gertler (1994) and Bernanke, Gertler and Gilchrist (1998).

Another model in the same vain has been presented by Greenwald and Stiglitz (1993). Take

$y^n$: net income; $Q(k)$: output; $k$: capital input;
$P_B$: probability of bankruptcy; $D$: debt; $b_o$ : the entrepreneur's own fund.

Then we may write $y^n = Q(k) - (1+r)(k - b_o) - cP_B$

By assuming that the probability of bankruptcy depends on the size of the loan (which is assumed in Greenwald and Stiglitz to be proportional to the capital stock) we thus have:

$$Q' = (1+r) + c\frac{\partial P_B}{\partial k}; \left(c\frac{\partial P_B}{\partial k} > 0\right).$$

This theory also gives us the same three results as above. Internal funds have lower cost than external finance, there is a financial hierarchy and lastly credit may be procyclical.

A multi-period model on the relation of finance and investment has been developed by Fazzari et al. (1988). It reads as follows:

$$V_{\max} = \int_0^{\infty} \left(\pi_t - (1+\Omega_t)V_t^N\right)e^{-\rho t}dt \tag{3.8}$$

where $\pi_t$ are profit flows, $\mathrm{V}_t^N$ the new equity issue, $\rho$ the discount rate, and $\Omega_t$ the premium cost for external finance. Here, too, the hierarchy of finance and the inverse relation of financial risk (default risk) and investment can be derived. Fazzari et al. (1988) also undertake an empirical test of the theory by regressing the investment on cash flows of firms for size classes of firms. Smaller firms are more likely to be credit constrained and thus their relation of cash flow and investment is expected to be strongest.

Another model with borrowing and lending and imperfect capital market is presented in Blanchard (1983*)*. In his model, there is an effect of debt on the utility of households, for example, of a country that borrows:

$$V_{\max} = \int_0^{\infty} U\left(c_t - G(b_t)\right)e^{-\rho t}dt \qquad G' > 0, G'' \geq 0 \tag{3.9}$$

s.t

$$\begin{aligned}\dot{k} &= i_t - \delta k_t \\ \dot{b} &= rb + \left(c + i(1+\varphi(i/k)\right) - f(k)\end{aligned}$$

with $\delta$ the depreciation rate of capital. The latter equation represents the debt dynamics with $f(k)$ a production function, investment, $i$ and adjustment cost of capital,

$\varphi(i/k)$. All variables are written in efficiency units, with $c$, consumption, $k$, capital stock, $b$, debt and $G(b)$, disutility of debt.

The Blanchard model includes the cost of debt in the utility functions. A variant of the Blanchard model will be discussed further below, see Chap. 4.4.

## 3.6 Imperfect Capital Markets: The Micro-Macro Link

The above models on imperfect capital markets and real activity — on the micro as well as macro level— mostly use a framework with one period and zero time horizon. A few exceptions that use an intertemporal framework were also briefly discussed. Next we want to present a model that shows the micro-macro link in an intertemporal framework. The model is based on Uzawa (1968) and is taken up in Asada and Semmler (1995). This model explores in particular the impact of debt on the asset price of the firm.[23] Whereas the former is still in the tradition of perfect capital markets, the latter explicitly takes imperfect capital markets into account.

In the standard model the capital market and thus finance does not really matter for the activity of the firm. The capital structure is irrelevant for the present value of the firm and thus the optimal investment is independent of capital markets. This appears as a common feature of prototype infinite horizon models of the firm. Since the finance of the firm's investment is nonessential, the model disregards an explicit specification of the evolution of the capital structure. Along the lines of Modigliani and Miller (1958), it is usually demonstrated that neither the type of equity financing (financing through retained earnings or issuance of new equity) nor the capital structure matter for the value of the firm and thus for investment. A kind of separation theorem holds, according to which decisions on investment are independent from financing practices and thus, the debt dynamics of the firm.

Equivalent results hold in models where a representative household's utility is maximized over time. The objective function of the firm is here replaced by a utility function. Formally, for example, in Blanchard and Fisher (1989: 58) a system with two state variables representing the evolution of capital stock and a debt equation, depicting the household's evolution of debt can be introduced. Here, too, if debt has no impact on consumption or investment behaviors, finance becomes irrelevant for optimal consumption, investment, and output. In fact, for the system's solution, one can disregard the evolution of debt (Blanchard and Fisher 1989: 63).

It was the development of the economics of information, as discussed in the previous sections, that led to the development of intertemporal models with imperfect capital markets. The above theory, as put forward by Stiglitz and others, for example, has initiated a change of perspective on finance and economic activity. The essential point for intertemporal models is that bankruptcy or default risk arising from the firm's financial structure may result in cross-effects between the real and financial sides and

[23] A more elaborate version of study of corporate debt and its impact on the asset value of the firm can be found in Grüne and Semmler (2005) who follow up a line of research first proposed by Merton (1974).

thus finance and financial structure matter for real activity. In this context, then, a second state equation representing the financial structure of the firm becomes relevant for the growth path of the firm, as does the financial hierarchy theory as aforementioned. When the debt burden of the firm and the associated probability of bankruptcy or default risk are present, the present value of the firm and investment are affected. This gives us the micro-macro link in intertemporal models that we are seeking.

There appear to be many variations of how bankruptcy risk affects the present value of the firm and thus investment in imperfect capital markets. All of them admit cross effects. More formally, we can distinguish three approaches that admit such cross effects.

In a first view, it is postulated that there is a unique relationship between the risk of the firm and the discount rate.[24] In fact, more formally, it can be shown that the discount rate is a monotonic increasing function of the risk a firm faces. In the limit when the discount rate approaches infinity, i.e. the risk approaches infinity, no resources are allocated to the future and a zero horizon optimization problem arises, see Sieveking and Semmler (1994). In general it is posited that the higher the risk of not receiving a cash flow next period, the lower investment.

A second view stresses that the bankruptcy or default risk will affect the value of the firm primarily through the cost arising from external finance, in particular, debt finance. The implicit cost of raising external funds is best summarized in a survey article by Myers: "Costs of financial distress include the legal and administrative costs of bankruptcy, as well as the subtler agency, moral hazard, monitoring and contracting costs which can erode firm value even if formal default is avoided" (Myers 1984: 581).[25] The cost of debt financed investments, resulting from a high leverage of the firm may have, as shown above, a direct (negative) effect on investment. In the models by Greenwald and Stiglitz (1993) the actual cost of raising external funds is a direct function of the debt the firm has incurred.[26] Along the line of the literature on adjustment costs of capital, the borrowing cost is hereby conceived to be a convex one.[27]

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[24] Non-constant discount rate models are increasingly discussed in modern finance theory. For further details on time varying or stochastic discount rates, see Chaps. 9-10. From an empirical perspective the discount rate is, however, rather unobservable. Therefore, proxies are now often used in econometric work for discount rates, see Shiller (1991, 2001).

[25] Note that the above formulation of the "cost of external finance" refers not solely to a higher interest rate for risky firms but rather to a whole set of factors eroding a firm's value. Also note that, as shown by Myers (1984), empirical tests on this matter are difficult to conduct.

[26] Greenwald and Stiglitz argue that the risk of bankruptcy depends on the firm's indebtedness. With debt service as a fixed obligation, the corresponding higher probability of bankruptcy is reflected in the value of the firm. As they argue, the financial market may reappraise the underlying probability of bankruptcy for firms with higher debt service.

[27] Auerbach (1984: 34) for example, states: "interest rate on debt .... is a convex, increasing function of the debt-capital ratio."

A third, more complete conception claims that risk or default cost will actually affect both the discount rate and the cost of external funds. The proposition that "the firm's riskiness increases with the degree of leverage" (Auerbach 1979: 438) can be translated into the view that the discount rate as well as the borrowing cost are a function of the degree of leverage.[28] Here it is often assumed that the interest rate is a convex function of the leverage. Although this line of thought represents the most comprehensive conception of how the firm's degree of leverage affects its value, it is analytically the least tractable formulation.

From all of the above three views one can conclude that a higher default risk of firms arising from debt finance will negatively covary with investment and output. We will summarize some results obtained by Asada and Semmler (1995) who followed the second view in an analytical study of such a dynamic model with credit markets.[29]

The starting point for such a model with capital market is the Uzawa (1968) model. His variant without credit reads

$$V_{\max} = \int_0^{\infty} e^{-\rho t} \left(r_t - \varphi(g_t)\right) K_t dt \tag{3.10}$$

s.t.

$$\frac{\dot{K}_t}{K_t} = g_t$$
$$0 = \varphi(g_t)K_t - RP_t - \dot{E}_{pt}$$

whereby $r$ is the rate of return on capital; $\varphi =$ costs of investment; $E_p$= equity; $RP$= retained profit.

The explicit extension to a model of a monopolistic firm, still without credit, reads with $\mathrm{E}_t$, effort, and, using the ratio $\frac{X_t}{K_t}$ (with $X_t$ output and $K_t$ capital stock)

$$\max_{E_t, g_t, s_{t_f}} = \int_0^{\infty} \left[\{p(E_t) - c\} E_t - \varphi(g_t)\right] K_t e^{-\rho t} dt \tag{3.11}$$

$$\dot{K} = g_t K_t; \qquad K_0 > 0 \tag{3.12}$$

$$\dot{d} = \varphi(g_t) - \left[\{p(E_t) - c\} E_t - i d_t\right] s_{t_f} - g_t d_t; \tag{3.13}$$

where $c$ is the production cost; $d_t$ the debt-capital stock ratio $\frac{D_t}{K_t}$; $i$ the interest rate (constant); and $\rho$ the discount rate.

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[28] For details, see Auerbach (1979).

[29] A detailed study of the impact of default risk on the asset value of the firm is undertaken in Grüne and Semmler (2005a). It is analytically the most tractable one and gives empirical predictions akin to the other views. Since the third view is analytically rather untractable only simulation results can be obtained; see Asada and Semmler (1995).

The debt equation is derived from:

$$\frac{\dot{d}_t}{d_t} = \frac{\dot{D}_t}{D_t} - \frac{\dot{K}_t}{K_t} = \frac{\varphi(g_t)K_t - [\{p(E_t) - c\}\, E_t - id_t]\, s_{t_f} K_t}{D_t} - g_t.$$

In this set up, as Asada and Semmler (1995) have shown the credit market has no effect on the value of the firm. The fact that the firm finances its investment through loans from the capital markets does not matter either. Finance and investment are still separated as in Modigliani and Miller (1958).

Next we introduce a feedback effect of debt finance on the value of the monopolistic firm.

$$\max_{E_t, g_t, s_{t_f}} V = \int_0^\infty [\{p(E_t) - c\}\, E_t - \varphi_1(g_t) - \varphi_2(id_t)]\, K_t e^{-\rho t} dt \tag{3.14}$$

$$\dot{K} = g_t K_t; \qquad K_0 > 0 \tag{3.15}$$

$$\dot{d} = \varphi_1(g_t) - [\{p(E_t) - c\}\, E_t - id_t]\, s_{t_f} - g_t d_t; \tag{3.16}$$

where we can take

$$\begin{aligned} \varphi_1(g_t) &= g_t + \alpha(g_t)^2,\ \alpha > 0 \\ \varphi_2(0) &= 0,\ \varphi_2'(id_t) > 0,\ \varphi_2''(id_t) > 0 \text{ for example} \\ \varphi_2(id_t) &= \beta(id_t)^2,\ \beta > 0. \end{aligned}$$

Hereby we have defined $\varphi_1$ the cost of investment and $\varphi_2$ the influence of bankruptcy risk on the firm value (convex).

The solution and the dynamics of the above model is studied in Asada and Semmler (1995) by using the Hamiltonian approach. It suffices to report the comparative static results. Herein an inverse demand function of the following type is assumed

$$X_t = A_t p_t^{-\eta} \Rightarrow X_t = BK_t p_t^{-\eta};$$
$$\text{where } B > 0 \text{ and } \eta > 1 \text{ (elasticity of demand)}$$

The inverse demand function is given by:

$$p_t = B^{\frac{1}{\eta}} E_t^{-\frac{1}{\eta}} = p(E_t) \tag{3.17}$$

From the necessary conditions for optimality using the Hamiltonian one obtains (for the control variables g, E and the state variables d, K and the co-state variables $\lambda_1, \lambda_2$) $g^*, d^*, E^* > 0$ (with $s_f$ prespecified) and an equation for the price-cost margin

$$B^{\frac{1}{\eta}} E_t^{1-\frac{1}{\eta}} - cE_t = (p(E_t) - c)\, E_t = \frac{1}{\eta - 1} cE^*.$$

The latter, E*, is the steady state of $E$ which is given by

$$E^* = \frac{(1 - \frac{1}{\eta})^\eta B}{c^\eta}.$$

**Table 3.1.** Comparative Static Results

| | | | | |
|---|---|---|---|---|
| 1. | $\rho \nearrow$ (discount rate) | $\Rightarrow$ | $g^* \swarrow$ $(d^* \swarrow)$ | rows $1-3$ |
| 2. | $\beta \nearrow$ (risk parameter) | $\Rightarrow$ | $g^* \swarrow$ $(d^* \swarrow$ | rows $4-6$ |
| 3. | $\eta \swarrow$ (demand with low elasticity) | $\Rightarrow$ | $g^* \nearrow$ $(d^* \nearrow)$ | rows $7-9$ |
| 4. | $i \nearrow$ (interest rate) | $\Rightarrow$ | $g^* \swarrow$ $(d^* \swarrow)$ | rows $10-12$ |
| 5. | $c \nearrow$ (costs) | $\Rightarrow$ | $g^* \swarrow$ $(d^* \swarrow)$ | rows $13-15$ |
| 6. | $s_f \nearrow$ (self-financing) | $\Rightarrow$ | $g^* \nearrow$ $(d^* \swarrow)$ | rows $16-18$ |
| 7. | $B \swarrow$ (reaction of demand to total demand) | $\Rightarrow$ | $g^* \swarrow$ $(d^* \swarrow)$ | rows $19-21$ |
| 8. | $\alpha \nearrow$ (adjustment cost of capital) | $\Rightarrow$ | $g^* \swarrow$ $(d^* \swarrow)$ | rows $22-24$ |

The results on investment (growth rate of capital stock) and the debt to capital stock ratio can be reported from a comparative-static study of the above model (rows refer to table 3.2):

Table 3.1 reports the parameters and the respective equilibrium values for the growth rates, the debt to capital stock ratios and the asset price of the firm.

Overall, the model predicts that investment (and thus the growth rate of capital) falls with higher discount rate, higher risk coefficient, higher interest rate, greater elasticity of demand, lower retention ratio and higher adjustment cost of capital. Those effects would in fact be expected from economic reasoning and studies on the determinants of firms' investment, growth and stock market value. On the other hand, as the model also shows in most cases, except for the case of declining self-financing, that the debt to capital stock ratio falls. These are steady state results and it thus might be reasonable as well to expect a lower debt to capital stock ratio for lower growth rates of the capital stock. Economic growth is accompanied by increasing demand for credit from the capital market. It is this effect that shows up in our steady state results. Bankruptcy cannot really occur in the above model but rather if there is a too strong credit expansion, the value of the firm will decline and with this the demand for credit declines. In fact the above model, as shown in Asada and Semmler (1995), may give rise to cyclical fluctuations rather than to bankruptcies of firms. As concerns the value of the firm, represented by the last column of table 3.2, there is not always a monotonic change of the value of the firm as parameters change since those parameters affect through their new equilibrium values, $g^*$ and $d^*$, and the asset price of the firm. Overall, the model portrays in a setup of a micro-macro link the interaction of credit market, credit financed investment, credit risk, the asset price of the firm and level of economic activity. A further, more detailed study of this type is pursued in Grüne and Semmler (2005a) where also a quantitative evaluation of the impact of default risk on the asset value of the firm is undertaken.

**Table 3.2.** Value of $g^*$, $d^*$ and $V^*$ for Different Parameter Constellations

| row | parameter | equilibrium values | | |
|---|---|---|---|---|
| | | g* | d* | V* |
| 1. | $\rho = 0.12^*$ | 0.0486 | 0.122 | 13.46 |
| 2. | $\rho = 0.115$ | 0.0504 | 0.210 | 13.63 |
| 3. | $\rho = 0.1$ | 0.0589 | 0.588 | 12.79 |
| 4. | $\beta = 40$ | 0.0476 | 0.069 | 13.65 |
| 5. | $\beta = 20^*$ | 0.0486 | 0.122 | 13.46 |
| 6. | $\beta = 4$ | 0.0523 | 0.303 | 12.79 |
| 7. | $\eta = 5.35$ | 0.0262 | -0.477 | 7.93 |
| 8. | $\eta = 4.35^*$ | 0.0486 | 0.122 | 13.46 |
| 9. | $\eta = 4.0$ | 0.0657 | 0.511 | 14.11 |
| 10. | $i = 0.06$ | 0.0473 | 0.061 | 13.71 |
| 11. | $i = 0.04^*$ | 0.0486 | 0.122 | 13.46 |
| 12. | $i = 0.02$ | 0.0520 | 0.257 | 12.85 |
| 13. | $c = 1.68$ | 0.0238 | -0.557 | 6.90 |
| 14. | $c = 1.38^*$ | 0.0486 | 0.122 | 13.46 |
| 15. | $c = 1.28$ | 0.0706 | 0.616 | 13.82 |
| 16. | $\mathrm{s}_f = 0.2$ | 0.0396 | 0.321 | 12.76 |
| 17. | $\mathrm{s}_f = 0.3^*$ | 0.0486 | 0.122 | 13.46 |
| 18. | $\mathrm{s}_f = 0.4$ | 0.0569 | -0.020 | 13.84 |
| 19. | $B = 4.3$ | 0.0268 | -0.457 | 8.20 |
| 20. | $B = 7.3^*$ | 0.0486 | 0.122 | 13.46 |
| 21. | $B = 10.3$ | 0.0842 | 0.903 | 11.98 |
| 22. | $\alpha = 7.5$ | 0.0658 | 0.503 | 10.95 |
| 23. | $\alpha = 11.5^*$ | 0.0486 | 0.122 | 13.46 |
| 24. | $\alpha = 15.5$ | 0.0416 | -0.093 | 14.09 |

## 3.7 Conclusions

This chapter has employed perfect and imperfect capital market theory and discussed the relation of credit market borrowing, credit risk, asset prices and economic activity. We also have shown how in a simple model of the firm the micro-macro link may work. In the next chapter we want to pursue the question of how to empirically test for credit risk of economic agents and its impact on economic activity.

# Part II

# The Credit Market and Economic Activity

CHAPTER 4

# Empirical Tests on Credit Market and Economic Activity

## 4.1 Introduction

In this chapter some key ideas on financial risk and economic activity will be tested. We are still mainly considering the credit market. If the lender faces credit risk, a risk of not recovering the loan from the borrower, this is because the borrower faces bankruptcy risk. Most modern financial analyses of financial risk in credit market use balance sheet variables of economic agents (households, firms, governments and countries) to derive some empirical measures for risk. In Sect. 4.2 we study the bankruptcy risk arising when firms borrow from capital markets to finance their activity. We will summarize some linear regression results. In Sect. 4.3. we introduce a nonlinear test of credit risk and economic activity using an econometric threshold model. In Sect. 4.4 we empirically study credit and bankruptcy risk when the intertemporal budget constraint is not fulfilled. The latter is undertaken in the context of a nonlinear intertemporal model.

## 4.2 Bankruptcy Risk and Economic Activity

### 4.2.1 Introduction: Measurement Problems

Here we will study the problem of financial risk from the point of view of the borrower in our case the firms are borrowing from the capital market for investments. Indeed, since firms' investments are to a considerable extent financed through the credit market, this market may have a forceful impact on economic activity of firms and thus on the performance of the macroeconomy. We use balance sheet variables of firms to study the impact of bankruptcy risk on economic activity. Most of the studies are undertaken with OLS regressions. Such regression studies are, at least in a first approximation, helpful in uncovering the relation of bankruptcy risk and economic activity.

The role of balance sheet variables to measure financial risk is essential in the above theory of imperfect capital markets. As discussed above, when invoking the theory of asymmetric information, it is assumed that lenders and borrowers of funds have different knowledge about the possible success of the investment project. Given this information structure, lenders of funds will be unable to screen borrowers perfectly. Low-quality firms, when competing with high-quality firms for funds, have

to pay a premium to obtain external funds. If overall bankruptcy risks or the spread between high- and low-quality firms increases in a downswing, agency and borrowing costs of external funds rise and this exerts a negative impact on investment. Net worth of low-quality firms is predicted to move pro-cyclically and borrowing costs of external funds counter cyclically, amplifying real economic disturbances.

The role of balance sheet variables to measure bankruptcy risk can also be found in Keynesian tradition, for example in the work of Minsky (1975, 1982, 1986) and, in particular, in the work of a contemporary of Keynes, namely Kalecki (1937a). Many scholars refer to him as an important source when studying the impact of the credit market and real activities. In Kalecki, particular emphasis is given to the risk that firms might find themselves exposed to when their activities are debt financed. Kalecki (1937a,b) referred to the role of real returns on investment, the interest rate, 'increasing risk' due to debt finance and a prospective rate of return as a determinant of investment. He then posited that the difference between the prospective and actual rate of return on capital is a measure of the risk incurred in the investment project. The cost of capital funds consists of the (real) interest rate and the cost of risk stemming from borrowing outside funds. The latter is considered to be an increasing function of the ratio at which investment is debt financed.[30] Thus, given default risk and a real interest rate, investment should vary with the expected rate of return or, for a given expected rate of return and real interest rate, investment is expected to vary (inversely) with the risk arising from the investment decisions. In any case the theory of imperfect capital markets presented in Chap. 3 as well as the Keynesian – Kaleckian tradition suggest a strong role for balance sheet variables in the activity of firms.[31] Yet, measuring bankruptcy risk of firms by using balance sheets, is not an unambiguous task. Moreover, testing for the influence of risk variables on firms' investment requires also to simultaneously control for other forces impacting investment.

To account for real forces affecting investment, often the accelerator principle, or some variant of it, has been employed. As the real variable we will use capacity utilization. This reflects the real accelerator and has traditionally been used in investment studies.[32] There are strong co-movements between the utilization of capacity and the actual rate of return on investment. The utilization of capacity is also used as the basic reference variable against which the contributions of other variables are measured.

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[30] Kalecki (1937a) argues that investment will be undertaken up to the point where the excess of prospective profits over the interest rate is equated to the bankruptcy risk arising from debt financing of the investment project. Therefore, the cost of funds consists of two components: the interest rate and the 'increasing risk' due to debt finance. References to Kalecki's work can also be found in the recent work on imperfect capital markets.

[31] The role of the financial variables for investment had partly been lost in the Keynesian literature in the post-war period.

[32] The subsequent study is based on Franke and Semmler (1997). This study also tests the impact of the real return on capital from firms' investment. The results are very similar to using the utilization of capacity.

Next we might want to take into account the movements of the (real) interest rate. Although (some) macro theories point in the direction of a lesser importance of the interest rate for investment,[33] we nevertheless prefer not to exclude the real interest rate as an independent variable.

Traditionally, empirical studies of bankruptcy risk have employed variables such as credit flow, the debt-asset ratio and the interest coverage ratio[34] as appropriate proxies for the default risk of firms. Other studies have proposed liquidity variables as proxies for risk rather then debt variables. An important variable to measure the default risk of firms is interest rate spreads. Low-quality firms – financially fragile firms – face a higher bankruptcy risk because their net worth is lower, external financing costs are higher than for high-quality firms. Thus, it is the financial market evaluation of firms' default risk that leads to interest rate spreads. Therefore, interest rate spread might be a very appropriate measure for bankruptcy risk. One thus expects, for example, in a stage of declining economic activity, an increasing interest rate spread and in particular, an increasing spread between low- and high-quality bonds. If lenders can accurately assess the default risk of individual firms or industries, the changes of risk will be reflected in interest rate spreads. As an aggregate measure of the spread to be used as proxy for risk we take the difference between the short-term commercial paper rate and the interest rate of Treasury bonds.[35]

We also suggest to including M2 as an additional factor in investment decisions. Money and money expansion by the monetary authority will provide liquidity for firms and ease the tension of default risk. More specifically, we employ the velocity of M2 money among the independent variables. If there is a strong endogenous money supply via banks in the business cycle then one might expect a counter cyclical movement in M2 velocity. Yet, the money supply is also affected by monetary policy. If a restrictive monetary policy is pursued during the late period of a boom, for example, and continued at the beginning of recessionary periods, this might contribute to a counter cyclical movement in M2 velocity, a liquidity crunch and thus to a (possibly lagged) positive correlation with investment.

A more difficult problem is to measure prospective profits. If the stock market was a good predictor for firms' prospective profits one could rely on Tobin's $Q$ as an important factor in investment decisions of firms. We do include Tobin's $Q$ among the independent variables. On the other hand, one might argue that investment decisions depend more directly on business prospects. This suggests that we employ variables such as, for example, the leading indicators for estimating firms' expected return.

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[33] See Greenwald and Stiglitz (1986).

[34] The debt-asset ratio and interest coverage ratio may be considered as important variables in Minsky type models.

[35] Friedman and Kuttner (1992) have already employed interest rate spreads as measures for financial fragility. There, however, other proxies for financial risk are left aside. Interest rate spread has also been proposed as an additional leading indicator by Stock and Watson (1989).

In the subsequent part we therefore add to the independent variables an aggregate form of the leading indicators. Also the arithmetic average of Tobin's $Q$ and that aggregate measure is invoked.[36]

We can now summarize the empirical measures that are employed in the following regressions. We study only a limited time period and use U.S. quarterly data for the period 1960.4-1982.4. Because of some non-stationarity the data is detrended. The trend deviations of the growth rate of capital stock is taken as the dependent variable. The variable is gkDev. The capital stock data are from Fair (1984). As interest rate variable we take the 6 months commercial paper rate, deflated by the growth rate of GDP deflator. It is called irealDev. Both are from Citibase (1989). As financial variables, from the balance sheets of firms, in a preliminary step we have explored credit market debt (stock variable), gscmdDev, liquid assets, liquDev, quick assets and a measure for working capital.[37] Because of insignificance in the regressions (or the high collinearity with the other variables) we have dropped the quick ratio and the working capital variables from our financial regressions. Moreover, we take as interest coverage ratio icovDev. For the M2 money stock variable we take the M2 – velocity of money which is called velocDev. All other variables are used as ratios over capital stock and then detrended by a segmented trend. Therefore, Dev stands for deviation. Data are from Citibase (1989). As said above, the interest rate spread is measured by the difference between the six months commercial paper rate and the six months Treasury bill rate (Citibase Data 1989), called sprdDev. Tobin's $Q$, called qsumDev, and an aggregate of the leading indicator, called deleadDev, as well as a linear combination (with equal weights) of qsumDev and dleadDev, called confDev, are added (as different variables) to measure expected returns.[38]

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[36] We want to point out here that Keynes, for example, never thought of a variable solely reflecting the financial evaluation of the firm, as being the most important determinant of investment. He more accurately referred to the 'state of confidence of investors' and business prospects when discussing the role of expectations (Keynes, 1936, Chaps. 5 and 12). This includes general business conditions, consumption behavior, credit conditions and financial market prospects. It is on these grounds that we will refer to financial as well as to business prospects of firms in our regressions.

[37] The above measures were used in real terms (in 1982 dollars) with the following definitions: (1) flow and stock of credit market debt = net flows of corporate debt instruments; (2) liquid asset = stock of liquid assets; (3) working capital = stock of working capital; (4) interest coverage ratio = cash flows over net interest paid by non-financial cooperations (Fair, 1984). The data for these variables are taken from the Flow of Funds Accounts (1989).

[38] In order to construct an aggregate predictor for expected returns we aggregate with equal weights the four leading indicators of Business Conditions Digest (Citibase Data, 1989). The leading indicators are *dleac* (composite index, capital investment), *dlead* (composite index, inventory investment and purchase), *dleap* (composite index, profitability) and *dleaf* (composite index, money and financial flows).

### 4.2.2 Some Empirical Results

The following table 4.1 summarizes the regression results for our test of the impact of financial risk on firms' investment. Here we exclude interest rate spread, sprdDev.

As observable from the table the real variable, uDev, has the strongest and always significant impact on firm investment. The impact of the real interest rate is mostly insignificant. The influence of the financial risk variables are very fragile, sometimes

**Table 4.1.** Test of the Role of Bankruptcy Risk Variables for Investment, 1960:4 to 1982:4, (Exclusive of Interest Rate Spread Variable) with gkDev as Dependent Variable

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $uDev_{-3}$ | .305 | | .236 | .243 | .234 | .261 | .256 | .248 | .138 |
| | (16.2) | | (9.3) | (9.7) | (8.7) | (13.7) | (12.0) | (8.2) | (3.6) |
| $liquDev_{-1}$ | | .342 | .087 | .011 | | | | .049 | |
| | | (2.9) | (1.1) | (.2) | | | | (.5) | |
| $velocDev_{-2}$ | | -.136 | -.038 | .009 | | | | -.032 | |
| | | (6.7) | (2.2) | (.5) | | | | (1.6) | |
| $gscmdDev_{-1}$ | | | .169 | .122 | | | .150 | .028 | .023 |
| | | | (3.0) | (2.1) | | | (2.5) | (.5) | (.8) |
| $gscmdDev_{-2}$ | | .348 | | | | | | | |
| | | (4.7) | | | | | | | |
| $irealDev_{-2}$ | | | | -.025 | | | -.053 | -.034 | -.006 |
| | | | | (.8) | | | (1.8) | (1.0) | (.3) |
| $ircovrDev_{-2}$ | | | | .004 | | | | -.0006 | |
| | | | | (2.2) | | | | (.3) | |
| $qsumDev_{-2}$ | | | | | .004 | | | | |
| | | | | | (.5) | | | | |
| $dleadDev_{-2}$ | | | | | .068 | | | | |
| | | | | | (4.1) | | | | |
| $confDev_{-2}$ | | | | | | .035 | .024 | .031 | .041 |
| | | | | | | (2.6) | (1.8) | (2.0) | (2.4) |
| $\rho$ | | | | | | | | | .87 |
| | | | | | | | | | (13.1) |
| $R^2$ adjusted | .76 | .58 | .79 | .80 | .79 | .77 | .79 | .78 | .89 |
| S.E. | .58 | .77 | .54 | .54 | .54 | .56 | .54 | .56 | .38 |
| DW | .62 | .64 | .78 | .85 | .60 | .60 | .80 | .60 | 1.70 |

significant, sometimes not, depending on what other variables are included. The profit expectations variable comes out mostly significant (except for qsumDev).[39]

Next we want to show the regression results for the inclusion of the interest rate spread variable sprdDev.

**Table 4.2.** Test of the Role of Bankruptcy Risk for Investment, 1960:4–1982:4 (Inclusive of Interest Rate Spread), with gkDev as Dependent Variable

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $uDev_{-1}$ | .237 | | .253 | .237 | .253 | .264 | .246 | .237 | .152 |
| | (16.2) | | (11) | (9.3) | (11.0) | (14.4) | (12.8) | (9.2) | (4.4) |
| $liquDev_{-1}$ | | .275 | -.003 | -.027 | | | | -.025 | |
| | | (2.3) | (0.5) | (.35) | | | | (3) | |
| $velocmDev_{-2}$ | | -.109 | .007 | .017 | | | | .018 | |
| | | (4.6) | (.4) | (.87) | | | | (9) | |
| $gscmdDev_{-1}$ | | | .147 | .167 | | | .140 | .170 | .041 |
| | | | (2.9) | (3.2) | | | (2.7) | (2.9) | (1.4) |
| $gscmdDev_{-2}$ | | .347 | | | | | | | |
| | | (4.7) | | | | | | | |
| $irealDev_{-2}$ | | | | .006 | | | .001 | .007 | .007 |
| | | | | (.22) | | | (.05) | (.2) | (.4) |
| $icvorDev_{-2}$ | | | | .002 | | | | -.003 | |
| | | | | (1.4) | | | | (1.3) | |
| $sprdDev_{-3}$ | -.118 | -.079 | -.115 | -.114 | -.100 | -.110 | -.110 | -.110 | -.070 |
| | (5.8) | (2.1) | (4.8) | (4.5) | (3.8) | (5.1) | (4.8) | (4.5) | (3.6) |
| $gsumDev_{-2}$ | | | | | .005 | | | | |
| | | | | | (.7) | | | | |
| $dleadDev_{-2}$ | | | | | .022 | | | | |
| | | | | | (1.2) | | | | |
| $confDev_{-2}$ | | | | | | .015 | .005 | .002 | 0.30 |
| | | | | | | (1.3) | (.5) | (.2) | (1.8) |
| $\rho$ | | | | | | | | | .83 |
| | | | | | | | | | (11.6) |
| $R^2$ adjusted | .83 | .60 | .84 | .84 | .83 | .82 | .83 | .83 | .90 |
| S.E. | .50 | .76 | .48 | .48 | .49 | .49 | .48 | .48 | .36 |
| DW | .75 | .73 | .84 | .85 | .68 | .72 | .80 | .86. | 1.88 |

[39] Note that the last column in this as well as the next table reports regression results that are corrected for auto-correlation.

As the results of table 4.2 show the interest rate spread variable, sprdDev, is always significant and its influence on investment is strong, no matter what other variables are included in the regression.

### 4.2.3 Conclusions

It is hard to accurately measure the impact of bankruptcy risk on firm activity. The real variable, capacity utilization, is the most dominant variable explaining firm investment. Some bankruptcy risk variables are also significant, but they are often replaced in their influence by other variables when they are included. In particular the interest rate spread variable is an important variable measuring bankruptcy risk and its impact on firm activity. We also want to note that this interest rate spread variable has also been proposed as an additional variable for a leading indicator by Stock and Watson (1989) who showed the relevance of this variable for indicating turning points of the business cycle. How well spread is measuring default risk is also discussed in Friedman and Kuttner (1992) and Bernanke and Blinder (1992) who point to the fact that the interest rate spread is also impacted by monetary policy. Kashyap, Stein and Wilcox (1993) use another variable to measure credit market conditions and its input on economic activity. They use a variable for financial mix measured as L/(L+CP) whereby L represents loans from banks and CP the loans from the commercial paper markets. Finally, we want to note that we have undertaken our analysis on an aggregate level. This may wash out some stronger effects that might be visible on the firm level. Balance sheet variables on the firm level would have been more appropriate to measure bankruptcy risk and its impact on firm activity. Finally, we want to note that the low DW indicates some remaining structure in the residuals. This might suggest using nonlinear models for the impact of the credit market on economic activity. This will be taken up next.

## 4.3 Liquidity and Economic Activity in a Threshold Model

### 4.3.1 Introduction

Bankruptcy arises when an economic agent is unable to repay a loan. Whether or not there is a threat that the borrower cannot repay the loan can only be judged in an intertemporal context. Only in the long run, when the intertemporal budget constraint cannot be met will the agent run into bankruptcy. This is an aspect of debt contracts that will be studied empirically in section 4.4. Yet, whenever the agent can obtain short term credit, i.e. if some creditor is willing to provide the agent with short term liquidity, the agent, at least temporarily can remain operative. What we consider next is under what conditions the agent might obtain liquidity. This problem of liquidity has been discussed in numerous economic studies.

From a micro perspective the credit market theory discussed in Chap. 3 is relevant to explain liquidity constraints for agents. As shown, if lenders are unable to

perfectly monitor borrowers' investment projects, the balance sheets of agents and the agents' collateral is important for obtaining credit.[40] Households' and firms' net worth appears to move with the business cycle. In fact net worth moves pro cyclically. Then there will be less credit constraints for households and firms in high levels of economic activity compared to low levels of economic activity.

As has been shown in economic studies[41] most households, for example, are in fact liquidity constrained and cannot borrow against future income. This implies a close connection between income and spending.[42] Though available liquidity for those types of households – for example liquid assets such as deposits and treasury bonds – may be dissociated to some extent from income and spending, credit constraints would, however, effectively still play a major role in those households' spending behavior in that the ease and tightness of credit constraints over the business cycle would accelerate the contractions and expansions. This is in strong contrast to intertemporal models of consumer behavior which allow for intertemporal borrowing and lending. Here spending is dissociated from current income. Those types of consumers can use assets as collateral for borrowing and smooth out spending.

If, however, the majority of households are credit constrained this would support the hypothesis that spending is constrained by the ease and tightness of credit. A related argument can be made, see for example Fazzari et al. (1988), with respect to firms and their investment spending. Large firms that are evaluated on the stock market may not face credit constraints as much as small firms that are credit dependent. For small firms, mostly credit or bank dependent firms, the degree of credit constraints may vary over the business cycle – depending on the net worth or availability of credit for those firms. This, in turn, similar to liquidity constrained households, may act as a magnifying force for economic activity. Thus, overall, these observations for households and firms – as much as liquidity constraints are valid for households and a large number of small firms – predict that swings in households' and firms' balance sheet variables will magnify fluctuations in spending in the business cycle.[43]

Such an interaction of liquidity and output in the business cycle have been explored empirically in a large number of macroeconomic studies, for example in the papers by Eckstein, Green and Sinai (1974), Eckstein and Sinai (1986); and also Friedman (1986) and Blinder (1989). Modelling liquidity effects in the tradition of

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[40] An increase in the marginal default risk is usually translated into higher cost of external compared to internal funds but we largely neglect here the cost of credit.

[41] See, for example, Campbell and Mankiw (1989) and Zeldes (1990).

[42] See Hubbard and Judd (1986) and also Deaton (1991). Both studies survey extensively the literature on excess sensitivity of spending with respect to income changes of liquidity constrained households. Deaton (1991), however, shows that, to some extent, this excess sensitivity is modified by precautionary savings of liquidity constrained households.

[43] We also want to note that liquidity and available credit may have smoothing effects on production or consumption at least for small shocks. Thus, actual economies may exhibit corridor-stability, see Semmler and Sieveking (1993). In this view small shocks do not give rise to deviation amplifying fluctuations but large shocks can lead to a different regime of propagation mechanism. Thus, only large shocks are predicted to result in magnified economic activities.

Keynesian theory have been undertaken within the context of IS-LM models.[44] Interesting nonlinear versions of IS-LM[45] macro dynamics can be found in Day and Shafer (1985) and Day and Lin (1991). Those types of models exhibit quite intriguing periodic and non-periodic fluctuations in macro aggregates. In Foley (1987), besides money, commercial credit is introduced where firms are free to borrow and lend. Banks provide loans and offer deposits so that the overall source of liquidity is commercial credit and deposits. Here, too, strong fluctuations in aggregates can arise.

Thus, there is a long tradition that predicts empirically that credit may impact economic activity in a nonlinear way.[46] As aforementioned we here employ a simple version of a nonlinear macro dynamic model, developed by Semmler and Sieveking (1993), to give some predictions of the behavior of variables such as liquidity and output particularly over the business cycle. The model employed here which is testable by time series data allows for state-dependency and regime changes.

Of course, as many economists have stressed, credit flows and liquidity also depend on monetary policy. The credit view of three transmission mechanisms of monetary policy maintains that it operates through the asset side of banks' balance sheets.[47] When reserves are reduced, and banks can only imperfectly substitute away from the reduced monetary base, then the volume of loans as well as the interest rates on loans and commercial papers are affected. Given the asymmetric information between borrowers and lenders, banks tend to become more careful in the selection of customers and this leads to an overall cutback in bank lending. Banks have to decrease the volume of loans, it is argued they will extend loans only to the most secure customers or to customers with sound balance sheets and good collateral when. Therefore, the ease and tightness of credit that firms and households face due to their own balance sheets and collaterals is easily seen to be accentuated by monetary policy. The importance of this credit channel for monetary policy has already been observed in earlier papers on the financial-real interaction. The papers by Eckstein et al. (1974, 1986) and Sinai (1992) are good examples. There it is shown that there are certain periods in the financial history of the U.S. where monetary contractions have led to credit crunchs and a worsening of the above described borrowing situation for households and firms.[48] On the other hand monetary policy has helped to give rise, after a recessionary period, to a reliquification of households and firms and an improvement in their balance sheets.[49]

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[44] Different variants of models on liquidity and output, for a growing economy, are discussed in Flaschel, Franke and Semmler (1997), Chap. 4.

[45] See Semmler (1989).

[46] An excellent survey of earlier theories are given in Boyd and Blatt (1988). The work of Minsky (1978) continues this theoretical tradition.

[47] For further details, see Chap. 10.

[48] The worsening of liquidity for firms and households with a restricted monetary policy is discussed in Sinai (1992).

[49] Sinai, for example, states: "Business upturns have almost always been associated with easier money and ample credit, lower interest rates ... increased liquidity for house-

In empirical studies that employ time series analysis, however, it turns out to be rather difficult to give quantitative evaluation of the link between liquidity and economic activity.[50] There are complicated lead and lag patterns in the liquidity and output interaction and thus it is not easy to identify the liquidity-output link in the data, particularly if only linear regression models are employed.

There is, however, plenty of indirect empirical evidence that credit moves pro-cyclically[51] and that a nonlinear relation of liquidity and output is likely to be to be found in the data. The liquidity-output relation may be state dependent and undergo regime changes depending on the phases of the business cycle. A model that captured those nonlinear interactions is introduced in Semmler and Sieveking (1993) and econometrically studied in Koçkesen and Semmler (1997). Here, the liquidity and output interaction are state dependent in the sense that the relation of the variables change as some variables pass through certain thresholds. Recently, a number of macro models have been proposed that exhibit state dependent reactions and regime changes with respect to the variables involved.[52]

A variety of eloborate univariate and multivariate statistical methods are capable of testing for state dependency and regime changes in time series data. Although there is no general agreement as to what type of nonlinear econometric model is best suited to modeling a given data series, important advances have been made. [53] Besides a direct test of state dependent reactions we can employ indirect methods such as the recently developed Smooth Transition Regression (STR) model that captures switching behavior and regime changes. The latter approach appears to be very useful to empirically study the dynamic interactions between variables. It has been applied with some success to the study of macroeconomic and financial time series. [54]

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holds, business firms and financial institutions, (and) improved balance sheets ... "(Sinai 1992: 1).

[50] In earlier times the effects of monetary shocks were discussed in VAR type of money-output models with rather inconclusive results. A recent evaluation of the success and failure of those VAR studies is given in Bernanke and Blinder (1992).

[51] Pro-cyclical credit flows are documented in Friedman (1983), and Blinder (1989). Blinder, by decomposing credit market debt, finds that private credit market debt, in particular trade credit moves strongly pro-cyclically.

[52] State dependent and threshold behavior of variables in economic and econometric models are frequently arising due to (non-convex) lumpy adjustment costs. Typical examples are the inventory, money holding and price adjustment models (Blanchard and Fischer 1989, Chap. 8), employment models with lumpy adjustment costs, but also monetary policy rules which are applied discretionarity after some variables have passed through their thresholds. The same may hold for employment policies of firms, for example, with firms adjusting to large deviations proportionally more than to small ones. For surveys of macroeconomic models of threshold type, see Flaschel, Franke and Semmler (1997).

[53] In the work by Tong (1990) a survey is given on many univariate models and Granger and Teräsvirta (1993) consider univariate as well as multivariate methods.

[54] For a survey and applications see Tong (1990), Granger and Teräsvirta (1993) and Granger, Teräsvirta and Anderson (1993), Ozaki (1986, 1987, 1994) and Rothman (1999).

In our case, although the estimation strategy is not limited to this case, there are two variables to be examined as variables for state dependency and regime changes. Although there might be a choice of several important financial variables (that interact with real activity in a nonlinear fashion), we report results from a model that focuses on liquidity and output. These appear to us as the most relevant variables to test for the short-run nonlinear interaction of financial and real variables. We employ post-war U.S. data.

### 4.3.2 A Simple Model

One can think of a nonlinear economic model on liquidity and economic activity as follows. Firms, households and banks may be represented by their balance sheets with assets on the left and liabilities on the right side. When the asset side, due to declining income flows, deteriorates, credit is harder to obtain and interest costs for the agents may rise. A rising interest rate may lead to an adverse selection problem and banks constrain or recall credit. Credit or credit lines represent liquidity for firms and households. So we will speak about liquidity as a general term representing (short term) credit. The following generic continuous time model, which is derived from an IS-LM model, see Koçkesen and Semmler (1997), may reveal our ideas:

$$\dot{\lambda} = \lambda f_1(\lambda, y) \tag{4.1}$$

$$\dot{y} = y f_2(\lambda, y) \tag{4.2}$$

where $\lambda = L/K$, $y = Y/K$, with $L$ denoting liquidity, $y$ income and $K$ the capital stock. We assume that, at least beyond a certain corridor about the steady state of the system (4.1)-(4.2), income and liquidity in equ. (4.1) positively affects liquidity and also positively impacts income in equ. (4.2).

The nonlinear relation of liquidity and economic activity means that the reactions of agents are amplitude dependent in that the spending of agents depends on the ease and tightness of credit. For liquidity constrained agents credit depends on net worth which moves pro-cyclically with risk falling in an economic boom and rising in a recession. Our model thus posits that spending accelerates (decelerates) when income and liquidity rises above (falls below) some threshold values. The same holds for liquidity. This dynamic is depicted in figure 4.1. Note that, of course, also credit cost – which depends on default risk; i.e. the wedge between cost of internal and external funds – moves counter cyclically.

The amplitude-dependent reactions can be made more explicit in the following specification

$$\dot{\lambda} = \lambda \left(\alpha - \beta y - \in_1 \lambda + g_1(\lambda, y)\right) \tag{4.3}$$

$$\dot{y} = y \left(-\gamma + \delta\lambda - \in_2 y + g_2(\lambda, y)\right) \tag{4.4}$$

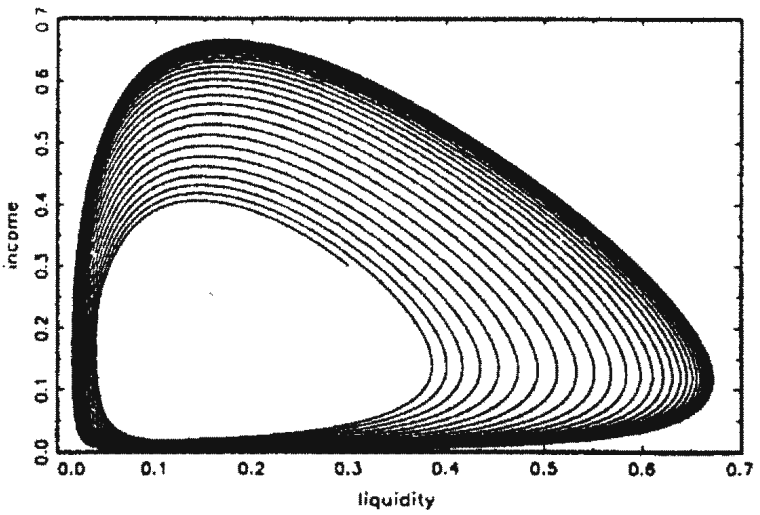



**Fig. 4.1.** Phase Portrait

where the $g_i$, with i=1,2 activate a regime change

$$g_i = g_i(\lambda, y) > 0 \text{ for } \begin{cases} \lambda > \mu_1 & \mu_1 > \lambda^* \\ \text{and} & \\ y > \nu_1 & \nu_1 > y^* \end{cases}$$

$$g_i = g_i(\lambda, y) < 0 \text{ for } \begin{cases} \lambda < \mu_2 & \mu_2 < \lambda^* \\ \text{and} & \\ y < \nu_2 & \nu_2 < y^* \end{cases}$$

In the upper regime there is a positive impact of liquidity and (or) spending whereas in the lower regime there is a negative perturbation of liquidity and (or) spending. The dynamic is depicted in figure 4.1. The following propositions can be shown to hold.

**Proposition 1.** *The System (4.3), (4.4) is asymptotically stable for* $g_i = 0$

**Proposition 2.** *If in system (4.3), (4.4) the perturbation terms* $g_1(\lambda, y), g_2(\lambda, y) \neq 0$ *are small enough it is asymptotically stable.*

**Proposition 3.** *For any* $g_1(\lambda, y), g_2(\lambda, y) \neq 0$ *system (4.3), (4.4) becomes unstable for* $\in_1 = 0, \in_2 = 0$. *The trajectories, however, remain in a positively invariant set for any* $\in_1, \in_2 > 0$ *even for large* $g_1(\lambda, y)$, $g_2(\lambda, y)$.

A proof of these propositions can be found in Semmler and Sieveking (1993).

There is an extensive literature that estimates such state dependent models. An important contribution has been made by Ozaki (1985) who estimates for example a van der Pol equation written in continuous time

$$\ddot{x} - b(x)\dot{x} + bx = \varepsilon$$
$$\varepsilon \ : \ \text{white noise}$$
$$b(x) \ : \ a(1 - x^2).$$

A discrete time nonlinear model approximation of such a continuous time locally self exciting and globally bounded system is

$$x_t = (\phi_1 + \Pi_1 e^{-x_{t-1}^2})x_{t-1} + (\phi_2 + \Pi_2 e^{-x_{t-1}^2})x_{t-2} + \varepsilon_t.$$

A further example of a system with threshold behavior is a piecewise linear model such as

$$\begin{aligned} x_t &= \Pi(T_1)x_{t-1} + \varepsilon_t && \text{for } x_{t-1} < T_1 \\ &= \Pi(x_{t-1})x_{t-1} + \varepsilon_t && \text{for } T_1 \le x_{t-1} < T_2 \\ &= \Pi(T_2)x_{t-1} + \varepsilon_t && \text{for } x_{t-1} \ge T_2. \end{aligned}$$

Models that are written in continuous time

$$\dot{z} = f(z, \theta, w)$$

with $\theta$ the parameter set and $w$ a noise term, can be discretized, for example, by the Euler approximation. The Euler method is[55]

$$z_{t+h} = z_t + hf(z/\theta) + \varepsilon_t$$

Popular discrete time nonlinear models are Threshold Autoregressive (TAR) models. They use local approximations to a nonlinear system by linear regimes via thresholds. A univariate TAR model is, for example,

$$Y_t = \alpha_i^{(j)} + \sum_{i=1}^{p} \alpha_i^{(j)} Y_{t-i} + \varepsilon_t^{(j)},$$
$$if \ r_{j-1} \le Y_{t-d} < r_j, \ j = 1, 2, .....k$$

$k =$ number of different regimes, $d =$ delay parameter, $\{r_j\} =$ threshold parameters.

A multivariate TAR model reads

$$Y_t = \alpha_0^{(j)} + \sum_{i=1}^{p} \beta_i^{(j)} X_{t-i} + \varepsilon_t^{(j)},$$
$$\text{IF } r_{j-1} \le X_{t-d} < r_j$$

[55] A different procedure is the method of local linearization used in Ozaki's work. There applications can be found to the van der Pol equation and random vibration systems, see Ozaki (1986, 1994).

A large number of applications to macroeconomic and financial data have been undertaken for theses types of models, for example to GNP, stock market returns and unemployment rates, see Potter (1993), Tong (1990), and Rothman (1999).
We report estimation results of the above nonlinear (threshold) model for liquidity and output dynamics with unconstrained lag structure. A linearity test is embedded in a nonlinear threshold model. Results of the linearity test are reported in Table 4.3.

**Table 4.3.** Linearity Tests

| **Equation** | **Variable** | $\rho$ | $\rho^3$ | $\rho^2$ | $\rho^1$ |
|---|---|---|---|---|---|
| Liquidity | $\rho_{t-6}$ | 0.06 | 0.43 | 0.33 | 0.008 |
| Rate of return | $\lambda_{t-7}$ | 0.001 | 0.005 | 0.31 | 0.01 |

where $p, p^3, p^2, p1$ refer to the probability values of certain (nested) hypotheses. For example, the linearity hypothesis is rejected at a 6% level of significance for the liquidity equation when the transition variable is $\rho_{t-6}$.

Results of the threshold estimations are:

$$\begin{aligned}\lambda_t = & \underset{(.0004)}{.0008} + \underset{(.06)}{.75}\,\lambda_{t-1} + \underset{(.09)}{.55}\,\lambda_{t-5} - \underset{(.05)}{.22}\,\rho_{t-5} + \underset{(.05)}{.18}\,\rho_{t-6} \\ & + \left( - \underset{(.14)}{.56}\,\lambda_{t-3} - \underset{(.07)}{.18}\,\rho_{t-4} + \underset{(.12)}{.36}\,\rho_{t-5} - \underset{(.10)}{.39}\,\rho_{t-6} \right) \\ & \times \left( 1 + \exp\left[ - \underset{(1.42)}{2.85} \times 72.59 \left( \rho_{t-6} - \underset{(.003)}{.005} \right) \right] \right)^{-1}\end{aligned}$$

$$R^2 = 0.85 \qquad SE = 0.0029 \quad LM(7) = 1.81\ (0.09)$$
$$ARCH(1) = 0.06\ (0.81)\ BJ = 1.45\ (0.48)$$

The LSTR model for rate of return is given by

$$\begin{aligned}\rho_t = & \underset{(.001)}{.004} + \underset{(.14)}{.76}\,\lambda_{t-1} - \underset{(.20)}{.58}\,\lambda_{t-5} + \underset{(.06)}{.63}\,\rho_{t-1} - \underset{(.11)}{.34}\,\rho_{t-5} + \underset{(.11)}{.36}\,\rho_{t-6} - \underset{(.06)}{.21}\,\rho_{t-8} \\ & + \left( \underset{(.003)}{.006} - \underset{(.27)}{1.02}\lambda_{t-4} + \underset{(.38)}{1.18}\lambda_{t-5} - \underset{(.39)}{.82}\,\lambda_{t-7} + \underset{(.26)}{1.10}\lambda_{t-8} + \underset{(.18)}{.57}\,\rho_{t-5} - \underset{(.19)}{.53}\,\rho_{t-6} \right) \\ & \times \left( 1 + \exp\left[ \underset{(20.38)}{18.81} \times 138.50 \left( \lambda_{t-7} - \underset{(.0004)}{.0002} \right) \right] \right)^{-1}\end{aligned}$$

$$R^2 = 0.79 \qquad SE = 0.007 \qquad LM(9) = 0.93\ (0.51)$$
$$ARCH(1) = 7.17\ (0.01)\ BJ = 0.093\ (0.95)\ LIN(\rho_{t-8}) = 2.26(0.01)$$

The results of the estimations are shown in figure 4.2.

As can be observed simulating the model with the estimated parameter values gives us the above shown figure. Details of the results are discussed in Koçkesen and Semmler (1998).

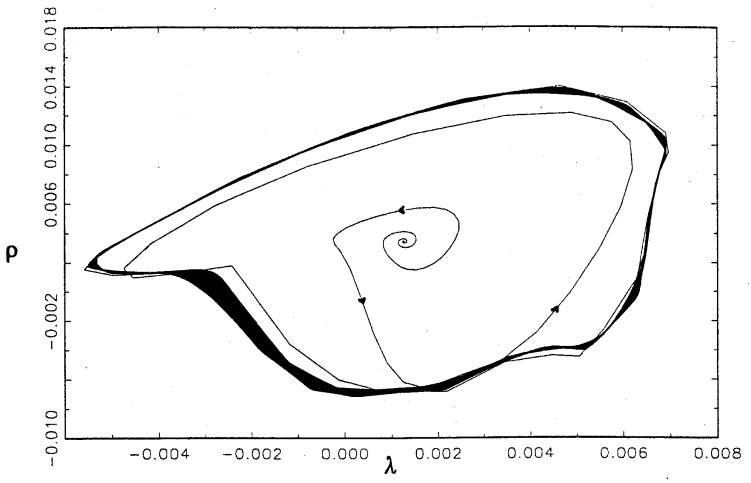



**Fig. 4.2.** Dynamics from a Threshold Model

### 4.3.3 Conclusions

Threshold models are very suitable to model nonlinear relationships between economic variables. The threshold methodology has found application in numerous other fields in economics, see the contributions in Rothman (1999), see also Granger and Teräsvirta (199´3). We have shown that there is convincing evidence for nonlinearities in the financial and real interaction, in particular, as studied in the interaction of liquidity and output. Thus nonlinearity might be especially relevant for the relationship of liquidity and output since short term credit is usually tightly connected to agents' balance sheet variables, e.g. short- and long-term debt, leverage and physical capital or liquid assets as collateral. All of them move with the level of economic activity and economic activity in turn is significantly impacted by credit conditions. There are likely to be thresholds that play a role in this interaction. Yet, from the long term perspective agents are usually screened by the lenders or credit agencies whether their intertemporal budget constraint is fulfilled. We now turn to the problem of how one can evaluate whether the agent's intertemporal budget constraint is not violated.

## 4.4 Estimations of Credit Risk and Sustainable Debt

### 4.4.1 Introduction

As aforementioned sustainable debt has to be discussed in an intertemporal context. Economic agents (households, firms, governments and countries) are creditworthy as long as the present value of their income does not fall short of the liabilities

that the agents face. Credit rating firms evaluate permanently the creditworthiness of creditors.[56] Debt sustainability and creditworthiness was at the root of the Asian financial crisis. A credit crisis can in fact trigger a financial crisis and large output losses. [57]

In this section we want to study and evaluate credit risk in the context of a dynamic economic model and propose an empirical test. More specifically we want to study borrowing capacity, creditworthiness and credit risk in the context of an economic growth model. In order to simplify matters we do not employ a stochastic version of a dynamic model but rather employ a deterministic framework.[58] Yet, our study might still be important for the issues of credit risk and management that have kept the attention of the financial economists since the Asian financial crisis.

Here we do not extensively discuss the diverse empirical variables and methods to evaluate credit risk and to compute default risk of bonds (see Benninga 1998, Chap. 17). Those methods are very useful in practice but have only little connection to a theory of credit risk and theoretical measures of creditworthiness. Measuring credit risk is also important in risk management and the value at risk approach. The latter approach works with expected volatility of asset prices ( for a survey, see Duffie and Pan 1997). Although our study has implications for credit risk analysis in empirical finance literature and risk management our approach is more specifically related to the literature that links credit market and economic activity in the context of intertemporal models. In recent times this link has been explored in numerous papers that take an intertemporal perspective.

In one type of paper, mostly assuming perfect credit markets, it is assumed that, roughly speaking, agents can borrow against future income as long as the discounted future income, the wealth of the agents, is no smaller than the debt that agents have incurred. In this case there is no credit risk whenever the non-explosiveness condition holds. Positing that the agents can borrow against future income, the non-explosiveness condition is equivalent to the requirement that the intertemporal budget constraint holds for the agents. Formally, the necessary conditions for optimality, derived from the Hamiltonian equation, are often employed to derive the dynamics of the state variables and the so called transversality condition is used to provide a statement on the non-explosiveness of the debt of the economic agents. Models of this type have been discussed in the literature for households, firms, governments and small open economies (with access to international capital markets).[59]

In a second type of paper, and also often in practice, assuming credit market imperfections, economists presume that borrowing is constrained. Frequently, borrowing ceilings are assumed which are supposed to prevent agents from borrowing an unlimited amount. Presuming that the agents' assets serve as collateral, a convenient way to define the debt ceiling is to then assume the debt ceiling to be a fraction of the

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[56] For a detailed description of credit rating practices, see Benninga (1998), Chap. 17.

[57] See the work by Milesi-Ferreti and Razin (1996).

[58] A stochastic version can be found in Sieveking and Semmler (1999).

[59] For a brief survey of such models for households, firms and governments or countries, see Blanchard and Fischer (1989, Chap.2) and Turnovsky (1995).

agents' wealth. The definition of debt ceilings have become standard, for example, in a Ramsey model of the firm, see Brock and Dechert (1985) or in a Ramsey growth model for small open economies; see, for example, Barro, Mankiw and Sala-i-Martin (1995). It has also been pointed out that banks often define debt ceilings for their borrowers, see Bhandari, Haque and Turnovsky (1990).

A third type of literature also assumes credit market imperfections but employs endogenous borrowing costs such as in the work by Bernanke and Gertler (1989, 1994) and further extensions to heterogeneous firms, such as small and large firms, in Gertler and Gilchrist (1994). State dependent borrowing costs have been associated with the financial accelerator theory. Here one presupposes only a one period zero horizon model and then shows that due to an endogenous change of a firm's net worth, as collateral for borrowing, credit cost is endogenous. For potential borrowers their credit cost is inversely related to their net worth. In parallel other literature has posited that borrowers may face a risk dependent interest rate which is assumed to be composed of a market interest rate (for example, an international interest rate) and an idiosyncratic component determined by the individual degree of risk of the borrower. Various forms of the agent specific risk premium can be assumed. Here, it is often posited to be endogenous in the sense that it is convex in the agent's debt.[60]

Recent extensions of the third type of work have been undertaken by embedding credit market imperfections and endogenous borrowing cost more formally in intertemporal models such as the standard stochastic growth model, see Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1998). Some of this literature has also dealt with the borrowing constraints of heterogeneous agents (households, firms) in an intertemporal general equilibrium framework. Although in our paper we stress intertemporal behavior of economic agents, we will not deal with the case of heterogeneous agents here.

We present a dynamic model with credit markets and asset prices that can be perceived as holding (true) for single agents or a country. In fact the set up of the model is undertaken in a way that reflects a country borrowing from abroad. Empirically we estimate instead the sustainability of foreign debt or assets of Euro-area countries where we readily have sufficient time series data available.

### 4.4.2 The Dynamic Model

First we give a formal presentation of the model that we want to estimate. In a contract between a creditor and debtor there are two problems involved. The first pertains to the computation of debt and the second to the computation of the debt ceiling. The first problem is usually answered by employing an equation of the form

$$\dot{B}(t) = \theta B(t) - f(t), \qquad B(0) = B_0$$

[60] The interest rate as function of the default risk of the borrower is posited by Bhandari, Haque and Turnovsky (1990) and Turnovsky (1995).

where $B(t)$ is the level of debt[61] at time $t$, $\theta$ the interest rate determining the credit cost and $f(t)$ the net income of the agent. The second problem can be settled by defining a debt ceiling such as

$$B(t) \leq C, \qquad (t > 0)$$

or less restrictively by

$$\sup_{t \geq 0} B(t) < \infty$$

or even less restrictively by the aforementioned transversality condition

$$\lim_{t \to \infty} e^{-\theta t} B(t) = 0. \tag{4.5}$$

The ability of a debtor to service the debt, i.e. the feasibility of a contract, will depend on the debtors source of income. Along the lines of intertemporal models of borrowing and lending[62] we model this source of income as arising from a stock of capital $k(t)$, at time $t$, which changes with THE investment rate $j(t)$ at time $t$ through

$$\dot{k}(t) = j(t) - \sigma\left(k(t)\right), \quad k(0) = k_0. \tag{4.6}$$

In our general model both the capital stock and the investment are allowed to be multivariate. As debt service we take the net income from the investment rate $j(t)$ at capital stock level $k(t)$ minus some minimal rate of consumption.[63] Hence

$$\dot{B}(t) = \theta B(t)) - f\left(k(t), j(t)\right), \; B(0) = B_0 \tag{4.7}$$

where $\theta B(t)$ is the credit cost. Note that the credit cost is not necessarily a constant factor (a constant interest rate). We call $B^*(k)$ the creditworthiness of the capital stock $k$. The problem to be solved is how to compute $B^*$.

If there is a constant credit cost factor (interest rate), $\theta = \frac{H(B,k)}{B}$, then, it is easy to see, $B^*(k)$ is the present value of $k$ or the asset price of $k$:

$$B^*(k) = \underset{j}{Max} \int_0^\infty e^{-\theta t} f\left(k(t), j(t)\right) dt - B(0) \tag{4.8}$$

---

[61] Note that all subsequent state variables are written in terms of efficiency labor along the line of Blanchard (1983).

[62] Prototype models used as basis for our further presentation can be found in Blanchard (1983), Blanchard and Fischer (1989) or Turnovsky (1995).

[63] In the subsequent analysis of creditworthiness we can set consumption equal to zero. Any positive consumption will move down the creditworthiness curve. Note also that public debt for which the Ricardian equivalence theorem holds, i.e. where debt is serviced by a non-distortionary tax, would cause the creditworthiness curve to shift down. In computing the "present value" of the future net surpluses we do not have to assume a particular interest rate. Yet, in the following study we neither elaborate on the problem of the price level nor on the exchange rate and its effect on net debt and creditworthiness.

s.t.

$$\dot{k}(t) = j(t) - \sigma\left(k(t)\right), \quad k(0) = k_s \tag{4.9}$$

$$\dot{B}(t) = \theta B(t) - f\left(k(t), j(t)\right), \quad B(0) = B_0. \tag{4.10}$$

The more general case is, however, that $\theta$ is not a constant. As in the theory of credit market imperfections we generically may let $\theta$ depend on $k$ and $B$.[64] Employing a growth model in terms of efficiency labor[65] we can use the following net income function that takes account of adjustment investment and adjustment cost of capital.

$$f(k, j) = k^{\alpha} - j - j^{\beta} k^{-\gamma} \tag{4.11}$$

where $\sigma > 0, \alpha > 0, \gamma > 0$ are constants.[66] In the above model $\sigma > 0$ captures both a constant growth rate of productivity as well as a capital depreciation rate and population growth.[67] Blanchard (1983) used $\beta = 2$, $\gamma = 1$ to analyze the optimal indebtedness of a country (see also Blanchard and Fischer 1989, Chap. 2).

Note that in the model (4.8)-(4.10) we have not used utility theory. However, as shown in Sieveking and Semmler (1998) the model (4.8)-(4.10) exhibits a strict relationship to a growth model built on a utility function, for example, such as[68]

$$Max \quad \int_0^{\infty} e^{-\theta t} u\left(c(t), k(t)\right) dt \tag{4.12}$$

s.t.

$$\dot{k}(t) = j(t) - \sigma\left(k(t)\right), \qquad k(0) = k. \tag{4.13}$$

$$\dot{B}(t) = \theta B(t) - f\left(k(t), j\right) + c(t), \qquad B(0) = B \tag{4.14}$$

with the transversality condition

$$\lim_{t \to \infty} e^{-\theta t} B(t) = 0 \tag{4.15}$$

which often turns up in the literature[69] among the "necessary conditions" for a solution of a welfare problem such as (4.12)-(4.15). In Sieveking and Semmler (1998) it

[64] The more general theory of creditworthiness with state dependent credit cost is provided in Grüne, Semmler and Sieveking (2004). Note that instead of relating the credit cost inversely to net worth, as in Bernanke, Gertler and Gilchrist (1998), one could use the two arguments, $k$ and $B$, explicitly.

[65] The subsequent growth model can be viewed as a standard RBC model where the stochastic process for technology shocks is shut down and technical change is exogenously occurring at a constant rate.

[66] Note that the production function $k^{\alpha}$ may have to be multiplied by a scaling factor. For the analytics we leave it aside here.

[67] For details, see Blanchard (1983).

[68] For details, see Blanchard (1983).

[69] See, for example, Bhandari, Hague and Turnovsky (1990). In our framework the equivalent transversality condition will be

$$\sup_{t \geq 0} B(t) < \infty$$

.

is shown that the problem (4.12)-(4.15) can be separated into two problems. The first problem is to find optimal solutions that generate the present value of net income flows and the second problem is to study the path of how the present value of net income flows are consumed. There also, conditions are discussed under which such separation is feasible. The separation into those two problems appears to be feasible as long as the evolution of debt does not appear in the objective function. If such separation is feasible we then only need to be concerned with the model (4.8)-(4.10). Yet instead of maximizing a utility function, the present value of a net income function is maximized.

The maximization problem (4.8)-(4.10) can be solved by using the necessary conditions of the Hamiltonian for (4.8)-(4.9). Thus we maximize

$$\underset{j}{Max} \int_0^\infty e^{-\theta t} f(k(t), j(t)) dt$$

s.t. (4.9).

The Hamiltonian for this problem is

$$\begin{aligned} H(k, x, j, \lambda) &= \max_j H(k, x, j, \lambda) \\ H(k, x, j, \lambda) &= \lambda f(k, j) + x(j - \sigma k) \\ \dot{x} = \frac{-\partial H}{\partial k} + \theta x &= (\sigma + \theta)\, x - \lambda f_k(k, j). \end{aligned}$$

We denote $x$ as the co-state variable in the Hamiltonian equations and $\lambda$ is equal to 1.[70] The function $f(k, j.)$ is strictly concave by assumption. Therefore, there is a function $j(k, x)$ which satisfies the first order condition of the Hamiltonian

$$f_j(k, j) + x = 0 \tag{4.16}$$

$$j = j(k, x) = \left(\frac{x - 1}{k^{-\gamma} \cdot \beta}\right)^{\frac{1}{\beta - 1}} \tag{4.17}$$

and $j$ is uniquely determined thereby. It follows that $(k, x)$ satisfy

$$\dot{k} = j(k, x) - \sigma k \tag{4.18}$$

$$\dot{x} = (\sigma + \theta)x - f_k(k, j(k, x)) \tag{4.19}$$

The isoclines can be obtained by the points in the $(k, x)$ space for $\beta = 2$ where $\dot{k} = 0$ satisfies

$$x = 1 + 2\sigma k^{1-\gamma} \tag{4.20}$$

and where $\dot{x} = 0$ satisfies

$$x_\pm = 1 + \vartheta k^{1-\gamma} \pm \sqrt{\vartheta^2 k^{2-2\gamma} + 2\vartheta k^{1-\gamma} - 4\alpha\gamma^{-1} k^{\alpha-\gamma}} \tag{4.21}$$

[70] For details of the computation of the equilibria in the case when one can apply the Hamiltonian, see Semmler and Sieveking (1998), appendix.

where $\vartheta = 2\gamma^{-1}(\sigma + \theta)$. Note that the latter isocline has two branches.

If the parameters are given, the steady state – or steady states, if there are multiple ones – can be computed and then the local and global dynamics studied. We scale the production function by $\alpha$ [71] and take $c = 0$. We employ the following parameters: $\alpha = 1.1, \gamma = 0.3, \sigma = 0.15, \theta = 0.1$.

For those parameters, using the Hamiltonian approach, there are two positive candidates for equilibria. The two equilibrium candidates are: (HE1): $k^* = 1.057$, $\mathrm{x}^* = 1.3$ and (HE2): $k^{**} = 0.21$, $\mathrm{x}^{**} = 1.1$. A third equilibrium candidate is $k = 0$.[72]

### 4.4.3 Estimating the Parameters

Next, we want to take our growth model with adjustment costs of investment to the data. It would be interesting to pursue this with time series data for Asian countries before the financial crisis 1997-98. Yet, there are no reliable long-term data sets available. We will thus use quarterly data from Euro-area countries. We could generate time series data for the relevant variables for most of the core countries of the Euro-area. For the purpose of parameter estimation we have to transform our dynamic equations into estimable equations. By presuming the version, where only a constant credit factor enters the debt equation, we can employ the Hamiltonian equation. This is justified in the case of Euro-area countries, since there are likely to be no severe idiosyncratic risk components in the interest rate. We can transform the system into estimable equations and employ time series data on capital stock and investment – all expressed in efficiency units – to estimate the involved parameter set.

Substituting the optimal investment rate (4.17) into (4.18) we get the following two dynamic equations

$$\dot{k} = \left(\frac{x-1}{k^{-\gamma}\cdot\beta}\right)^{\frac{1}{\beta-1}} - \sigma k \tag{4.22}$$

$$\dot{x} = (\sigma+\theta)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1)}. \tag{4.23}$$

Next, we transform the above system into observable variables so that we obtain estimable dynamic equations.

From (4.22) we obtain

$$\hat{k} = j/k - \sigma \tag{4.24}$$

with $\overset{\wedge}{k} = \dot{k}/k$. Note that from the determination of $j$ in (4.22) we can get

$$x = 1 + \beta j^{\beta-1}k^{-\gamma}. \tag{4.25}$$

---

[71] We have multiplied the production function by $a = 0.30$ in order to obtain sufficiently separated equilibria.

[72] We want to stress again that from the Hamiltonian equation one can only obtain candidates for equilibria.

Taking the time derivative with respect to $j$ we obtain[73]

$$\dot{x} = \left(\beta\left(\beta-1\right)j^{\beta-2}k^{-\gamma}\right)\cdot\dot{j} \tag{4.26}$$

and using (4.23) we have

$$\left(\beta\left(\beta-1\right)j^{\beta-2}k^{-\gamma}\right)\cdot\dot{j} = \left(\sigma+\theta\right)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1).}$$

Thus

$$\dot{j} = \frac{\left(\sigma+\theta\right)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1)}}{\beta\left(\beta-1\right)j^{\beta-2}k^{-\gamma}} \tag{4.27}$$

or

$$\overset{\wedge}{j} = \frac{\dot{j}}{j} = \left(\frac{\left(\sigma+\theta\right)x - \alpha k^{\alpha-1} - j^{\beta}\gamma k^{(-\gamma-1)}}{\beta\left(\beta-1\right)j^{\beta-2}k^{-\gamma}}\right)/j \tag{4.28}$$

Substituting (4.25) into (4.27) we get as estimable equations in observable variables (4.24) and (4.28) which depend on the following parameter set to be estimated.

$$\varphi = (\theta, \sigma, \beta, \gamma, \alpha, a)$$

The estimation of the above parameter set is undertaken by aggregating capital stock and investment for the core countries of the Euro-area. The data are quarterly data from 1978.1 - 1996.2. Although aggregate capital stock data, starting from 1970.1 are available, we apply our estimation to the period 1978.1 - 1996.2. This is because the European Monetary System was introduced in 1978 whereby the exchange rates between the countries where fixed within a band. This makes the cross-country aggregation of capital stock and investment feasible. The aggregate capital stock series is gross private capital stock and the investment series is total fixed investment. Both are taken from the OECD data base (1999). The series for gross capital stock and investment represent aggregate real data for Germany, France, Italy, Spain, Austria, Netherlands and Belgium. Since we are employing a model on labor efficiency each country's time series for capital stock and investment is scaled down by labor in efficiency units measured by the time series $L_t = L_0 e^{(n+g_{y/l})t}$ where $n$ is average population growth and $g_{y/l}$ average productivity growth. As to our estimation strategy we employ NLLS estimation and use a constrained optimization procedure.[74] The results are shown in Table 4.4.

**Table 4.4.** Parameter Estimates for Euro-Area Countries (1978.1-1996.2)

| $\theta$ | $\sigma$ | $\beta$ | $\gamma$ | $\alpha$ | $a$ |
|---|---|---|---|---|---|
| 0.035 | 0.092 | 0.312 | 0.116 | 0.385 | 3.32 |

[73] Note that in order to obtain a simple estimable equation we only take the time derivative with respect to $j$.

[74] The estimations were undertaken in GAUSS for which the constrained optimization procedure recently provided by GAUSS was used.

The parameters obtained from the historical data are quite reasonable.[75] Overall one can observe that the adjustment costs of investment are not very large since the exponents $\beta$ and $\gamma$ are small.

Using the estimated parameters one can again compute through (4.16) - (4.20) the steady states for the capital stock. Doing so numerically, it turns out that for our parameter estimates (table 4.4) the steady state is unique and we obtain a $k^* = 37.12$. This coincides, roughly, with the mean of the historical series of capital stock for Euro-area countries. This gives a steady state net income of $f(k, j) = 8.799$, computed from (4.11) at the steady state of $k^* = 37.12$. Moreover, for the present value of the net income at the steady state we obtain $V(k^*) = 244.4193$.

Using the estimated parameters figure 4.3 shows the computed output, investment (including adjustment costs of investment) and the net income.

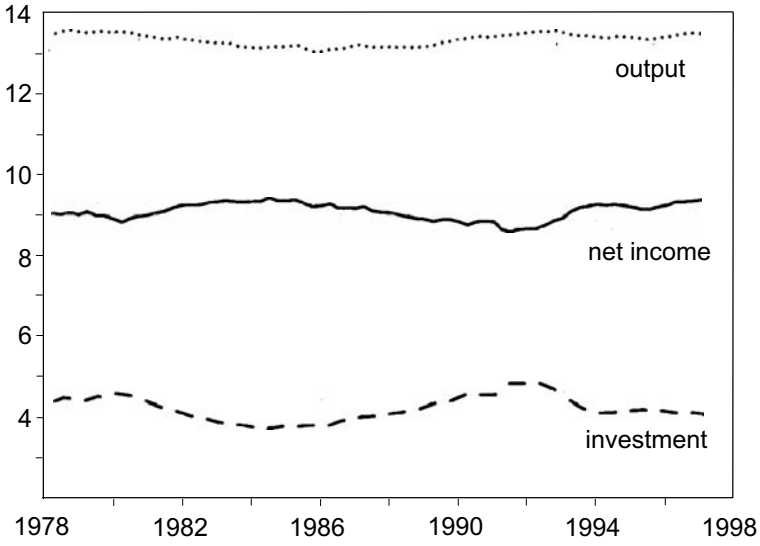



**Fig. 4.3.** Net Income, Investment and Output

As figure 4.3 shows, since we are using aggregate variables in efficiency units, the output in efficiency units tends to be stationary and the net income moves inversely to investment (the latter including adjustment costs).

Finally, note that with those parameter estimates given in table 4.4 we could now easily compute the present value outside the steady state and thus the critical debt curve by using the Hamilton-Jacobi-Bellman equation, see Grüne, Semmler and Sieveking (2004). Since, however, there is no external debt of Euro-area countries but rather external assets, as shown in the next section, the result of such an exercise

[75] We want to note that standard errors could not be recovered since the Hessian in the estimation was not non-negative definite.

will not be very instructive. The balance sheets of banks and firms, as discussed in Krugman (1999a,b) and Mishkin (1998), will presumably show no sign of deterioration, since the Euro-area countries have net claims vis-a-vis the rest of the world. Our methods of computing the present value of net income could, however, be fruitfully undertaken for other countries with deteriorating external debt and balance sheets of banks and firms.[76] Note, however, that the above method gives us only asymptotic results, i.e. if $t \to \infty$. Next, for Euro-area countries we pursue another method – for a finite number of observations – to compute the sustainability of external assets.

### 4.4.4 Testing Sustainability of Debt

Next, following Flood and Garber (1980) and Hamilton and Flavin (1986) a NLLS estimate for the sustainability of external debt can be designed for a finite number of observations. Similar to the computation of the capital stock and investment for our core countries of Euro-area countries we have computed the trade account, the current account and the net foreign assets of those core countries for the time period 1978.1-1998.1. Since we want to undertake sustainability tests for certain growth regimes, we have computed monthly observations. In our computation we had to eliminate the trade among the Euro-area countries.[77] We consider the time series for the entire period 1978.1-1998.12 and in addition subdivide the period into two periods 1978.1-1993.12. and 1994.1-1998.12. The break in 1994 makes sense since the exchange rate crisis of September 1992 lead to a reestablishment of new exchange rates with a wider band in 1993. Thus, the sustainability tests will be undertaken for those two subperiods.

In a discrete version the foreign debt can be computed as follows. Starting with initial debt $B_0$ one can compute in a discrete time way the stock of debt as follows. By assuming a constant interest rate we have

$$B_t = (1 + r_{t-1})B_{t-1} - TA_t \tag{4.29}$$

where $TA_t$ is the trade account and $B_{t-1}$ the stock of foreign debt at period $t-1$ and $r_{t-1}$ the interest rate. As interest rate we took the Libor rate. The initial stock of foreign debt $B_0$ for 1978.1 was estimated. This way, the entire time series of external debt and trade account could be generated.

From equ. (4.29) we can develop a discrete time sustainability test. For reason of simplicity let us assume a constant interest rate. Equ. (4.29) is then a simple first

[76] Of course, one would have to consider also the exchange rate regime under which the country borrows and in particular the fact whether the country (banks, firms) borrows in foreign currency. In this case an exchange rate shock will exacerbate the deterioration of the balance sheets, see Mishkin (1998) and Krugman (1999 a,b).

[77] A similar attempt to compute external debt of countries and regions, following a similar methodology as suggested above, has been recently undertaken by Lane and Milesi-Ferreti (1999). Their results for the Euro-area core countries show similar trends as our computation. Their results are, however, less precise since they do not eliminate intra-Euro-area countries trade.

order difference equation that can be solved by recursive forward substitution leading to

$$B_t = \sum_{i=t+1}^{N} \frac{TA_i}{(1+r)^{i-t}} + \frac{(1+r)^t B_N}{(1+r)^N}. \tag{4.30}$$

In the equ. (4.30) the second term must go to zero if the intertemporal budget constraint is supposed to hold. Then equ. (4.30) means that the current value of debt is equal to the expected discounted future trade account surplus

$$B_t = E_t \sum_{i=t+1}^{\infty} \frac{TA_i}{(1+r)^{i-t}}. \tag{4.31}$$

Equivalent to requiring that equ. (4.31) be fulfilled, is the following condition:

$$E_t \lim_{N\to\infty} \frac{B_N}{(1+r)^N} = 0. \tag{4.32}$$

The equation is the usual transversality condition or No-Ponzi game condition as discussed in section 4.4.2.

If the external debt is constrained not to exceed a constant, $A_{0,}$ on the right hand side of (4.30), we then have

$$B_t = E_t \sum_{i=t+1}^{\infty} \frac{TA_i}{(1+r)^{i-t}} + A_0(1+r)^t \tag{4.33}$$

The NLLS test proposed by Flood and Garber (1980) and Hamilton and Flavin (1986) and Greiner and Semmler (1999) can be modified for our case. It reads:

$$TA_t = b_1 + b_2 TA_{t-1} + b_3 TA_{t-2} + b_4 TA_{t-3} + \varepsilon_{2t} \tag{4.34}$$

$$\begin{aligned} B_t = {} & b_5(1+r)^t + b_6 + \frac{(b_2 b + b_3 b^2 + b_4 b^3)TA_t}{(1 - b_2 b - b_3 b^2 - b_4 b^3)} \\ & + \frac{(b_3 b + b_4 b^2)TA_{t-1}}{(1 - b_2 b - b_3 b^2 - b_4 b^3)} + \frac{(b_4 b)TA_{t-2}}{(1 - b_2 b - b_3 b^2 - b_4 b^3)} + \varepsilon_{1t} \end{aligned} \tag{4.35}$$

with $b = \frac{1}{1+r}$. We want to note, however, that following Wilcox (1989) it might be reasonable to compute trade account surplus and debt series as discounted time series. We have also undertaken the computation of those discounted time series by discounting both the trade account and the external debt with an average interest rate and performed the above (4.34)-(4.35) sustainability test.

Figure 4.4 shows the undiscounted and discounted time series for external assets of the Euro-area.

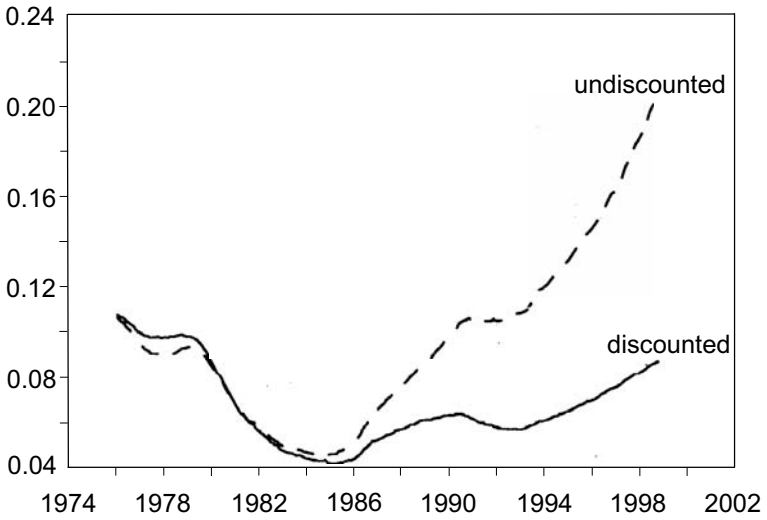



**Fig. 4.4.** Undiscounted and Discounted Net Foreign Assets

**Table 4.5.** Sustainability Test of Euro Debt, 1978.1-1998.12

| | undiscounted | | discounted | |
|---|---|---|---|---|
| Param | Estim | t-stat | Estim | t-stat |
| $b_1$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $b_2$ | 0.76 | 0.05 | 0.53 | 0.04 |
| $b_3$ | 0.45 | -0.02 | 0.37 | 0.02 |
| $b_4$ | -0.51 | -0.05 | -0.06 | -0.07 |
| $b_5$ | -0.07 | -1.20 | -0.002 | -0.04 |
| $b_6$ | 0.0051 | 0.06 | -0.064 | -0.88 |

Table 4.5 reports test results for both types of time series for the entire time period 1978.1-1998.12.

Table 4.6 reports our estimation results for subperiods again for both undiscounted and discounted trade account and debt service. The results of estimation of the coefficients as to the relevance of non-sustainability of foreign assets for the Euro-area are not very conclusive. The coefficient $b_5$, which is the relevant coefficient in our context, has the correct sign but is always insignificant.

Next we compute the estimate (4.34)-(4.35) for the two subperiods. Table 4.6 reports the results for undiscounted and discounted variables respectively.

**Table 4.6.** Sustainability Test of Foreign Debt for Euro-Area Countries, 1978.1-1993.12 and 1994.1-1998.12

| | undiscounted | | | | discounted | | | |
|---|---|---|---|---|---|---|---|---|
| | 1978.1-1993.12 | | 1994.1-1998.12 | | 1978.1-1993.12 | | 1994.1-1998.12 | |
| | Estim | t-stat | Estim | t-stat | Estim | t-stat | Estim | t-stat |
| $b_1$ | 0.00 | 0.00 | 0.001 | 0.53 | 0.00 | 0.00 | 0.001 | 0.39 |
| $b_2$ | 0.423 | 0.04 | 0.308 | 0.16 | 0.498 | 0.04 | 0.365 | 0.14 |
| $b_3$ | 0.338 | 0.02 | -0.200 | -0.10 | 0.401 | 0.03 | -0.266 | -0.09 |
| $b_4$ | 0.048 | 0.01 | -0.318 | -0.19 | -0.062 | -0.01 | -0.548 | -0.22 |
| $b_5$ | -0.042 | -0.72 | -0.203 | -25.86 | 0.018 | 0.30 | -0.061 | -13.08 |
| $b_6$ | -0.025 | -0.32 | 0.277 | 17.61 | -0.086 | -1.07 | 0.056 | 5.91 |

As can clearly be seen from the coefficient $b_5$ both the undiscounted and the discounted trade time series show a rapid built-up of net foreign assets of the Euro-area that do not seem to be sustainable. Our tests imply there is a build-up of foreign assets that particularly occurred after the currency crisis 1992-1993.

In sum, we have shown, here, that sustainable debt in models with borrowing and lending may typically be state constrained. In order to control credit risk the lender needs to know the debt capacity of the borrower at each point in time. This knowledge seems to be necessary if one wants to move beyond an one period debt contract. We explore the problem of critical debt and creditworthiness by applying the Hamiltonian. Using those methods we analytically and numerically can demonstrate the region in which the borrower remains creditworthy. Imposing a ceiling on borrowing may lead to a loss of welfare if it is set to low. On the other hand, if the ceiling is set too high the non-explosiveness condition may not hold and creditworthiness may be lost.

By using this method we study the debt capacity of a borrower, the role of debt ceilings for lending and borrowing behavior.[78] The computation of these creditworthiness curves serves to determine sustainable debt for any initial capital stock $k_0$ and thus to control credit risk for any point in time. We note that there are, of course, numerous empirical approaches to control for credit risk by approximating sustainable debt by empirical indicators.[79] Our attempt was, however, to show how one can compute sustainable debt based on a dynamic economic model without having to refer to the numerous indicators for credit risk that rating companies use.

[78] We also can show that multiple equilibria may arise if there are state dependent credit costs. The multiple equilibria may be important since there may be cut-off points which decide whether the agent or the economy moves to high or low level steady states.

[79] In a series of papers Milesi-Ferretti and Razin (1996, 1997, 1998) have addressed the empirical issue of how to obtain proxies for measuring sustainable debt. Whether or not the agent can keep the debt under control also depends on the agent's reaction to rising debt ratios. The reaction may be time or state dependent. A sustainability test with time varying response to debt is presented in Greiner, Köller and Semmler (2005).

## 4.5 Conclusions

In this chapter we have undertaken some empirical tests on credit market and economic activity. After introducing some linear regression tests on bankruptcy risk and economic activity we have tested for nonlinear relationships in the financial-real interaction. Both types of tests have used U.S. time series data. It seems to us that, in particular, nonlinear relationships are present in the short-run. These can be detected by looking at short time scales in the relationship between liquidity and output.

We then presented an intertemporal model with a credit market and applied it to an open economy problem where a country borrows from abroad. We estimate that model with time series data. We have managed to transform the dynamic model into an empirical model so that it can be taken to the data. Given the parameter estimates we can, for actual economies, compute the borrowing capacity and debt ceiling of actual countries that should hold for the long-run. This was undertaken for the core countries of the Euro-area. We have also shown that one can compute the sustainability of debt for actual economies by using time series methods. As it turns out the result for the Euro-area is that the Euro-area does not have liabilities but rather owns net assets vis-a-vis the rest of the world.[80]

Finally we note that our time series methods (Chap. 4.4.) to compute sustainable debt could be applied to any agent's debt. It could be a firm, a household, a government or country. An application of the above proposed time series methods to government debt is given in Greiner and Semmler (1999) and Greiner, Köller and Semmler (2005). Debt sustainability is an important issue in credit rating of private and sovereign debt and the above method can be applied to provide estimates of the long-run debt sustainability and credit risk. A practical empirical method to evaluate credit risk and its impact on returns, using Markov transition matrices, can be found in Benninga (1998).

[80] The implication of this computation is that the Euro as a currency should be rather stable in the long-run since currency runs should not be expected.

CHAPTER 5

# Approaches to Stock Market and Economic Activity

## 5.1 Introduction

The interaction of the stock market and economic activity has recently become an important topic in empirical finance as well as in macroeconomic research. The research has pursued two directions.

A large number of papers have studied the impact of the stock market on real activity. Here particular emphasis is given to the relationship of the volatility of the stock market and output. The research studies the impact of wealth, as evaluated on the stock market, on borrowing, lending and spending behavior of banks, firms and households. The argument may go like this. An increase in wealth through the appreciation of stocks increases spending directly since people feel wealthier. At the same time the appreciation of stocks increases the collateral for borrowing by firms and households. Credit may expand and thus spending is likely to increase. A depreciation of stocks lets spending decrease and devaluates the collateral and credit contractions followed by large output loss may occur. Thus, large stock price swings can easily be seen to impact economic activity. Often Tobin's Q is employed to study this impact of stock market appreciation or depreciation on firm investment. Of course, other financial variables such as interest rates, interest rate spreads, the term structure of the interest rates and credit constraints, as discussed in Chaps. 1-4 are also important for household and firm spending. Thus, beside the real variables, asset prices and financial variables are also important for economic activity and, moreover, have often been good predictors for turning points in economic activity and business cycles.[81]

On the other hand, another important line of research is to show how real activity affects asset prices and returns. Often, proxies for economic fundamentals are employed to show that fundamentals drive stock prices and returns. The two main important variables for stock prices are the expected cash flows (and dividend payments) of firms and discount rates. Both are supposed to determine asset prices in a fundamental way. Empirical researchers have used numerous macroeconomic variables as proxies for news on expected returns, future cash flows and discount rates.[82] In addition variables with leads and lags are studied for their impact on asset pricing

[81] For details of the role of financial variables as predictors for business cycles, see Friedman and Kuttner (1992), Stock and Watson (1989), Estrella and Hardouvilis (1991), Estrella and Mishkin (1997) and Plosser and Rouwenhorst (1994), Lettau and Ludvigson (2001 a,b, 2002).

[82] See Fama and French (1988), Fama and French (1989), Fama (1990).

and returns. In general, econometric literature has shown that good predictors of stock prices and returns have proved to be dividends, earnings and growth rate of real output[83] Moreover, financial variables such as interest rate spread and the term structure of interest rates have also been significant in predicting stock prices and stock returns (Fama (1990), Schwert (1990)). Other balance sheet variables, such as firms' leverage ratio, net worth and liquidity have also successfully been employed (Schwert 1990).

Presently discussed approaches in the empirical literature have primarily stressed either of the above mentioned two strands of research. Subsequently we will present some approaches, the relevant stylized facts for those approaches and some empirical results of the studies. Thereafter, we will present models that deal with the interaction of macroeconomic factors and the stock market. We will also discuss some empirical results on such models as well.

## 5.2 The Intertemporal Approach

Currently, the best known approach is the market efficiency hypothesis. Theoretically it is based on the capital asset pricing model (CAPM) and its extension to a multi-period consumption based capital asset pricing model (CCAPM). Details of these models will be presented in Chaps. 9-10 and Chaps. 15-16. In terms of economic models researchers, nowadays, often employ the capital asset pricing model for production economies, a stochastic optimal growth model of RBC type, for studying the relationship between asset markets and real activity. Intertemporal decisions are at the heart of the RBC methodology and it is thus natural to study the asset market-output interaction in the context of such a model since it also includes production. Some advances have been made by using stochastic growth models to predict asset prices and returns, see Chaps. 10 and 15. Here short summaries of stylized facts frequently cited in connection with this approach as well as a survey of empirical results, may suffice. Details are postponed to Chaps. 10 and 15. The intertemporal equilibrium model is often measured against the stylized facts as reported in table 5.1.

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[83] See Fama and French (1988), Fama and French (1989), Fama (1990) and to some extent inflation rates, Schwert (1989).

[84] For the U.S. real variables are measured in growth rates, 1970.1-1993.3. Data are taken from Canova and Nicola (1995). Asset market data represent real returns and are from Lettau, Gong and Semmler (2001) and represent 1947.1-1993.3. All data are of quarterly frequency. Asset market units are per cent per quarter. The T-bill rate is the 3 month T-bill rate. The Sharpe-ratio is the mean of equity divided by it's standard deviation. For Europe real variables are also measured in growth rates, 1970.1-1993.3. Data are taken from Eurostat (1997). Following Canova and Nicola (1995) for each of the variables a European variable is obtained by employing a weighted average of the respective variables for Germany, France, Italy and the U.K, where GNP ratios are taken as the weight. This holds also for the 3 month T-bill rate. In the case of the U.K. the T-bill rate was obtained by averaging short term rates.

**Table 5.1.** Stylized Facts on Real Variables and Asset Markets: U.S. and European Data[84]

| Variable | U.S. | | Europe | |
|---|---|---|---|---|
| | mean | std.dev. | mean | std.dev |
| GNP | | 0.97 | | 0.65 |
| Consumption | | 0.77 | | 0.61 |
| Investment | | 2.88 | | 1.40 |
| Employment | | 0.46 | | 0.32 |
| T-bill | 0.18 | 0.86 | 0.43 | 0.89 |
| Stock-return | 2.17 | 7.53 | 1.81 | 7.37 |
| Equity premium | 1.99 | 7.42 | 1.38 | 7.04 |
| Sharpe-ratio | 0.27 | | 0.19 | |

Recently it has become customary to contrast historical time series with a model's time series and to demonstrate to what extent the model's time series can match historical data. Models are required to match statistical regularities of actual time series in terms of the first and second moments and the cross correlation with output.

In the above table we present summary statistics of time series for U.S. and Europe on GNP, consumption, investment, employment, the treasury bill rate, equity return and the Sharpe-ratio. The latter measure of financial market performance has recently become a quite convenient measure to match theory and facts, since, as a measure of the risk-return trade-off, the Sharpe-ratio captures both excess returns and excess volatility.[85] Yet, we want to mention that the Sharpe-ratio might also be time varying. This will be discussed in Chaps. 9-10 and Chap. 15.

As shown in table 5.1, the hierarchy of volatility measured by the standard deviation is common for U.S. as well as European data. As shown, stock returns exhibit the strongest volatility. The second strongest volatility is exhibited by investment followed by consumption. Employment has the lowest volatility.

In addition, as can be seen for U.S. as well as European data, the equity return carries an equity premium as compared to the risk free interest rate. This excess return was first stated by Mehra and Prescott (1985) as the equity premium puzzle. As can be observed the market return by far exceeds the return from the risk-free rate. As shown in a variety of recent papers,[86] the intertemporal models, in particular the RBC model insufficiently explains the equity premium and the excess volatility of equity return and thus the Sharpe-ratio. Standard RBC asset market models employ the Solow-residual as technology shocks – or impulse dynamics. For a given variance

[85] See Lettau and Uhlig (1997 a,b) and Lettau, Gong and Semmler (2001) where the Sharpe-ratio is employed as a measure to match theory and facts in the financial market.

[86] See, for example, Rouwenhorst (1995), Danthin, Donaldson and Mehra (1992), Boldrin, Christiano and Fisher (1996, 2001 ), Lettau (1997), Lettau and Uhlig (1997) and Lettau, Gong and Semmler (2001).

of the technology shock, standard utility functions and no adjustment costs, asset market facts are hard to match with the standard model. For details see Chap. 9 and Chap. 15.

In sum, for the actual time series compared, for example, with the standard RBC model, we observe a larger equity return and stronger volatility of equity prices in contrast to the risk-free rate. These two facts are measured by the Sharpe-ratio which cannot be matched by the standard RBC model.[87] Moreover, it is worth noting that in the stochastic growth model there is only a one-sided relationship. Real shocks affect stock prices and returns but shocks to asset prices – or overreaction of asset prices relative to changes in fundamentals – have no effects on real activity. The asset market is always cleared and there are no feedback mechanisms to propagate financial shocks to the real side.

The asset market implications of the above mentioned intertemporal models – and also the RBC model – are, for example, studied in Rouwenhorst (1995), Danthine, Donaldson and Mehra (1992), Lettau and Uhlig (1997 a,b), Lettau, Gong and Semmler (2001), Wöhrmann, Semmler and Lettau (2001), Boldrin, Christiano and Fisher (2001), Grüne and Semmler (2004b,c, 2005b). There, the baseline model with technology shocks as the driving force for macroeconomic fluctuations as well as some extensions of the base line model are employed to attempt to replicate the above summarized basic stylized facts of the stock market such as the excess volatility of asset prices and returns, the excess return,[88] the spread between asset returns the risk-free rate, and the Sharpe-ratio. The general result is that the baseline model has failed to replicate the above stylized facts. Details for both consumption as well as production based asset pricing models are evaluated in Chaps. 9-10 and Chap. 15. As mentioned above, there are numerous extensions that have been developed to overcome some of the deficiencies of the representative agent model to asset pricing. Some success can be found in a recent attempt by Boldrin, Christiano and Fisher (2001), yet there are still deficiencies, see Chap. 15.

## 5.3 The Excess Volatility Theory

Other theories and macro econometric studies depart from the market efficiency hypothesis and pursue the overreaction hypothesis when employing macro variables as predictors for stock prices and stock returns (Shiller 1991, Summers 1986, Poterba and Summers 1988). Moreover, in this tradition the role of monetary, fiscal and external shocks are seen to be relevant. Although in the long run stock prices may revert to their mean as determined by macroeconomic proxies of fundamentals in the short-run, speculative forces and the interaction of trading strategies of heterogeneous

[87] Danthine et al. who study the equity return also state:"To the equity premium and risk free rate puzzles, we add an excess volatility puzzle: the essential inability of the RBC model to replicate the observation that the market rate of return is fundamentally more volatile than the national product"(Danthine et.al. 1992: 531).

[88] See Mehra and Prescott (1985).

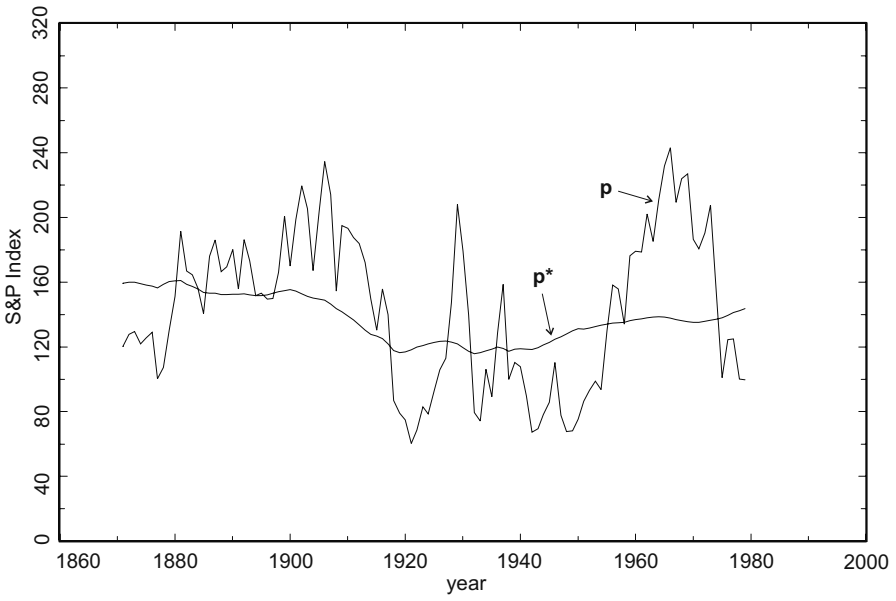



**Fig. 5.1.** Excess Volatility

agents may be more relevant than fundamentals. The latter view has been, with some success, tested in the mean reversion hypothesis of Poterba and Summers (1988).[89]

Let us use the following notations: $p$, the real S&P composite stock price index, and $p^*$ the ex post rational expectations price (detrended by a exponential growth factor).[90]

Shiller (1991) has devised an empirical test of the present value model with constant discount rate. There he defines excess volatility. He uses a variance bound test such as

$$\sigma(D) \geq \sqrt{2r}\sigma(\Delta p).$$

This variance bound test means that the standard deviation must be at least $\sigma(D)$ to justify the volatility of stock prices. Take, for example, $r = 0.07$; $\sigma(\Delta p) = 8.2$; $\sigma(D)$ must be 3.07; but $\sigma(D)$ is only $0.76$ for the data Shiller employs.

Recently, because of some criticism of using a constant discount rate, the excess volatility theory has been extended and researchers have used stochastic discount factors, see Shiller (1991). For details of this criticism and discussions on time varying discount rates, see Cochrane (2001, Chap. 21). Still the basic puzzle, namely the

[89] The overreaction of equity prices in relation to news on fundamentals originates, in this view, in positive feedback mechanisms operating in financial markets. Details are discussed in Chap. 5.4.

[90] For the methodology of how to compute the ex post rational expectation price, see Shiller (1991). The data are from http://www.econ.yale.edu/∼shiller/data.htm.

excess volatility remains. A study of excess volatility of the stock price for an industry is undertaken in Mazzucato and Semmler (1999).

## 5.4 Heterogeneous Agents Models

In recent times numerous researchers have developed models of heterogeneous agents and heterogeneous expectations to explain waves of optimism and pessimism, excess volatility – of the above mentioned type – and the statistical properties that characterizes asset price dynamics such as volatility clustering and time varying volatility. In principle models of heterogeneous expectations are well suited to explain those phenomena. Yet there are also some short comings of those models, see Chap. 7.

Important contributions have been made that study, as suggested by Shiller's excess volatility theory, the social interaction of heterogeneous asset traders. There are models of interaction of fundamentalists, who may incur a cost of exploring future trends of fundamentals, and chartists who extrapolate past experiences of asset prices and returns, see Day and Huang (1990). Other researchers postulate the existence of arbitrageurs and noise traders as heterogeneous trading groups, see DeLong, Shleifer, Summers and Waldmann (1990). A further model postulates agents with heterogeneous expectations and beliefs impacting investors' behavior, see Flaschel, Franke and Semmler (1997, Chap. 12).

The research into the dynamics of asset pricing resulting from the interaction of heterogeneous agents exhibiting different attitudes to risk and having different expectations has recently become quite important with the work by Brock and Hommes (1998), Franke and Sethi (1998), Levy et al (1995) and Chiarella and He (2001). Often those models build on the replicator dynamics of evolutionary theory. This implies that those agents with the best strategy and the highest asset returns will increase there wealth fastest and thus dominante the market in the long run. In consumption-based asset pricing models heterogeneous agents models have also become important, see Cochrane (2001, Chap. 21).

Such models are elegantly summarized in a recent work by Chiarella and He (2001). A stylized model of this type may read as follows. The expected return is defined as

$$E_t(\rho_{t+1}) = r + \delta + d\overline{\rho}_t$$

with $E_t(\rho_{t+1})$ the expected return on an asset, $\delta$ the equity premium, here taken as a constant, and, $d$ the weight of the chartists, who may be trend followers $(d > 0)$ or contrarians $(d < 0)$, with $d = 0$ for fundamentalists, and

$$\overline{\rho}_t = \frac{1}{L}\sum_{k=1}^{L} \rho_{t-k},$$

the expected trend formulated by the chartists and contrarians. The variance of the return, $V(\rho_{t+1})$, may be time varying. The expected return by the fundamentalist investors $r + \delta$, is assumed to be constant, with $r$ the risk-free interest rate and $\delta$

the equity premium and thus for the fundamentalists holds $d = 0$. Chiarella and He (2001) extend the model by allowing each group of agents to perceive specific equity primia such that one can have different $\delta_i$ for each group of agents.

The wealth proportions of the different types of investors evolve over time, depending on their relative success in predicting the return. The model can replicate, depending on the parameters chosen, the above mentioned statistical properties of actual asset markets such as volatility clustering and thus time varying variance, but since the equity premium is given exogenously the model does not attempt to replicate the equity premium or the Sharpe-ratio which empirically also appears to be time varying. An attempt to study the forces determining the equity premium and Sharpe-ratio is made in Chap. 6 and by the approaches studied in Chaps. 9-10. Further agent based and evolutionary modeling of asset markets is pursued in Chap. 14.

As indicated above, although the heterogeneous trading strategies of the different groups of investors may generate overshooting and quite complex asset price dynamics, we want to point out that it is presumably the interaction of the trading strategies and the varying perception of what the fundamentals are – and what their trend is – which explains the actual asset price dynamics. A model of this type is discussed in Chap. 7.

## 5.5 The VAR Methodology

In general it is well recognized that the studies of the interaction of financial and real variables have difficulties in fully capturing the lead and lag patterns in financial and real variables when tested econometrically. To overcome this deficiency, the use of the VAR framework to test for lead and lag patterns has been appealing. A first application of a VAR methodology to European data sets can be found in Canova and Nichola (1995).

Employing a VAR on stock price, interest rate and output (using a linear structure of the model for U.S. time series data 1960.1 - 1993.10), Chiarella, Semmler and Mittnik (2002) obtain results as depicted in the following figure.

Figure 5.2 shows the cumulative impulse-responses for U.S. time series data 1960.01-1993.10 with the three variables: output (growth rates of the monthly production index, Prod), real T-bill (monthly T-bill, TB) and monthly real stock price (nominal stock price deflated by the consumer price index, ST). The solid lines are the impulse responses and the broken lines the error bands.

Overall, we can summarize the following results: A one standard deviation shock to output has a positive effect in the next periods and keeps output persistently up. The T-bill rises and the stock prices falls. This is the direction of change in real and financial variables that one would expect. On the other hand a shock to the T-bill keeps the T-bill up and there is, as one also would expect, a fall in the stock price. Yet, the output is only insignificantly affected (and also has the wrong sign). If there is a shock to the stock price, the stock price is persistently higher, yet the T-bill and output are only insignificantly affected.

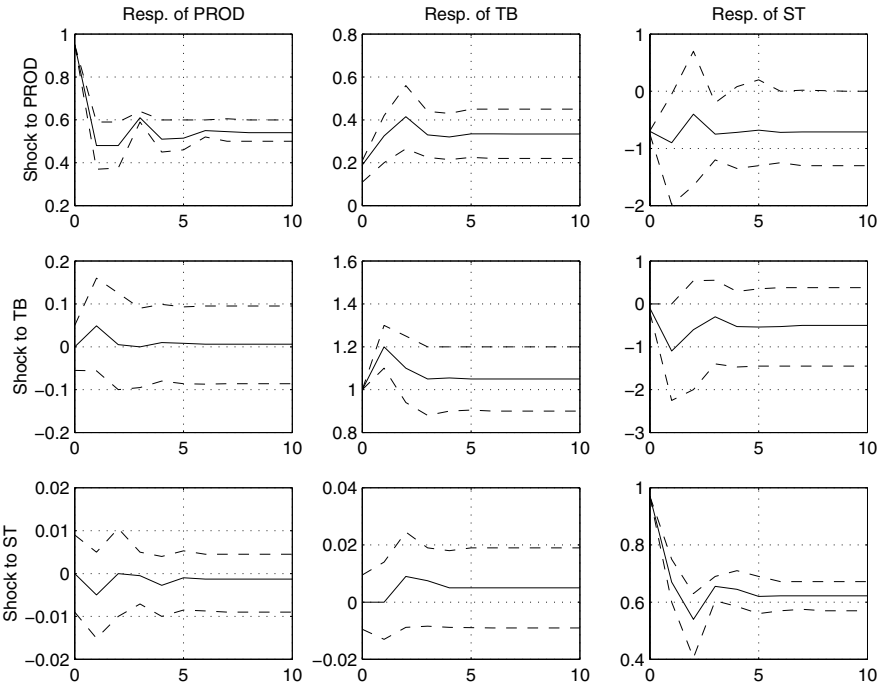



**Fig. 5.2.** Cumulative Impulse-Responses

Overall, a shock to output is affecting output and may impact the interest rate and stock price with a delay. Concerning the interest rate, the T-Bill, one usually expects an immediate impact effect on the stock prices and on output with a delay. We see in the above study a very small effect (yet of incorrect sign) but this may come from the fact that output, as discussed in Chap. 2, will most likely respond to long-term interest rates and less to short-term interest rates. On the other hand, we correctly see what one would expect that a shock to the stock price affects the stock price, but output and interest rates are only marginally or not affected. This appears to hold at least in linear VAR studies. Yet one might want to predict that if there is a large shock to stock prices, as, for example, the shock of October 1987 in the U.S. and the stock market shock 1997-98 in Asian countries, the effects on output may be larger. In linear VAR studies the response is always proportional to shocks and the effects of large shocks – shocks beyond a certain threshold initiating credit contractions and bank failures— cannot be captured. This can be studied in threshold models and models that also take into account other financial markets. A study of this type is pursued in Chap. 12. Moreover, a more complete VAR study of the stock market and its interaction with other variables may also take into account inflation rates and exchange rates.

Overall, one might argue that the VAR methodology is strong in capturing lead and lag patterns in the interaction of the variables but it does not reveal important structural relations, in particular if nonlinearities prevail in the interaction of the variables. Moreover, dynamic macro models may be needed to provide some rationale for the use of structural relationships and to highlight relevant restrictions on empirical tests. This is undertaken in Chap. 6.

## 5.6 Regime Change Models

There is some econometric work on the nonlinear interaction of stock market and output. The major type of models are built on Hamilton's regime change models. The Hamilton idea (Hamilton, 1989) that output follows two different autoregressive processes depending on whether the economy is in an expanding or contracting regime, is extended to a study of the stock market in Hamilton and Lin (1996).[91] Connecting to the above work by Schwert it is presumed that periods of high volatility may interchange with periods of low volatility of stock returns depending on whether the economy is in a recession or expansion. On the other hand, an important factor for the output at business cycle frequency appears to be the state of the stock market. In their version Hamilton and Lin (1996) show some predictive power embeded in the stock market for output and conversely, using a regime change model, the state of the economy as predictor for the volatility of stock returns.

The fact that the volatility is following two different regimes – recessions and expansions– is documented in the following figure where the space between the broken and dotted lines indicates recessions.

In the lower part of the figure, where the squared returns are shown, we can nicely observe that volatility of stock returns follows two different regimes.

Another nonlinear test for stock prices and output can be undertaken by using the STR methodology as introduced in Chap. 4.3 for liquidity and output. One can obtain interesting results using a bivariate STR estimate, for details, see Chiarella, Semmler and Koçkesen (1998). There, a delayed stock price acts as threshold for the output variable and a delayed output variable acts as threshold for the stock price.

The above mentioned studies of threshold (or business cycle) dependent volatility points to the possibility that returns and volatility may not be constant but time varying, i.e. vary with the business cycle. This gives rise to the conjecture that the above stated assumption in Chap. 5.2 of a constant risk-free rate, equity premium and Sharpe-ratio — often referred to in RBC models – might not be quite correct. One should rather attempt to match models with time varying financial characteristics such as equity premium and Sharpe-ratio. This is, with some success, undertaken in Cochrane (2001, Chap. 21), see also Wöhrmann and Semmler and Lettau (2001) and Chap. 15.

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[91] For further regime change models, see Rothman (1999).

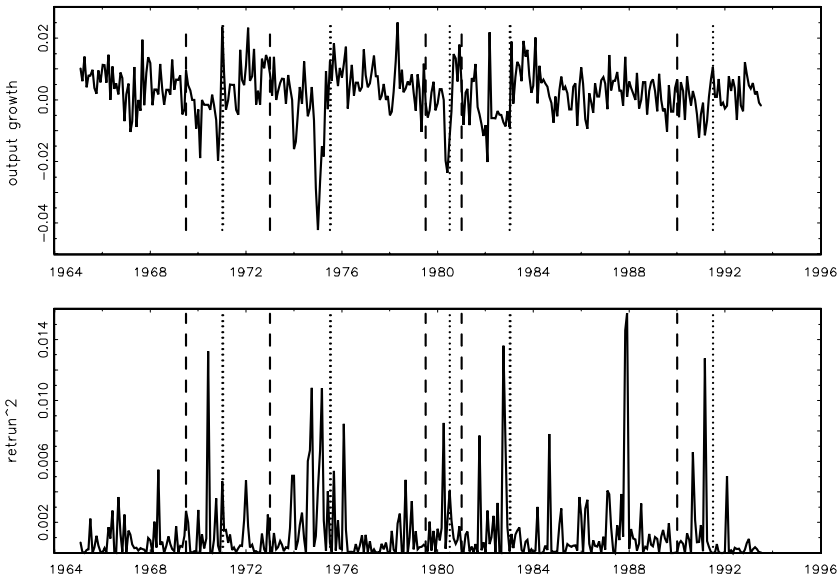



**Fig. 5.3.** Volatility Regimes

## 5.7 Conclusions

Our review of empirical approaches to study the interaction of stock prices and output –or in some cases stock prices, other financial variables and output— should be viewed as an introduction to modelling asset markets and economic activity. In Chap. 6 macro factors impacting stock prices are studied and a macro model that takes account of the interaction of macro variables and asset prices is introduced and empirical results reported. In Chap. 7 we explore the effects of new technology on asset prices and returns. Thereafter standard asset pricing models, in particular the capital asset pricing and intertemporal capital asset pricing models, are considered in detail and some estimation results are reported as well.

# Part III

# The Stock Market and Economic Activity

CHAPTER 6

# Macro Factors and the Stock Market

## 6.1 Introduction

Dynamic macro models of Keynesian type can be used to explain the interaction of stock prices, interest rates and output. From empirical research we know that those macro factors are strongly interacting. Such a macro approach has been introduced by Blanchard (1981) where he studies the interaction of stock price, interest rate and aggregate activity. In the Blanchard (1981) model, unlike in the RBC model, there are in principle, cross effects between asset prices and real activity. Along the line of Tobin (1969, 1980) it is presumed that output, through consumption and investment functions, is driven by real activity as well as stock prices. Output demand is determined by consumption and investment behaviors. As empirical studies have shown a contemporaneous relation of investment and the stock price may be weak. Yet, when lags are introduced and Tobin's Q is measured as marginal Q, as some studies do (Abel and Blanchard 1984), or the discount factor is approximated by a time varying risk premium (Lettau and Ludvigson, 2001), the relationship appears to improve.

On the other hand, since the Blanchard macro model is in a sense, a rational expectations model. The solution of this model is characterized by saddle path stability, shocks to macroeconomic variables cause stock prices to jump whilst keeping the output fixed (rather than allowing it to adjust gradually). Thus because stock prices jump there is still no feedback effect on output. Once the stock price is on the stable branch of the saddle paths output also gradually adjusts.[92] The stock price overshoots its steady state value during its jump and then decreases thereafter. Blanchard's macro model thus predicts that unless unanticipated shocks occur, the stock price moves monotonically toward a point of rest or if it is there it will stay there. In fact, only exogenous shocks will move stock prices. This line of research has been, as above mentioned, econometrically pursued a generation of papers, for example by Summers (1986), Cutler, Poterba and Summers (1989) and McMillin and Laumas (1988). As in other rational expectations models, in its basic version, feedback

[92] Blanchard states: "Following a standard if not entirely convincing practice, I shall assume that q always adjusts so as to leave the economy on the stable path to the equilibrium" (Blanchard 1981: 135); see also p. 136 where Blanchard discusses the response of the stock price to shocks, for example, unanticipated monetary and fiscal shocks. For a detailed discussion on policy shocks in the context of the Blanchard model, see McMillin and Laumas (1988).

mechanisms still do not exist that can lead to an endogenous propagation of shocks and fluctuations.

Below the Blanchard model will appropriately be modified to allow for such feedback effects. We modify and extend the Blanchard model and econometrically study the interaction of stock price, interest rate and output. The Blanchard variant is as mentioned, a perfect foresight model which exhibits saddle path stability. In Chiarella, Semmler, Mittnik and Zhu (2002) the perfect foresight jump variable technique is replaced by gradual adjustments, in particular gradual expectations adjustments based on adaptive expectations. The limiting behavior of such a model which admits, among others, cyclical paths generates the Blanchard model when expectations adjust infinitely fast to yield perfect foresight as a limiting case.[93] The model is solved through discrete time approximation and empirically estimated for time series data.

Based on the Blanchard variant[94] in this chapter a modelling strategy is pursued for the relationship of stock market, interest rate and real activity that overcomes weaknesses of both the RBC model and the rational expectations version of a macro model. In our model, unlike in the RBC type stochastic growth model, the stock price will impact the real activity, and different from the Blanchard model, stock price jumps to their stable paths are avoided by positing gradual adjustments of stock prices, interest rates and output. This, in turn, may better explain the endogenous propagation mechanism the fluctuations of both stock prices and output as well as the equity premium and Sharpe ratio.

## 6.2 A Dynamic Macro Model

In our notations, we follow Blanchard (1981). Output prices are fixed. $q$ is an index of the stock price, $y$ is output, $g$ the index of fiscal expenditure, $d$ is aggregate expenditure

$$d = aq + \beta y + g \qquad (\, a > 0, \quad 0 \leq \beta < 1). \tag{6.1}$$

Output adjusts to changes in aggregate expenditure with a delay according to

$$\begin{aligned} \dot{y} &= \kappa_y (d - y) \\ &= \kappa_y (aq - by + g), \end{aligned} \tag{6.2}$$

where $b \equiv 1 - \beta$ so that $0 < b < 1$ and the speed of output adjustment $\kappa_y > 0$.

From the standard assumption of an LM-equilibrium in the asset market we can write

$$i = cy - h(m - p) \qquad (c > 0, h > 0), \tag{6.3}$$

where $i$ denotes the short term rate of interest, $m$ and $p$ the logarithms of nominal money and prices respectively.

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[93] Further models of this macroeconomic modelling tradition that include the financial market can be found in Flaschel, Franke and Semmler (1997).

[94] The subsequent section is based on Chiarella, Semmler, Mittnik and Zhu, (2002).

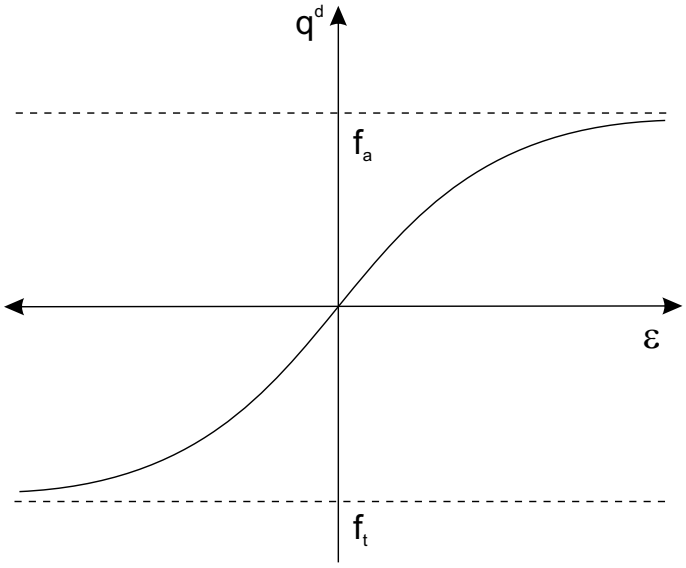



**Fig. 6.1.** Excess Demand for Stock

Real profit is given by

$$\pi = \alpha_0 + \alpha_1 y, \tag{6.4}$$

so that $(x+\alpha_0+\alpha_1 y)/q$ is the instantaneous expected real rate of return from holding shares, $x$ denotes the expected change in the value of the stock market. Hence the excess return is (which may allow for a constant risk premium on equity, see below)

$$\epsilon = \frac{x + \alpha_0 + \alpha_1 y}{q} - i. \tag{6.5}$$

In Blanchard $\epsilon$ is always zero. This assumes perfect substitutes and no arbitrage is possible. With imperfect substitutability between the two assets, the excess demand for stocks ($q^d$) is a positive but bounded function

$$q^d = f(\epsilon) \quad (f(0) = 0), \tag{6.6}$$

with $f$ as in the following figure.

If we allow for an equity premium, as discussed in Chap. 5, $\bar{\epsilon}$, with the equity premium a constant this would give rise to

$$q^d = f(\epsilon - \bar{\epsilon}) \quad (f(0) = -\bar{\epsilon}),$$

instead of (6.6) and the function, as depicted in the above figure, would shift to the left.

We further assume that the stock market adjusts to the excess demand according to

$$\dot{q} = \kappa_q f(\epsilon) \text{ or} \tag{6.7}$$

$$\dot{q} = \kappa_q f(\epsilon - \bar{\epsilon}) \tag{6.8}$$

where $\kappa_q (> 0)$ is the speed of adjustment of the stock market to excess demand for stocks. If we assume that $\kappa_q = \infty$ then from the above we recover

$$\frac{x + \alpha_0 + \alpha_1 y}{q} = i, \text{ or} \tag{6.9}$$

$$\frac{x + \alpha_0 + \alpha_1 y}{q} = i + \bar{\epsilon} \tag{6.10}$$

the Blanchard model.

For the formation of expectations we assume the adaptive expectations scheme.

$$\dot{x} = \kappa_x(\dot{q} - x), \tag{6.11}$$

where $\kappa_x (> 0)$ is the speed of revisions to expectations. The inverse $\kappa_x^{-1}$ may be interpreted as the time lag in the adjustment of expectations. By assuming this time lag to be zero (i.e. $\kappa_x = \infty$ ) the above equation reduces to the perfect foresight case

$$x = \dot{q}, \tag{6.12}$$

which is also a key assumption in Blanchard's model.

The generalized Blanchard model as worked out in Chiarella, Semmler, Mittnik and Zhu (2002) consists of

$$\dot{y} = \kappa_y(aq - by + g), \tag{6.13}$$

$$\dot{q} = \kappa_q f\left(\frac{x + \alpha_0 + \alpha_1 y}{q} - cy + h(m - p)\right), \tag{6.14}$$

$$\dot{x} = \kappa_x\left(\kappa_q f\left(\frac{x + \alpha_0 + \alpha_1 y}{q} - cy + h(m - p)\right) - x\right) \tag{6.15}$$

or with the equity premium $\bar{\epsilon}$

$$\dot{y} = \kappa_y(aq - by + g), \tag{6.16}$$

$$\dot{q} = \kappa_q f\left(\frac{x + \alpha_0 + \alpha_1 y}{q} - cy + h(m - p) - \bar{\epsilon}\right), \tag{6.17}$$

$$\dot{x} = \kappa_x\left(\kappa_q f\left(\frac{x + \alpha_0 + \alpha_1 y}{q} - cy + h(m - p) - \bar{\epsilon}\right) - x\right). \tag{6.18}$$

The equilibrium of the system is given by

$$\dot{y} = 0, \dot{q} = 0, \dot{x} = 0 \tag{6.19}$$

and the values $(\bar{y}, \bar{q})$ that solve

$$\begin{aligned} &aq - by + g = 0, \\ &\frac{\alpha_0 + \alpha_1 y}{q} = cy - h(m - p) \qquad \text{or} \\ &aq - by + g = 0, \\ &\frac{\alpha_0 + \alpha_1 y}{q} = cy - h(m - p) + \bar{\epsilon}. \end{aligned}$$

Without the equity premium we will write $\delta \equiv h(m-p)$. For time varying real balances we denote $\delta_t = h(m_t - p_t)$ and with equity premium we may redefine $\delta_t = h(m_t - p_t) - \bar{\epsilon}$.

For the above system we first show how the original Blanchard model can be recovered from it. First we assume perfect foresight by letting $\kappa_x \to \infty$ which by the above yields

$$\dot{q} = x. \tag{6.20}$$

Then we assume instantaneous adjustment to excess demand in the stock market by letting $\kappa_q \to \infty$ in the above. Hence with no equity premium we obtain

$$\frac{x + \alpha_0 + \alpha_1 y}{q} = cy - h(m-p). \tag{6.21}$$

Combining the last two equations yields the differential equation for $q$

$$\dot{q} = q[cy - h(m-p)] - \alpha_0 - \alpha_1 y. \tag{6.22}$$

The differential equations for $y$ and $q$ constitute the dynamical system studied by Blanchard. The equilibria from the above system are saddle points in this perfect foresight case. If the jump-variable procedure which is used by Blanchard is not adopted then the global dynamics need to be considered. This means that we have to study the above three dimensional system, see Chiarella, Semmler, Mittnik and Zhu (2002). This system can be transformed into an estimable two dimensional system. It is then estimated in Chiarella, Semmler, Mittnik and Zhu (2002).

## 6.3 Empirical Results

When the system with three variables is transformed into a system with two variables we can directly estimate the nonlinear bivariate system by NLLS estimation using the Euler approximation method. We undertake this for US data 1960.01-1993.10.[95] In Chiarella, Semmler, Mittnik and Zhu (2002) we also present estimations for European Data (1974:02-1993.06).

We directly estimate the parameters of a discrete time nonlinear bivariate system with a constrained number of lags by using the Euler scheme. The estimated parameters, obtained from the BP- filtered data, are reported in table 6.1. Direct estimation, using the Euler scheme for the transformed system (6.13)-(6.15) in bivariate form, are as follows:

[95] Results of a regime change model of STR type with an unconstrained lag structure are undertaken in Chiarella, Semmler and Koçkesen (1998) and compared to the direct estimation below.

**Table 6.1.** Parameter Estimates, US: 1960.01-1993.10, Detrended Data[96]

| Economic Structure | Speeds of Adjustment | Government Policy |
|---|---|---|
| $a = 0.122$ | $\kappa_y = 0.185$ | $g = 0.000$ |
| $b = 0.370$ | $\kappa_q = 0.240$ | $\delta = -6.670$ |
| $\alpha_0 = 0.065$ | $\kappa_x = 1.120$ | |
| $\alpha_1 = 6.620$ | | |
| $c = 1.568$ | | |
| $\lambda = 0.036$ | | |
| $\bar{f} = 0.205$ | | |

It is noticeable from table 6.1 that all parameters have the predicted sign, except $\delta$ which is estimated without the equity premium. Note also that $\delta$ is taken as a constant. One can observe the hierarchy in the speed of adjustments that also other studies would suggest. Since here the term $\delta = h(m - p)$ is fixed. We next take the time series of real balances $\delta_t = h(m_t - p_t)$ as exogenous sequence.[97] This assumes that the history of monetary policy is important, in that it affects $m_t$, for the path of interest rates. In addition, we account for a term that indicates an equity premium. We here posit that the equity premium is a constant showing up as a parameter in the model. The results with real balances and an equity premium are presented in table 6.2. For this purpose the system (6.16)-(6.18) has also been transformed into an estimable bivariate system.[98]

Here, too, all parameters are reasonable and the hierarchy of adjustment speeds is reasonable as well. There is now an equity premium, $\bar{\epsilon}$, which has the expected sign, although, since we have used detrended data, the size of it is hard to interpret.

Overall the direct estimation of the model performs reasonably well. Moreover, in Chiarella, Semmler, Mittnik and Zhu (2002) stochastic simulations with the estimated parameters of table 6.2 are reported that show that this type of dynamic macro model can explain reasonably well the excess volatility of the stock price, the equity premium and the Sharpe-ratio. A further extension of this type of a macro model of the real-financial interaction can be found in Chiarella, Flaschel and Semmler (2004) where the reactions of the agents are made state dependent.

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[96] We employ monthly data on stock price and an index of industrial production which are taken from the Hamilton and Lin (1996) data set.

[97] The data for money $M$ and price level $P$ are obtained from Citibase (1998).

[98] For details see Chiarella, Semmler, Mittnik and Zhu (2002).

[99] In the estimations above we have prefixed c and h.

**Table 6.2.** Parameter Estimates, US: 1960.01–1993.10, Detrended Data[99]

| Economic Structure | Speeds of Adjustment | Government Policy |
|---|---|---|
| $a = 0.122$ | $\kappa_y = 0.285$ | $g = 0.000$ |
| $b = 0.370$ | $\kappa_q = 1.998$ | |
| $\alpha_0 = 0.397$ | $\kappa_x = 1.798$ | |
| $\alpha_1 = 0.05$ | | |
| $c = 0.400$ | | |
| $h = 0.100$ | | |
| $\bar{f} = 0.025$ | | |
| $\bar{\epsilon} = 0.035$ | | |

## 6.4 Conclusions

The above suggested model that links the stock market, interest rates and output can be seen as a prototype model to understand the interaction of asset prices and real activity in modern economies. The model is still rudimentary in the sense that it lacks price dynamics, the term structure of interest rates, the effects of monetary monetary and fiscal policies and the impact of exchange rate fluctuations on both stock prices and output (see Chap. 12). Yet, such a type of model can be considered as a working model for numerous extensions. For further variants of this type of model, see Chiarella, Flaschel and Semmler (2001, 2004) and Chiarella et al. (2005). However, in the above model there is still no evolution of new technology that may impact both asset prices as well as output. A model of the stock market including the latter is presented next.

CHAPTER 7

# New Technology and the Stock Market

## 7.1 Introduction

In the previous model there was no evolution of new technologies that could impact productivity, output and asset prices. Recent models explicitly consider the relationship between new technology and stock prices, see (Greenwood and Jovanovic (1999), Hobijn and Jovanovic (1999) and Mazzucato and Semmler (1999, 2002)). This type of work studies the effect of the evolution of new technologies on stock prices. The main hypothesis here is that the perceived emergence of new technology makes the asset price of existing technology, operated by incumbents, fall and the asset price of the innovators, the newcomers, rise.

## 7.2 Some Facts

The fall of the aggregate stock price in the 1980s and then the rapid rise of the stock price in the 1990s in the U.S. has often been used as an historical example to exemplify those two opposing effects of new technology on stock prices. Greenwood and Jovanovic (1999) show that the stock price of the incumbents first fell in the 1970s and then remained flat in the 1980s and 1990s whereas the stock price of all firms, driven by the innovators, has been rising since the end of the 1980s.

Next we may look at the investment into new technology and stock prices. Figure 7.1 shows the share of information technology investment of total investment in equipment and the rise of the Nasdaq in the 1990s. As Greenwood and Jovanovic (1999) show the Nasdaq was mainly driven by new IT start up firms. As figure 7.1, which is based on the data by Hobijn and Jovanovic (1999), demonstrates there is a strong comovement of the Nasdaq and investment new technology.

Greenwood and Jovanovic (1999) and Hobijn and Jovanovic (1999) use the Lucas (1978) consumption-based asset price model in order to explain the phenomena depicted in figure 7.1. In their model, however, there are no firms, and the dividend payments are exogenously given as in the Lucas (1978) model.

The following model, developed by Semmler and Greiner (1996), starts with firms and earnings of firms and may be more suitable for understanding such periods of major technological change and the associated stock price movements. At the center of this model are two types of firms, incumbent firms and innovators. Whereas the incumbent firms cling to old technologies, at least for a while, it takes time until the innovators add value to the stock market.

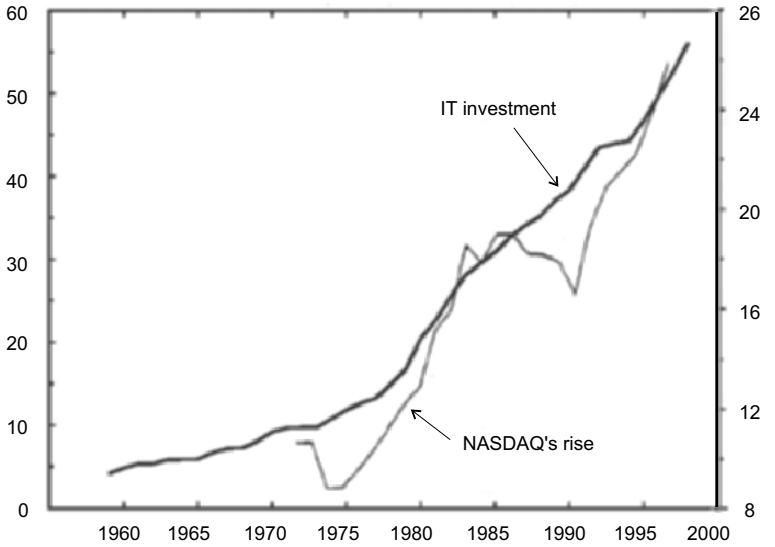



**Fig. 7.1.** Nasdaq Stock Price and Investment in Computer Equipment

In the model one group of firms is presumed to actively innovate and the other group, the incumbents who operate existing technology, passively respond to changes in the technological environment. Innovating firms usually expect a return from committing resources and undertaking inventive investment. They may compute the net present value of their revenue from the innovation. While the innovators aim at capturing excess profit when the technology is implemented, the second group, the incumbents, may, under competitive pressure, learn to improve their efficiency and profit by being second movers. We posit that the new technology will be created at a certain cost, an innovation cost. The total cost for operating the new technology is assumed to be dependent on the effort spent to obtain the new technology (independent of the number of firms) and a cost proportional to the number of firms operating it.

We do not, however, presume that perfectly competitive conditions hold so that the profit for the innovators is instantly dissipating. It is reasonable to presume that the new technology is employed monopolistically. When the innovators expect gains from innovations – which can be expressed as the present value of future profit flows – the innovating firms will expand. While anticipation of the innovators earnings make stock prices rise the perceived out-dated vintages of capital goods of the incumbents make their stock prices fall. Although, there might be entry into the group of innovating firms, encouraged by excess profits, there may also be occuring exits due to negative profits, see Hobijn and Jovanovic (1999). We might assume that the process of compressing the profit is slow.

By borrowing from evolutionary theory of the replicator dynamics we may assume different types of interaction effects between the firms: a *predator-prey* relation

between the innovators and incumbents, a *cooperative effect*; and a *competition (or crowding) effect*. The predator-prey relation occurs when innovators grow at the expense of the incumbents. The competitive effect results when the new technology dissipates. The excess profit falls because of reduced prices and compressed markups. We may use an inverse demand function to specify this effect. The two groups of firms also gain from each others success. There is a cooperative effect (spillover or learning effect) that bounds the number of incumbents away from zero, so that, although firms exit, complete extinction of incumbents does not occur. With a costly new technology, the innovators most likely will have an unprofitable period when the new technology is introduced and thus the stock price will not rise yet. On the other hand, the forward looking stock market may anticipate net income gains and stock prices may rise. Innovative firms face a period when they can enjoy technological rent and rising stock prices. Later, firms may lose their profit due to the subsequent competitive effect as a result of an increase in the capacity to produce and the incumbents capability to adopt the new technology.

## 7.3 The Model

A small scale model of two types of firms modelling the behavior of the innovators and the incumbents is posited

$$V_{\max} = \int_0^\infty e^{-rt} g(x_2, u) dt; u \in \Omega_+$$

s.t.

$$\dot{x}_1 = k - a x_1 x_2^2 + b x_2 - x_1 e/\mu \tag{7.1}$$

$$\dot{x}_2 = x_2(a x_1 x_2 + v g(x_2, u) - \beta) \tag{7.2}$$

with $g(x_2, u) = \mu(x_2, u) x_2 u - cu - c_0 x_2,\ \mu = \alpha/(\Phi + x_2 u)$, where $k, \alpha, \beta, e, c, \Phi$ and $v > 0$ are constants, $x_1$ is the number of incumbents, $x_2$ the number of innovators and $u$ a decision variable related to the introduction of new technology. The decision variable $u$ indicates the level of effort spent to create the new technology. This can mean the hiring of engineers, running research laboratories or purchasing information on new technologies. This investment is usually risky since there is considerable uncertainty and risk involved. We limit our model to a deterministic version.[100]

The cost per unit of effort is denoted by $c$. The cost $cu$ is independent of the number of firms and there is a cost proportional to the number of firms, $c_0 x_2$. Thus, $cu + c_0 x_2$ is the amount of resources that innovators have to devote to the innovation. The term $\mu(\cdot)$ is the (net) price, or markup, received for the product produced by the new technology, where $\mu(\cdot) x_2 u$ is the net revenue. When the innovators attempt to maximize the earnings arising from new technology $g(\cdot)$, facing a revenue $\mu(\cdot) x_2 u$ and the cost $cu + c_0 x_2$ excess profit will increase their number. In equ. (7.2) the term

[100] For a stochastic version, see Semmler (1994).

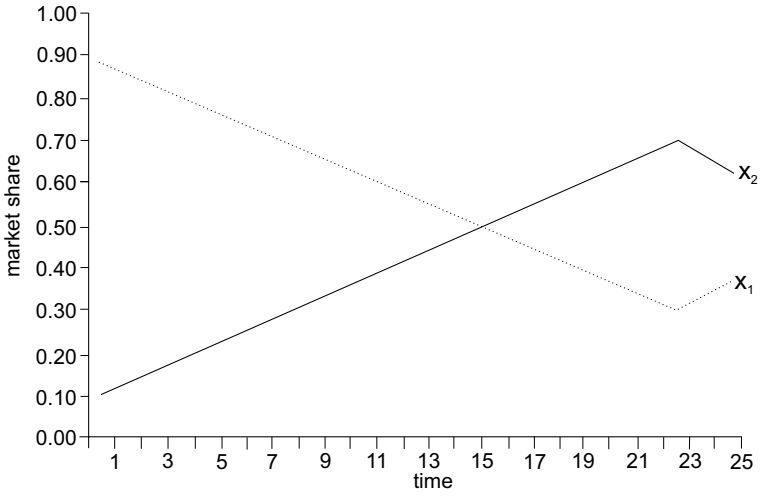



**Fig. 7.2.** Market Share Dynamics of Innovators and Incumbents

$vg(\cdot)$ with $v$ a constant, means that there is an increase in the number of innovators which is proportional to their excess profit.

The term $ax_1x_2^2$ represents the predator-prey interaction where the adoption of the new technology is supposed to take place proportionally to the product of $x_1$ and $x_2x_2$ (a common assumption for the spread of information in sales-advertising models). This implies that as the number of firms applying the new technology grows, so does the accessibility to that technology for the incumbents as well. This way the rate of decrease of the incumbents in (7.1) may be translated into an increase of the innovators in equ. (7.2).

It is reasonable to posit that information about the new technology leaks out faster the larger the number of firms that apply the new technology. Our assumption means that the diffusion speed accelerates by $x_2$. The term $bx_2$ in equ. (7.1) reflects the cooperative effect of $x_2$ on $x_1$. This represents learning by the incumbents to improve their performance when information about the new technology spreads and the competitive pressure from the new technology on the incumbents increases. The last term $x_1e/\mu$ in (7.1) is the crowding effect for $x_1$, with $\mu$ the mark-up from an inverse demand function which also appears in equ. (7.2). [101]

Figure 7.2 depicts the relative market shares of the incumbents, $x_1$ and the innovators $x_2$, for certain initial conditions. It shows that innovating firms that undertake an optimal inventive investment may succeed and increase their market share, but the incumbents still coexist side by side with the innovators operating the new technology. They may even gain back some market share at a later period.

[101] Semmler and Greiner (1996) show that there might be multiple equilibria of the model depending on parameters.

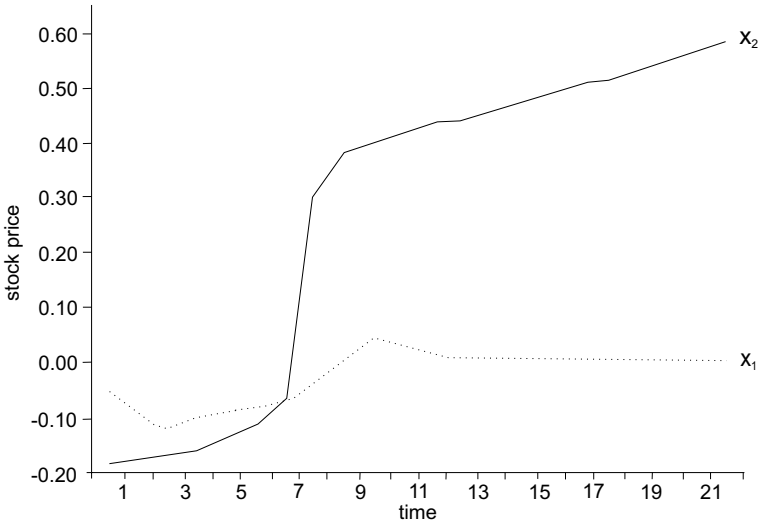



**Fig. 7.3.** Stock Price Dynamics of Innovators and Incumbents

In figure 7.3 the present value of the incumbents (dashed line) and the innovators (solid line) are depicted. The stock price of the incumbents is obtained by discounting their current profit flows and the innovators' stock price is obtained solving the model of equs. (7.1)-(7.2).[102]

Figures 7.2 and 7.3 replicate the aforementioned stylized facts on the innovators' expansion of the market share, the rise of its stock price, the long-run upswing of the innovators' stock price and the decline and low level of the incumbents' stock price due to the fact that they are technologically lagging. They also may cease to be valued by the financial market. This is what seems to have happened in the U.S. in the 1980s and 1990s.

An analysis similar to ours on innovative effort, market share and stock price dynamics for the U.S. can be found in Jovanovic and Greenwood (1999) and Hobijn and Jovanovic (1999). Yet, as aforementioned they are using a consumption-based asset pricing model, the Lucas (1978) asset price model. An asset price model of the above type that includes production is employed in Mazzucato and Semmler (2002) where the stock price dynamics of the early U.S. automobile industry is studied. In either case such a dynamic view of technology evolution allows one to connect industry dynamics and stock price volatility.

In general, as Hobijn and Jovanovic (1999) have shown firms that fail to innovate successfully may fall prey to the more successful innovators – being the object of

[102] Value function iteration using dynamic programming is employed to solve the above model, see Semmler and Greiner (1996). There it is also shown that, if the start-up cost for the innovator is too high compared to the expected returns, the innovator may end up with a negative present value and thus may go bankrupt.

mergers and acquisitions -or are forced to exit, possibly leading to a large exit (shake-out) of firms. While stock prices may already be very volatile in the innovation period where it is unclear who are the winners and losers, stock prices of firms that fail to innovate may rapidly drop after a shake-out and exits of firms. Similarly, in more generic terms, a stock price crash may be triggered by a more general slowdown of innovations.[103]

## 7.4 Conclusions

Overall, we want to stress that long swings and short run volatility of stock prices in an economy with rapid technological change cannot be interpreted solely as excess volatility resulting. For example, swings and volatility, resulting from the mood, the strategy and herd behavior of stock market traders, as we have discussed in Chap. 5.3, but real determinants are important as well when a new technology arises. It is the turnover of the leading firms, the uncertain prospects of firms, their innovative potentials the fluctuations in real earnings and dividends and the market share instability that are also strongly driving stock market volatility. Given those uncertainties about the real winners and losers there is excess volatility and occasional under- and over-valuation of the firm's, the industry's and economy's aggregate stock price.[104] Yet, as shown in Chap. 5.3, waves of optimism or pessimism are important in this context as well. In the interaction of heterogeneous traders' social psychology becomes important when the fundamentals are uncertain in the presence of rapid technological change. Those waves of optimism or pessimism are neglected in some recent studies that propose that stock prices are driven solely by fundamentals (see Hobijn and Jovanovic, 1999).

[103] A model of shake-out and stock price fall is presented in Barbarino and Jovanovic (2005). They argue that the stock price boom in the 1990's and the subsequent crash need not reflect an irrational exuberance as Shiller (2001) has argued.

[104] For a reasonable method to separate the volatility component that is driven by "fundamentals" and the excess volatility resulting, for example, from overoptimism or pessimism or from the psychology and social interaction of traders in a very uncertain environment, see Mazzucato and Semmler (1999).

CHAPTER 8

# Static Portfolio Theory: CAPM and Extensions

## 8.1 Introduction

This chapter discusses theoretical foundations and empirical evidence for the most prominent asset pricing theory: the Capital Asset Pricing Model (CAPM). It represents a pricing model for risky assets. The CAPM has been extended to the multi–factor model (MFM) and arbitrage pricing theory (APT). The motivation of the latter has been to overcome the problems associated with the market portfolio in the CAPM by introducing a multi–factor approach. The subsequent chapter is kept simple and refers to the CAPM only. In some additional remarks crucial assumptions regarding investors' preferences and stock return distributions are relaxed. Currently the debate whether exact pricing restrictions in the MFM and APT imply the return to the "good old CAPM" is still going on. By introducing state dependent relationships as well as general nonlinearities in a CAPM model one can try to unify the different views but we will not elaborate on such extensions. A more extensive treatment of static and dynamic portfolio theories is given in Part VI of the book.

## 8.2 Portfolio Theory and CAPM: Simple Form

First we want to give some definitions. We denote by $V_1$ the portfolio market value at the end of the interval; $V_0$ the portfolio market value at the beginning; $D$ the cash distributions; $N$ the number of intervals (monthly); $C_i$ the cash flow (net); $R_D$ the internal rate of return and; $r_i$ the monthly returns.

The investment return is

$$R_p = \frac{V_1 - V_0 + D_1}{V_0}.$$

The arithmetic average rate of return is

$$R_A = \frac{R_{p1} + R_{p2} + ...R_{pn}}{N}.$$

The continuously compounded rate of return affecting the price, $P$, can be denoted by

$$P_t = P_{t-1}e^{rt} \quad (r = \text{rate of return during t-1,t})$$
$$P_{t_{12}} = P_0 e^{r_1+r_2+r_3...+r_{12}} \quad (r_i = \text{returns for month i})$$

and the average monthly return is: $r = (r_1 + r_2...)/12$

The internal rate of return, $R_D$, is defined as

$$V_0 = \frac{C_1}{(1+R_D)} + \frac{C_2}{(1+R_D)} + \frac{C_N + V_N}{(1+R_D)^n}$$

Next, let us introduce the theory of static portfolio decision. The mean-variance principle is here our guiding principle. It means that a portfolio of assets (securities) is supposed to maximize the return for the investors for some level of risk they are willing to accept. This gives us the Markowitz efficient portfolio. The assumption is that investors are risk adverse.[105] The two fund theory comprises a risk free asset (short term government bonds) and risky assets (stocks).

Let us illustrate a portfolio risk. Take the following data[106]

**Table 8.1.** Portfolio Risk

| outcome | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| possible return | 50 | 30 | 10 | -10 | -30 |
| probability | 0.1 | 0.2 | 0.4 | 0.2 | 0.1 |

The expected portfolio return, formally stated, is

$$\begin{aligned} E(R_p) &= P_1R_1 + P_2R_2 + ... + P_nR_n \\ &= \sum_{j=1}^{n} P_jR_j \end{aligned}$$

Using the above example we obtain $E(R_p) = 10\%$. Hereby the $P_j$ are the associated probabilities.

Two typical distributions of asset returns are shown in figure 8.1.

The variance of the returns is

$$\begin{aligned} Var(R_p) &= P_1(R_1 - E(R_p))^2 + P_2(R_2 - E(R_p))^2 + ....P_n(R_n - E(R_n))^2 \\ &= \sum_{j=1}^{n} P_j(R_j - E(R_p))^2 \end{aligned}$$

From the above example we have a variance of 480%. This gives a standard deviation $\sigma = \sqrt{480} = 22\%$.

An asset price is said to follow a random walk if the expected future price change is independent of past price changes. Risk for a longer horizon (volatility) is defined as $\sigma \cdot \sqrt{N}$ whereby $N$ are the time periods ahead.

[105] For a more detailed treatment of the investors preference, see Chap. 14.

[106] The subsequent examples can be found from Fabozzi and Modigliani (1997, Chap. 8).

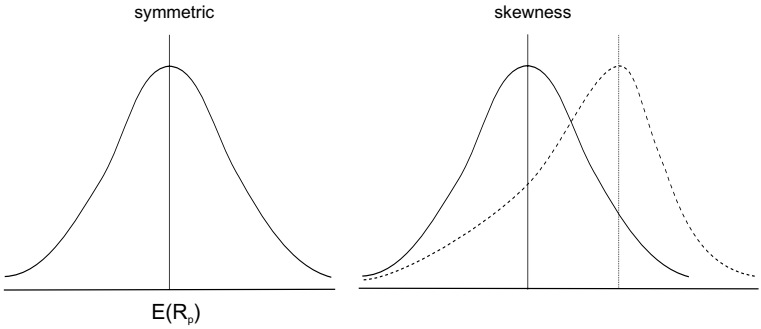



**Fig. 8.1.** Different Types of Risk

Diversification serves the purpose of constructing a portfolio to reduce portfolio risk without sacrificing return. Diversification does not systematically affect the return of the portfolio. It is equal to the weighted average of individual security returns. Yet, diversification reduces the standard deviation of returns. The standard deviation, $\sigma_p$, decreases the less there is a correlation among securities. We define the correlation coefficient as

$$R = \frac{cov(R_i, R_M)}{\sigma_{R_i} \cdot \sigma_M}$$

with the covariance,

$$cov = \frac{1}{N} \sum_t (R_{i_t} - E(R_i))(R_{M_t} - E(R_M)).$$

The next example shows how risk falls with increasing diversification.[107] In fact the next table shows the risk versus diversification for randomly selected portfolios (June 1960-May 1970).

From table 8.2 we can observe the following results. First, the average return is unrelated to the number of securities, second there is a decline in portfolio risk (with the number of securities). Third, there is an increasing correlation with the index of NYSE stocks. Fourth, the $R^2$, measuring the return of the portfolio with market return ($0 < R^2 < 1$) rises with the degree of portfolio diversification (a well diversified portfolio has a high $R^2$). Thus, as Figure 8.2 indicates, unsystematic risk tends to be diversified with the number of holdings, but systematic risk is not.

Portfolio mean returns are

$$E(R_p) = \gamma_1 E(R_1) + \gamma_2 E(R_2). \tag{8.1}$$

The portfolio variance ($\sigma^2_{R_p}$) for two assets is

[107] The data of the following table are based on the data reported in Fabozzi and Modigliani (1997, Chap. 8).

**Table 8.2.** Covariance and Diversification

| Number of Securities in Portfolio | Average Return (%/mo.) | Standard Deviation of Return (%/mo.) | Correlation Coefficient with Market R | Coefficient of Determination with Market $R^2$ |
|---|---|---|---|---|
| 1 | 0.88 | 7.0 | 0.54 | 0.29 |
| 2 | 0.69 | 5.0 | 0.63 | 0.40 |
| 3 | 0.74 | 4.8 | 0.75 | 0.56 |
| 4 | 0.65 | 4.6 | 0.77 | 0.59 |
| 5 | 0.71 | 4.6 | 0.79 | 0.62 |
| 10 | 0.68 | 4.2 | 0.85 | 0.72 |
| 15 | 0.69 | 4.0 | 0.88 | 0.77 |
| 20 | 0.67 | 3.9 | 0.89 | 0.80 |

$$\sigma^2_{R_p} = \gamma_1^2\sigma^2_{R_1} + \gamma_2^2\sigma^2_{R_2} + 2\gamma_1\gamma_2\sigma_{R_1}\sigma_{R_2} corr\ (R_1, R_2). \tag{8.2}$$

The portfolio variance is the sum of the weighted variances of the two assets plus the weighted correlation between the two assets.

In general we have

$$\sigma^2_{R_p} = \sum_{g=1}^{G} \gamma_g^2\sigma^2_{R_g} + \sum_{g=1}^{G}\sum_{b=1}^{G} \gamma_g\gamma_b cov(R_g, R_b).$$

Markowitz efficient portfolios are defined by using the mean-variance methodology. This is illustrated by the following. There are two approaches

1) One fund theory refers to risky assets only.
2) Two fund theory refers to risky assets and a risk free asset.

In the above figure the capital market line (CML) or the Sharpe Ratio,

$$SR = \frac{E(R_p) - R_F}{\sigma_p},$$

is an important measure in capital asset pricing. From the figure 8.3 we also observe that a portfolio $P_B$ will dominate a portfolio $P_A$. Yet, we want to note that recent empirical research shows that the Sharpe-ratio and thus the expected equity premium and risk, measured by the standard deviation of the equity premium, are time varying, see Lettau and Ludvigson (2001b). In particular the Sharpe-ratio is moving countercyclically, see Wöhrmann, Semmler and Lettau (2001). This, in turn, means that there is a predictable component in the equity premium, since risk is not fully priced in the equity premium.

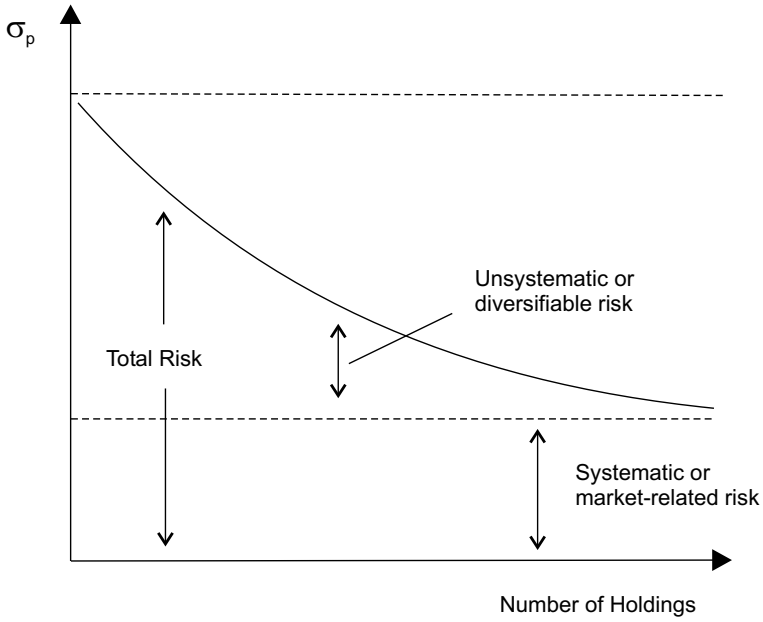



**Fig. 8.2.** Distribution of Asset Returns

For the two fund portfolio we can derive

$$E(R_p) = \gamma_F R_F + \gamma_M E(R_M),\ \gamma_F = 1 - \gamma_M \tag{8.3}$$
$$E(R_p) = (1 - \gamma_M) R_F + \gamma_M E(R_m) \tag{8.4}$$
$$E(R_p) = R_F + \gamma_M \left[E(R_M) - R_F\right]. \tag{8.5}$$

Using (8.2) gives (note that $R_F$ carries no risk)

$$\sigma^2_{R_p} = \gamma^2_M \sigma^2_{R_M} \tag{8.6}$$

and

$$\sigma_{R_p} = \gamma_M \sigma_{R_M} \Rightarrow \ \ \gamma_M = \frac{\sigma_{R_p}}{\sigma_{R_M}}. \tag{8.7}$$

Equ. (8.5) represents the capital market line and the investor's risk preference would determine a position on this line. A greater $\gamma_M$ reflects a greater preference for risk.

The standard form of the Capital Asset Pricing Model, the CAPM, can be written as follows

$$E(R_i) = R_F + \beta_i \left[E\left(R_M\right) - R_F\right];\ \beta_i = \frac{cov(R_i, R_M)}{\sigma^2_{R_M}}. \tag{8.8}$$

The $\beta_i$ represent the price of risk for a security $R_i$ or a portfolio $R_p$. The model (8.8) is therefore also called a beta pricing model.

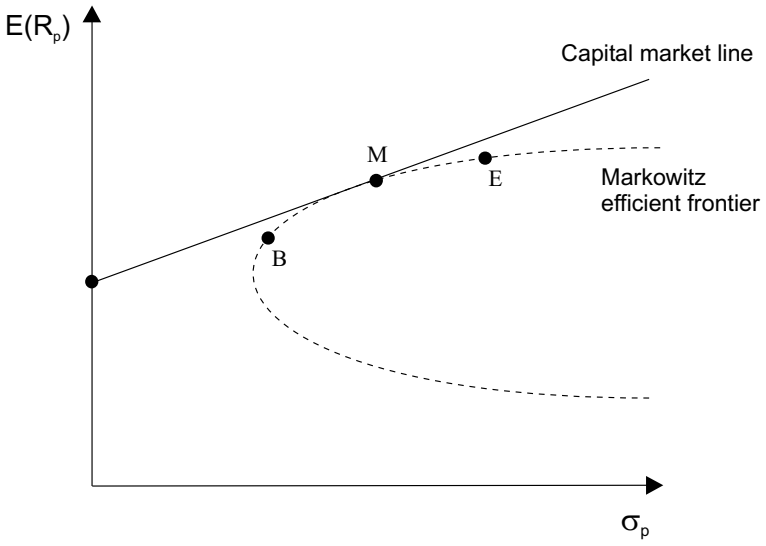



**Fig. 8.3.** Markowitz Efficient Frontier

## 8.3 Portfolio Theory and CAPM: Generalizations

In general if the economic decisions of consumers are guided by the mean-variance methodology they maximize the discounted expected returns. The assumptions here are one, that the investors only consider the first two moments of the distribution of stock returns and two, have homogenous beliefs. Given a mean portfolio return, investors (price takers) choose the one with lowest variance. The investment horizon is one period. There are perfect markets. Investments are infinitely divisible, there are no transaction costs and taxes and there are no short-sale restrictions.

The risk and return trade-off for a portfolio of $n$ assets considers a portfolio $P$ with $n$ assets, weights $\omega' = (\omega_1, \ldots, \omega_n)$ with $\omega' \underline{1} = 1$, $\underline{1}$ a vector of ones, returns $r' = (r_1, \ldots, r_n)$, mean returns $\mu_r$ and variance-covariance matrix $\Sigma = \{\sigma_{ij}\}_{i,j=1,\ldots,n}$. The mean of the portfolio return is $\mu_{r_P} = \omega' r$ and the variance $\sigma^2_{R_P} = \omega' V \omega$ with $V = Cov(\sigma)$. The minimum variance portfolios and the efficient frontier is given by

$$\min_{\omega} \omega' V \omega \tag{8.9}$$

$$s.t. \quad \omega' r = \bar{\mu}_{r_P}$$
$$\omega \underline{1} = 1.$$

The mean-variance portfolio produces the efficient frontier as shown in figure 8.3. For computing the graph of the efficient frontier in figure 8.3 for each mean return $\bar{\mu}_{r_P}$ the minimizing variance is computed and the relationship to the mean return plotted. The computation of the efficient frontier is undertaken with a quadratic programming

approach[108], see also Benninga (1998). Further details on the computational aspects of the CAPM, and the multifactor approach arbitrage pricing theory (APT), can be found in Benninga (1998) and Campbell, Lo and MacKinlay (1997, Chaps. 5-6).

The CAPM, according to Black (1972), presumes no risk-free asset. Thus:

$$r_i - r_Z = \beta_i(r_M - r_Z) \quad \text{with} \quad \beta_i = \frac{Cov(r_i, r_M)}{Var(r_M)}.$$

Iternatively Sharpe (1964) and Lintner (1965) do presume the existence of a risk-free asset, $r_Z = r_f$. Their assumptions are that investors are risk-averse, rational, and have homogenous expectations. There exists a risk-free asset, perfect markets, and a unique equilibrium.

Then we obtain the capital market line as

$$SR = \frac{E(r_M) - r_f}{\sigma(r_M)}$$

The risk-return relationship is

$$E(r_i) = r_f + \underbrace{[E(r_M) - r_f]}_{\text{market risk}} \underbrace{\frac{Cov(r_i, r_M)}{Var(r_M)}}_{\text{sensitivity}\beta_i} \tag{8.10}$$

whereby the second term on the right hand side of (8.10) represents the risk premium.

A survey of empirical tests of the CAPM are presented in Campbell, Lo and MacKinlay (1997, Chap. 5). For the expected return, $\tilde{r}_i$ and $\tilde{r}_m$, researchers have employed time series regression of the type

$$\tilde{r}_i = r_t + \beta_i(\tilde{r}_m - r_t) \tag{8.11}$$

and the the estimated $\bar{\beta}_i$ regressed on the expected return, $\tilde{r}_i$

$$\tilde{r}_{it} = \alpha_0 + \alpha_1\bar{\beta}_i. \tag{8.12}$$

Yet, the empirical results are usually very disappointing, see Benninga (1998, Chap. 8). For further empirical studies see also Fabozzi and Modigliani (1997), and Fama and French (1988). The above simple test of the CAPM equs. (8.11)-(8.12) have often failed to confirm the CAPM. Yet, as shown in Eisenbeiss et al. (2004) estimating a time varying $\rho$ with a non-parametric approach is more promising. The failure of the CAPM has given rise to the development of the multifactor model (MFM) and arbitrage pricing theory (APT).

Advanced tests of the CAPM are undertaken that show that broad stock market indices are no adequate proxies for the market portfolio. Many researchers argue that other assets such as real estate, land and human capital might need to be included in the

[108] A Gauss program to solve the above quadratic programming problem for a portfolio with no short-sale restrictions is available upon request. A program that allows for short-sale restrictions is also available in Gauss.

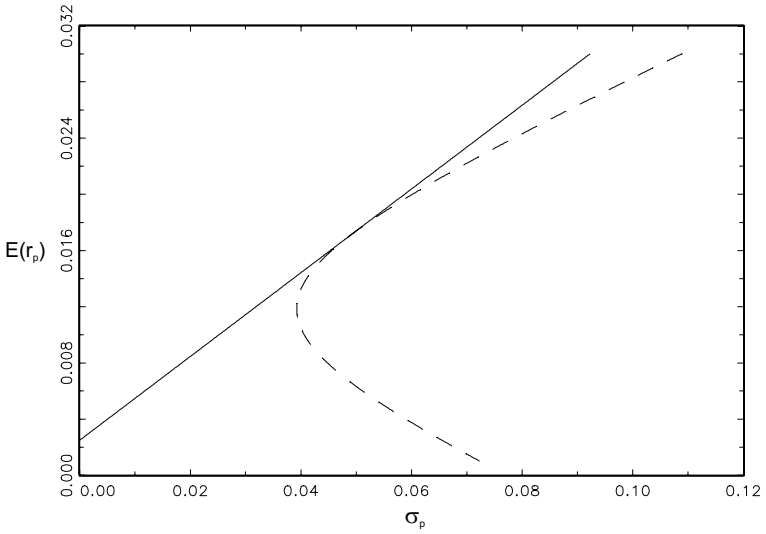



**Fig. 8.4.** Efficient Frontier for an Equity Portfolio of US Firms

measure of wealth, Benninga (1998). It is also shown that betas systematically vary over the business cycle. Tests for anomalies and nonlinearities in the beta pricing model can be found in Fama and French (1992). A more fundamental criticism of the CAPM from a dynamic perspective has been presented by Campbell and Viceira (2002).

## 8.4 Efficient Frontier for an Equity Portfolio

In the next step we compute the Markowitz efficient frontier for an equity portfolio with no short-sales restrictions using the aforementioned Gauss program to solve the quadratic programming problem involved in computing the efficient frontier. We take monthly data for the stock price of twelve US corporations for the sample period of 1972-1991.12. Except the month of October 1987 this is quite a tranquile period for stock market prices.

Figure 8.4 shows the computed Markowitz efficient frontier for a portfolio of twelve US corporations. The horizontal axis represents the standard deviations and the vertical axis the expected return of the equity portfolio. In addition, the straight line represents the locus of all combinations of the risk free asset, here chosen as monthly T-bill, 0.03/12, and the return from the risky equity portfolio. This line is also called the capital market line. The investor's risk preference would determine a point on the capital market line. Moreover, the tangency point of the capital market line and the efficient frontier represents a portfolio with risky assets only and any point beyond the tangency point requires borrowing from credit market, at the risk free rate to invest in a portfolio of risky assets.

## 8.5 Conclusions

So far we have only considered static portfolio theory. Some further extensions and generalizations of the static portfolio theory are given in Campbell, Lo and MacKinlay (1997). The above summary on the CAPM and the brief survey on the empirical work on the CAPM should serve here as an introduction to dynamic portfolio theory. Dynamic portfolio theory builds on the intertemporal framework. An introduction to this is given in the subsequent two chapters and again taken up in Chaps. 15-16.

# Part IV

# Asset Pricing and Economic Activity

CHAPTER 9

# Consumption Based Asset Pricing Models

## 9.1 Introduction

In this chapter we give an introduction to the intertemporal consumption based dynamic asset market models using common preferences for the household. These models employ an intertemporal framework and thus represent dynamic asset pricing theories. In the subsequent chapter we will study a prototype asset pricing model that includes production and is based on the stochastic growth or RBC model. Although both chapters dealing with modern asset pricing theory employ utility functions, in some versions of the consumption based asset pricing theory the dividend stream is frequently exogenously given whereas in the model with production the dividend is endogenously generated from the firms' income. However, we want to note that there are other production based asset pricing models that do not use utility theory, see Cochrane (1991, 1996).

Before introducing specific models we need to briefly define the present value of an asset used in this context. Note that the CAPM assumes that investors are concerned with the mean and the variance of the returns. In the consumption based capital asset pricing model (CCAPM) investors are concerned with mean and variance of consumption. Subsequently, the optimal consumption path of a representative household is derived from the first-order condition of an intertemporal maximization problem with a CCAPM model. This gives us the Euler equation, the stochastic discount factor, the risk-free interest rate, the equity premium and the Sharpe ratio from a dynamic asset pricing model.

## 9.2 Present Value Approach

First, let us introduce some terms related to the present value approach. A return of an asset is defined as

$$r_{t+1} = \frac{p_{t+1} - p_t + d_{t+1}}{p_t}$$

whereby $p_t$ is the price of the asset at the end of period $t$, $p_{t+1}$ is its price at period $t+1$, $d_{t+1}$ the dividend payment at period $t+1$. The dividend stream may be exogenous or generated through production activities. Subsequently we will explore

both possibilities. With a constant discount rate we have

$$p_t = E_t\left(\frac{p_{t+1} + d_{t+1}}{1+r}\right).$$

The solution by forward iteration of the above formula gives us the efficient market hypothesis.

$$p_t = \underbrace{E_t\left[\sum_{i=1}^{k}\left(\frac{1}{1+r}\right)^i d_{t+i}\right]}_{p_t \text{ = fundamental value}} + \underbrace{E_t\left[\left(\frac{1}{1+r}\right)^k p_{t+k}\right]}_{= 0 \text{ for k} \to \infty.}$$

Note that for the efficient market hypothesis to hold the second term in the above formula has to go to zero as time goes to infinity. A special case is the Gordon growth model. Here the dividend is expected to grow at a constant rate $g$. This model takes account of the fact that the second term goes to zero. For the dividend stream it is assumed

$$\begin{aligned} E_t(d_{t+i}) &= (1+g)E_t(d_{t+i-1}) \\ &= (1+g)^i d_t. \end{aligned}$$

Substituting this into the first term of the above equation for the asset price we get, for a constant discount rate,

$$p_t = \frac{E_t(d_{t+1})}{r-g} = \frac{(1+g)d_t}{r-g}.$$

This is the Gordon growth model which shows that the stock price is very sensitive to a change of the discount rate, $r$. The hypothesis that the expected stock return is constant through time is called the martingale property of stock prices. Of course, the dividend stream will change over time and the empirical realistic assumption is that the dividend stream is persistent in the sense that it follows a strong autoregressive process. For the stock price movement in a production based asset pricing theory, as shown in Chap. 3.6, the cash flows of firms – and the dividend stream as formulated above – result from an optimal investment strategy of the firm and it does not grow at a constant rate, except at some steady state solution. This, at least, would result from a dynamic asset pricing model with production. Asset pricing for a dynamic consumption based asset pricing theory is studied next. Central to this study is the Euler equation and the different methods to solve it.

## 9.3 Asset Pricing with a Stochastic Discount Factor

Basic for the consumption based asset pricing model is the utility function of the investor. The utility function captures the fundamental desire for more consumption

rather than the intermediate objectives of the mean and the variance of portfolio returns as studied in the previous chapter. The consumption based asset pricing model that we subsequently derive reads as follows[109]

$$p_t = E_t\Big[\beta\frac{U'(C_{t+1})}{U'(C_t)}x_{t+1}\Big] \tag{9.1}$$

whereby $x_{t+1}$ is equal to $p_{t+1} + d_{t+1}$, with $p_t,\ p_{t+1}$ the price of the asset at time period $t$ and $t+1$, $\beta$ the subjective discount factor, $x_{t+1}$ the pay off and $d_{t+1}$ the dividend.

The above asset pricing equ. (9.1) may be derived from a model with two periods

$$U(C_t, C_{t+1}) = U(C_t) + \beta E_t(U(C_{t+1}))$$

For the preferences $U(C_t)$, usually one employs a power utility function such as

$$U(C_t) = \frac{1}{1-\gamma}C_t^{1-\gamma}$$

This is a utility function with constant relative risk aversion, $\gamma$. We obtain, with $\gamma \to 1$, log utility $U(C) = ln(C)$.

Let $\varepsilon$ be the amount of the asset to be chosen and $e$ the endowment

$$\begin{aligned}\max_{\varepsilon} U(C_t) + E_t[\beta U(C_{t+1})]& \\ \text{s.t. } C_t &= e - p_t\varepsilon \\ C_{t+1} &= e_{t+1} + x_{t+1}\varepsilon\end{aligned}$$

Substitute $C_t$ and $C_{t+1}$ into the first equ. and take the derivative with respect to $\varepsilon$. This gives

$$\begin{aligned}U(C_t)(-p_t) + E[\beta U'(C_{t+1})x_{t+1}] &= 0 \\ p_t U'(C_t) = E[\beta U'(C_{t+1})x_{t+1}]& \end{aligned} \tag{9.2}$$

from which we obtain equ. (9.1), the fundamental asset price equation with a stochastic discount factor.

Equ. (9.2) represents the loss of utility for one unit of the asset which is equal to the increase in discounted utility from the extra pay off at time period $t+1$. We thus obtain that the marginal loss is equal to the marginal gain.

Next we show the relation between the stochastic discount factor and the marginal rate of substitution. Let us use

$$p = E(mx)$$

and

$$m_{t+1} = \beta\frac{U^{'}(C_{t+1})}{U'(C_t)}. \tag{9.3}$$

[109] For further details, see Cochrane (2001, Chap. 1).

Equ. (9.3) is called the stochastic discount factor which is derived from some first order condition.

The basic dynamic asset pricing model of equ. (9.1) can then be written

$$p_t = E_t(m_{t+1}x_{t+1}). \tag{9.4}$$

With no uncertainty we get the risk-free rate:

$$p_t = \frac{1}{R^f}x_{t+1}. \tag{9.5}$$

Riskier assets are valued by using a risk-adjusted discount factor

$$p_t^i = \frac{1}{R^i}E(x_{t+1}^i). \tag{9.6}$$

Hereby we denote $R^f = 1 + r^f$, with $R^f$ the risk free rate, $\frac{1}{R^f}$ the discount factor and $\frac{1}{R^i}$ the risk corrected discount factor.

The marginal rate of substitution (or pricing kernel) is $m_{t+1}$. Thus $m_{t+1}$ is the rate at which the investor is willing to substitute consumption at time $t+1$ for consumption at time $t$.

The risk-free rate can be derived as follows. We can write from (9.5):

$$R_{t+1} = \frac{x_{t+1}}{p_t}$$

or

$$1 = E(mR)$$

Thus

$$R^f = 1/E(m)$$

and $R = \frac{x}{p}$, since $1 = E(mR^f)$ and more specifically $1 = E(m)R^f$.

For the power utility function and turning off uncertainty, with $U'(C) = C^{-\gamma}$, we get

$$R^f = \frac{1}{\beta}\Big(\frac{C_{t+1}}{C_t}\Big)^{\gamma}.$$

It follows that $R^f = \frac{1}{E(m)}$, thus

$$R_t^f = \frac{1}{E_t\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}\Big]}.$$

Next we introduce the risk-correction for the equity return. For risky assets we have $1 = E(mR^i)$.

We use the following definition

$$cov(m, x) = E(mx) - E(m)E(x). \tag{9.7}$$

From $p = E(mx)$, we obtain then

$$p = E(m)E(x) + cov(m, x).$$

Using the definition of $R^f$ (from (9.5)) we get

$$p = \underbrace{\frac{E(x)}{R^f}}_{\text{present value without risk}} + \underbrace{cov(m, x)}_{\text{risk-adjustment}} \tag{9.8}$$

$$p = \frac{E(x)}{R^f} + \frac{cov[\beta U'(C_{t+1}), x_{t+1}]}{U'(C_t)}. \tag{9.9}$$

In (9.8) it is shown that an asset whose pay off covaries with the discount factor has its price raised. On the other hand, one might say, as depicted in (9.9) that an asset price is lowered if its pay-off covaries positively with consumption. Note that marginal utility $U'(C)$ declines as $C$ rises.

Next we derive the equity premium.

Using $1 = E(mR^i)$ again and (9.7), the covariance decomposition, we have

$$1 = E(m)E(R^i) + cov(m, R^i)$$

and using $R^f \equiv \frac{1}{E(m)}$ we have

$$E(R^i) - R^f = -R^f cov(m, R^i)$$

$$E(R^i) - R^f = -\frac{cov[u'(c_{t+1}), R^i_{t+1}]}{E[u'(c_{t+1})]} \tag{9.10}$$

The equ. (9.10) represents the equity premium. Here too we can observe that risky assets have an expected return equal to the risk free rate plus a term that represents the adjustment for risk. Assets whose returns covary positively with consumption make consumption more volatile and thus needs to promise higher expected returns to induce the investors to hold them. Finally, we can write for the Sharpe ratio

$$SR = \frac{E(R^i) - R^f}{\sigma_{R^i}}$$

which gives us our measure of the return-risk trade-off.

## 9.4 Derivation of some Euler Equations

### 9.4.1 Continuous Time Euler Equation

We first derive the Euler equation in the context of a continuous time model. We take a standard intertemporal model.[110] Note that here all variables are here written

[110] For details, see Flaschel, Franke and Semmler (1997, Chap. 5) and Romer (1996, Chap. 7.).

in efficiency units, with $c$, consumption; $n$, population growth; $\mu$ the exogenous productivity growth; $f(k) = k^\beta$ the production function and $\gamma$ the coefficient of relative risk aversion. The latter is the inverse of the elasticity of substitution between consumption at different dates.

The optimization problem we consider is

$$\max_c \int_0^\infty e^{-\rho t} \frac{c^{1-\gamma}}{1-\gamma} dt$$

s.t.

$$\dot{k} = f(k) - c - (n+\mu)k.$$

We employ the Hamiltonian

$$H(c,k,\gamma) = u(c) + \lambda(k^\beta - c - (n+\mu)k)$$

and derive the first-order condition

$$\frac{\partial H}{\partial c} = 0 \text{ which gives } c^{-\gamma} = \lambda. \tag{9.11}$$

Taking the time derivatives on both sides of (9.11) we obtain $-\gamma c^{-\gamma-1} \cdot \dot{c} = \dot{\lambda}$. Dividing both sides by $\lambda$ gives

$$-\gamma \frac{\dot{c}}{c} = \frac{\dot{\lambda}}{\lambda}. \tag{9.12}$$

On the other hand from the derivative of the Hamiltonian with respect to $k$ we obtain

$$\frac{\dot{\lambda}}{\lambda} = (\rho - \beta k^{\beta-1} + n + \mu) \tag{9.13}$$

whereby $\beta k^{\beta-1}$ is the marginal product of capital, which we denote by $r$. Then (9.12) and (9.13) give us

$$\frac{\dot{c}}{c} = \frac{(\beta k^{\beta-1} - \rho - n - \mu)}{\gamma} = \frac{r - \rho - (n+\mu)}{\gamma}. \tag{9.14}$$

Equ. (9.14) describes the optimal consumption path of the representative household obtained in feedback form from the evolution of capital stocks. This is the form the Euler equation takes in the above continuous time model. A survey of empirical tests of such a type of Euler equation based on the power utility function can be found in Campbell et al. (1997, Chap. 8). Euler equations for growing economies are derived in Greiner, Semmler and Gong (2004). Next we describe the discrete time counter part for a two period model.

### 9.4.2 Discrete Time Euler Equation: 2-Period Model

Subsequently, we present a model with a household that lives for two periods. We denote by $w_t$ the wage; $A_t = (1+g)A_{t-1}$ productivity growth and $r_{t+1}$ the return from an asset, and $C_t, C_{t+1}$ the level of consumption.

The first and the second period consumption are related by

$$C_{2t+1} = (1 + r_{t+1})(w_t A_t - C_{1t}). \tag{9.15}$$

Then

$$U_t = \frac{C_{1t}^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho}\frac{C_{2t+1}^{1-\gamma}}{1-\gamma} \tag{9.16}$$

which gives us the two periods' utility.

Then we maximize (9.16) s.t. (9.15) by using the Lagrangian

$$L = \frac{C_{1t}^{1-\gamma}}{1-\gamma} + \frac{1}{1+\rho}\frac{C_{2t+1}^{1-\gamma}}{1-\gamma} + \lambda(A_t w_t - C_{1t} - \frac{1}{1+r_{t+1}}C_{2t+1}).$$

The first-order conditions with respect to $C_1, C_2$ read

$$C_{1t}^{-\gamma} = \lambda \tag{9.17}$$

$$\frac{1}{1+\rho}C_{2t+1}^{-\gamma} = \frac{\lambda}{1+r_{t+1}}. \tag{9.18}$$

Substituting (9.17) into (9.18) gives:

$$\frac{1}{1+\rho}C_{2t+1}^{-\gamma} = \frac{1}{1+r_{t+1}}C_{1t}^{-\gamma} \qquad \text{or} \tag{9.19}$$

$$1 = \frac{1+r_{t+1}}{1+\rho}\left(\frac{C_{2t+1}}{C_{1t}}\right)^{-\gamma} = (1+r_{t+1})\,\beta\left(\frac{C_{2t+1}}{C_{1t}}\right)^{-\gamma} = 1 \tag{9.20}$$

whereby $\beta\left(\frac{C_{2t+1}}{C_{1t}}\right)^{-\gamma}$ represents the deterministic form of the discount factor.

From (9.20) we obtain the optimal consumption path

$$\frac{C_{2t+1}}{C_{1t}} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{1/\gamma}. \tag{9.21}$$

For $\mathrm{r}_{t+1} > \rho$ it follows that $C_{2t+1}$ will increase, given $\frac{1}{1+\rho} = \beta$.

### 9.4.3 Discrete Time Euler Equation: n-Period Model

For an n-period model we have a budget constraint

$$\sum_{t=1}^{T}\left(\frac{1}{1+r_t}\right)^t C_t \leq A_0 + \sum_{t=1}^{T}\frac{1}{(1+r)^t}Y_t. \tag{9.22}$$

Assuming a power utility function:

$$\sum_{t=1}^{T} \frac{1}{(1+\rho)^t} \cdot \frac{C_t^{1-\gamma}}{1-\gamma} \tag{9.23}$$

then from (9.22) and (9.23) we again get

$$\frac{1}{(1+\rho)^t} C_t^{-\gamma} = (1+r)\frac{1}{(1+\rho)^{t+1}} C_{t+1}^{-\gamma}.$$

Thus

$$1 = (1+r)\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{9.24}$$

and

$$\frac{C_{t+1}}{C_t} = \left(\frac{1+r}{1+\rho}\right)^{1/\gamma} \tag{9.25}$$

which gives us the optimal consumption path.

## 9.5 Consumption, Risky Assets and the Euler Equation

Next, we introduce risky assets and discuss the Euler equation for the stochastic case as an example. In the stochastic case the Euler equation reads

$$U'(C_t) = \frac{1}{1+\rho} E_t \left[\left(1 + r_{t+1}^i\right) U'\left(C_{t+1}\right)\right]. \tag{9.26}$$

Here again, the right hand side represents the gain in expected marginal utility from investing the dollar in asset $i$, selling it at time $t+1$ and consuming the proceeds. We can write

$$1 = E_t \left[\left(1 + r_{t+1}^i\right) \beta \frac{U'\left(C_{t+1}\right)}{U'\left(C_t\right)}\right] \tag{9.27}$$

where $\beta = \frac{1}{1+\rho}$ and $\beta \frac{U'(C_{t+1})}{U'(C_t)} = m_{t+1}$ which is our stochastic discount factor or pricing kernel.

Since the expectation of the product of two variables equals the product of their expectations plus their covariance we can rewrite (9.26) as

$$U'(C_t) = \frac{1}{1+\rho} E\left[1 + r_{t+1}^i\right] E_t\left[U'\left(C_{t+1}\right)\right] + Cov_t(1 + r_{t+1}^i, U'\left(C_{t+1}\right)) \tag{9.28}$$

where the latter expression, $Cov_t(1 + r_{t+1}^i, U'\left(C_{t+1}\right))$, is as discussed above, a relevant factor in the CCAPM.

For example, with quadratic utility $U(C) = C - \frac{aC^2}{2}$ we have (see Romer, 1996, Chap. 7.5)

$$E_t(1 + r^i_{t+1}) = \frac{1}{E_t U'(C_{t+1})} \left[ (1+\rho)\, U'(C_t) + a\, Cov_t(1 + r^i_{t+1}, C_{t+1}) \right]. \tag{9.29}$$

The higher the covariance of an asset's payoff with consumption the higher its expected return must be.

Next let us assume a risk-free asset return. The payoff is certain therefore we have $Cov_t(1 + r^i_{t+1}, C_{t+1}) = 0$ and

$$1 + r^f_{t+1} = \frac{(1+\rho)\, U'(C_t)}{E_t \left[ U'(C_{t+1}) \right]} \tag{9.30}$$

or

$$1 + r^f_{t+1} = 1/E_t(m_{t+1}). \tag{9.31}$$

Next subtracting (9.30) from (9.29) we get the equity premium

$$E_t(r^i_{t+1}) - r^f_{t+1} = \frac{a\, Cov_t(1 + r^i_{t+1}, C_{t+1})}{E_t \left[ U'(C_{t+1}) \right]}. \tag{9.32}$$

The (expected) return premium is proportional to the covariance of its return with consumption whereby

$$Cov_t(1 + r^i_{t+1}, C_{t+1}) = \text{ consumption } \beta \text{ in CCAPM}.$$

The central prediction of the CAPM is that the premium that assets offer are proportional to their consumption beta.

Table 9.1[111] presents some stylized facts on consumption and asset returns. Rows 1-2 show the difference of the equity and risk-free returns. The next five rows illustrate the low volatility of the risk-free rate, consumption growth and dividend growth as compared to the large volatility of the stock returns. As to the correlation of consumption and dividend growth with the stock return, one can observe a weak correlation but one can in particular note a very weak covariance of consumption growth with stock returns (see the right hand side of row eight in table 9.1). Although such a strong co-variance is postulated in theory, to match the equity premium, only a very weak co-variance is observable in the data.

[111] The data in the following table is from Campbell (1998) and Campbell et al. (1997).

**Table 9.1.** Stylized Facts on CCAPM U.S. Data: 1947.2-1993.4

| | |
|---|---|
| mean of real return on stocks | 7.2% annual return |
| mean of riskless rate | 3 months T-Bill is 0.7% per year |
| volatility of real stock returns | annualized standard deviation is 15.8% |
| volatility of riskless rate | annualized standard deviation of Tbill is 1.8%, most of it due to inflation risk |
| volatility of real consumption growth | standard deviation of growth rate of real consumption of nondurables is 1.1% |
| volatility of real dividend growth | volatility at short horizon is 29% annualized, standard deviation with quarterly data, at annual frequency is 7.3% |
| correlation of real consumption growth and real dividend growth | a weak correlation of 0.05 for quarterly data, at 2-4 year frequency it increases to 0.2 |
| correlation of real consumption growth and real stock return | at quarterly frequency it is 0.21, at 1-year frequency it is 0.34 and declines at longer horizon, covariance is 0.0027, $\sigma_{\Delta c} = 0.033, \sigma_{R^i} = 0.167$ |
| correlation of real dividend growth and real stock return | a weak correlation of 0.04 at quarterly data, for 1-year horizon it is 0.14, 2-year horizon 0.28 |

Making the assumption of log-normality for asset prices one can derive from an Euler equation such as (9.27) for preferences with power utility the equity premium in simple terms (see Campbell 1997), using $\gamma$ as the coefficient of relative risk aversion. The equity premium can then be written as[112]

$$E_t \left[ r^i_{t+1} - r^f_{t+1} \right] + \frac{\sigma_i^2}{2} = \gamma \sigma_{ic} \tag{9.33}$$

with $\sigma_i^2 =$ variance of asset returns; $\sigma_{ic} =$ covariance of asset returns with consumption growth.

Using the equ. (9.33) one can discuss the empirical components that might explain the equity premium puzzle. This is shown in the next table which summarizes some results from Campbell (1998). Taking the equity premium, the variance of the asset and the covariance of the return with consumption growth, $\sigma_{ic}$ as given, we see that there is, with some exception, for most countries a very large $\gamma$ required to explain the equity premium. Here the $\gamma$ is computed by dividing column 3 by column 5, multiplied by 100. Column 4 is the annualized standard deviation of excess stock returns.[113]

[112] For details of the derivation and econometric implications, see also Campbell et al. (1997, Chap. 8.2).

[113] The data in the following table is from Campbell (1998).

**Table 9.2.** The Equity Premium Puzzle in International Data

| Country | Sample Period | $E_t\left[r^i_{t+1} - r^f_{t+1}\right] + \frac{\sigma_i^2}{2} = \gamma\sigma_{ic}$ | $\sigma_i$ | $\sigma_{ic}$ | $\gamma$ |
|---|---|---|---|---|---|
| AUL | 1970.1-1994.2 | 3.687 | 24.080 | 9.025 | 40.858 |
| CAN | 1970.1-1994.2 | 2.439 | 17.209 | 5.849 | 41.689 |
| FR | 1970.2-1994.2 | 6.763 | 23.060 | -3.712 | $< 0$ |
| GER | 1978.4-1994.2 | 6.596 | 21.331 | 2.480 | 265.960 |
| ITA | 1971.2-1993.2 | 2.100 | 28.172 | -0.225 | $< 0$ |
| JAP | 1970.1-1993.3 | 7.181 | 21.689 | 5.463 | 131.442 |
| NTH | 1977.2-1994.2 | 9.368 | 16.189 | 2.578 | 363.328 |
| SP | 1974.2-1993.2 | -0.309 | 25.668 | -0.986 | 31.310 |
| SWD | 1970.1-1994.1 | 9.537 | 23.892 | 0.167 | 5699.045 |
| SWT | 1975.4-1994.2 | 8.852 | 18.726 | -1.013 | $< 0$ |
| UK | 1970.1-1994.2 | 8.282 | 22.413 | 5.500 | 150.583 |
| USA | 1970.1-1993.3 | 6.245 | 17.842 | 3.878 | 135.255 |
| USA | 1947.2-1993.3 | 7.693 | 15.597 | 3.166 | 243.014 |
| SWD | 1919 - 1992 | 5.207 | 18.721 | 8.385 | 62.108 |
| UK | 1919 - 1992 | 8.525 | 21.802 | 21.833 | 39.048 |
| USA | 1890 - 1991 | 6.211 | 18.768 | 30.079 | 20.650 |

Thus we also can see that $\gamma$ does not seem to be a very robust parameter and it is, where positive, excessively large.

From the Euler equation (9.27) based on the power utility function, one can also derive the following testable equation[114] postulated to hold in empirical data for risky assets and consumption growth ($\Delta c$)

$$r^i_{t+1} = \mu + \gamma \Delta c_{t+1} + \mu_{t+1} \tag{9.34}$$

with $c$ the log of consumption. Campbell et al. (1997, Chap. 8.2) report results for U.S. data that are not supportive of equ. (9.34). Campbell (1998) shows the failure of (9.34) also for international data.

As aforementioned, the most common utility function used in economics is the power utility function

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{9.35}$$

for which the Euler equation is

$$1 = E\left[\left(1 + r^i_{t+1}\right)\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]. \tag{9.36}$$

[114] See also Campbell et al. (1997, Chap. 8.2).

The power utility function is time separable. A non-separable utility function is given by

$$U_t = \left\{ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left( E[U_{t+1}^{1-\gamma}] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{9.37}$$

$$\theta = \frac{1-\gamma}{(1-1/\psi)}.$$

For details of such a function that originates in Epstein and Zin (1989, 1991) and the derivation of its Euler equation, see Campbell et al. (1997, Chap. 8).

Another non-separable utility function frequently used in economics builds on habit formation. It can be written either as ratio model[115]

$$U_t = E \left\{ \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j}/X_{t+j})^{1-\gamma} - 1}{1-\gamma} \right\} \tag{9.38}$$

or as difference model

$$U_t = E \left\{ \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma}}{1-\gamma} - 1 \right\} \tag{9.39}$$

with $X_t = C_{t-1}^*$ or $X_t = \bar{C}_{t-1}^*$

where $C_{t-1}^*$ is the agent's past consumption and $\bar{C}_{t-1}^*$ the aggregate past consumption. See Campbell et al. (1997: 327) for derivation of the Euler equation for those types of utility functions. For a discussion on the use of habit formation to study the equity premium and the time varying Sharpe-ratio, see Wöhrmann, Semmler and Lettau (2001). For a further study of the habit formation model, as well as models that move beyond the consumption based asset pricing model, see Chap. 15. There we will discuss recent advances in asset pricing that are coming closer to solve the equity premium puzzle.

## 9.6 Conclusions

In studying the consumption based dynamic asset pricing theory we have first presumed that there is an exogenously given dividend stream which is equal to the consumption stream of the agent whose utility function could take on different forms. For the case of simple utility functions we have also derived the Euler equation as the essential equation to study dynamic asset pricing. Appendix 2 derives the Euler equation from dynamic programming. As we also have shown, using preferences such as

[115] See Abel (1990, 1996).

log or power utility, the equity premium and the Sharpe ratio cannot match the equity premium and the Sharpe ratio of actual time series data. For those preferences a too high parameter of risk aversion and/or a strong covariance of consumption growth with asset returns are required which one does not find in the data. The question thus remains whether models that more explicitly take into account production activities or rely on other types of preferences may be able to provide a better match of theory and the data. Asset pricing for production economies is taken up next. Other preferences are considered in Chap. 15.

CHAPTER 10

# Asset Pricing Models with Production

## 10.1 Introduction

Asset pricing models[116] for production economies have been studied on the basis of stochastic growth models. A prototype stochastic growth model is the Real Business Cycle (RBC) Model, which has become one of the standard macroeconomic models. It tries to explain macroeconomic fluctuations as equilibrium reactions of a representative agent economy with complete markets. Many refinements have been introduced since the seminal papers by Kydland and Prescott (1982) and Hansen (1985) improved the model's fit with the data. Often the implications for asset prices are spelled out for RBC models. Asset prices contain valuable information about the intertemporal decision making of economic agents a mechanism is at the heart of the RBC methodology. Here, we summarize results from the work by Lettau, Gong and Semmler (2001) that uses a log-linear variant of the RBC model developed by Campbell (1994) and estimate the parameters of a standard RBC model by taking its asset pricing implications into account.

In fact, modelling asset prices and risk premia in models with production is much more challenging than in exchange economies. Most of the asset pricing literature has followed Lucas (1978) and Mehra and Prescott (1985) in computing asset prices from the consumption based asset pricing models with an exogenous dividend streams. Production economies offer a much richer, and realistic environment. First, in economies with an exogenous dividend stream and no savings consumers are forced to consume their endowment. In economies with production where asset returns and consumption are endogenous consumers can save and hence transfer consumption between periods. Second, in economies with an exogenous dividend stream the aggregate consumption is usually used as a proxy for equity dividends. Empirically, this is not a very sensible modelling choice. Since there is a capital stock in production economies, there is a more realistic modelling of equity dividends is possible.

Christiano and Eichenbaum (1992) use a Generalized Method of Moments (GMM) procedure to estimate the RBC parameters. Their moment restrictions only concern the real variables of the model. Semmler and Gong (1996) estimate the model using a Maximum Likelihood method. The purpose of these sections is to take restrictions on asset prices implied by the RBC model into account when exploring the parameters of the model. One can introduce asset pricing restrictions step-by-

[116] The subsequent part is based on Lettau, Gong and Semmler (2001), see also Gong and Semmler (2006, Ch. 6). For another type of asset pricing model with production not employing a utility functions, see Cochrane (1991, 1996).

step to clearly demonstrate the effect of each new restriction. As will become clear, the more asset market restrictions are introduced, the more difficult it becomes to empirically match the model with the data. First we report estimations of the model that uses only real variables, as in Christiano and Eichenbaum (1992) and Semmler and Gong (1996). We can report parameters like risk aversion, the discount rate and depreciation. The first additional restriction is the risk-free interest rate. We attempt to match the observed 30-day T-bill rate to the one-period risk-free rate implied by the model. We find that the estimates are fairly close to those obtained without the additional restriction suggesting that the model's prediction of the risk-free rate is broadly consistent with the data.

The second additional asset pricing restriction concerns the risk-return tradeoff implied by the model as measured by the Sharpe-ratio, or the price of risk as discussed in Chap. 8. This variable determines how much expected return agents require per unit of financial risk. Hansen and Jagannathan (1991) and Lettau and Uhlig (1997 a,b) show how the Sharpe-ratio can be used to evaluate the ability of different models to generate high risk premia. Introducing a Sharpe-ratio as a moment restriction to the estimation procedure requires an iterative procedure in order to estimate the risk aversion parameter. More importantly, the model cannot be estimated any more since the parameters become unbounded. In other words, the model cannot fulfill moment restrictions concerning real variables and the Sharpe-ratio simultaneously. The problem is that matching the Sharpe-ratio requires high risk aversion which on the other hand is incompatible with the observed variability of consumption. This tension which is at the heart of the model makes it impossible to estimate. We experiment with various versions of the model, e.g. fixing risk aversion at a high level and then estimating the remaining parameters. Here, too, we are not able to estimate the model while simultaneously generating sensible behavior on the real side of the model as well as obtaining a high Sharpe-ratio.

The theoretical framework of this undertaking is based on Lettau and Uhlig (1997 a,b). He presents closed-form solutions for risk premia of equity and long real bonds, the Sharpe-ratio as well as the risk-free interest rate for the loglinear RBC model of Campbell (1994). These equations can be used as additional moment restrictions in the estimation of the RBC model. The advantage of the log-linear approach is that the closed-form solutions for the financial variables can be directly used in the estimation algorithm. No additional numerical procedure to solve the model is necessary. This reduces the complexity of the estimation substantially.

The estimation technique used here follows the Maximum Likelihood (ML) method in Semmler and Gong (1996). However, the algorithm has to be modified to allow for a simultaneous estimation of the risk aversion parameter and the Sharpe-ratio. For our time series of real variables we employ the data set provided by Christiano (1988).[117]

[117] The estimation is conducted through a numerical procedure that allows us to iteratively compute the solution of the decision variables for given parameters and to revise the parameters through a numerical optimization procedure so as to maximize the Maximum Likelihood function, see Semmler and Gong (1996).

**Table 10.1.** Stylized Facts of Asset Markets: US and European Data (Unconditional Mean and Variance)

| Variable | U.S. | | Europe | |
|---|---|---|---|---|
| | mean | std.dev. | mean | std.dev |
| T-bill | 0.18 | 0.86 | 0.43 | 0.89 |
| Stock-return | 2.17 | 7.53 | 1.81 | 7.37 |
| Equity premium | 1.99 | 7.42 | 1.38 | 7.04 |
| Sharpe-ratio | 0.27 | | 0.19 | |

Next we introduce some stylized facts. Then we discuss the log-linearization of the baseline RBC model and the closed-form solutions for the financial variables as computed in Lettau, Gong and Semmler (2001). We present some results for the different variants of our RBC model and interpret our results contrasting the asset market implications of our estimates to the stylized facts of the asset market.

For further literature on asset price implications of the RBC model, see Cooley (1995), Campbell (1994, 1997), Canova and De Nicolo (1995), Danthine et al. (1992), Rouwenhorst (1995), Lettau and Uhlig (1997 a,b), Jerman (1998), Wöhrmann, Semmler and Lettau (2001) and Boldrin et al. (2001).

## 10.2 Stylized Facts

Before we report some results of our production based asset pricing models we again want to present some stylized facts that will help the reader to judge the success of the subsequent models. Table 10.1 reports the appropriate stylized facts of asset markets.[118]

## 10.3 The Baseline RBC Model

We use the notation $Y_t$ for output, $K_t$ for capital stock, $A_t$ for technology, $N_t$ for normalized labor input and $C_t$ for consumption. Using power utility the maximization problem of a representative agent is assumed to take the form

[118] For the US asset market data represent real returns and are from Lettau, Gong and Semmler (2001), 1947.1-1993.3. All data are at quarterly frequency. Asset market units are percent per quarter. The T-bill rate is the 3 months T-bill rate. For Europe data are taken from Eurostat (1997), all data 1970.1-1993.3, are at quarterly frequency. Asset market units represent real returns and are percent per quarter. The Sharpe-ratio is the mean of equity prices divided by their standard deviation. Following Canova and Nicola (1995) for each of the variables a European variable is obtained by employing a weighted average of the respective variables for Germany, France, Italy and the U.K, where GNP ratios are taken as the weight. This holds also for the 3 months T-bill rate. In the case of the U.K. the T-bill rate was obtained by averaging short term rates.

$$\text{Max } E_t \sum_{t=0}^{\infty} \beta^i \left[ \frac{C^{1-\gamma}}{1-\gamma} + \theta \log(1 - N_{t+i}) \right], \tag{10.1}$$

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t, \tag{10.2}$$

with $Y_t = (\mathrm{A}_t N_t)^{\alpha} K_t^{1-\alpha}$, log A: $a_t = \phi a_{t-1} + \varepsilon_t$. The latter is a stochastic process for the technology shock.

According to Campbell (1994), from the first order condition of this maximization problem one obtains two decision rules.

The first is the optimal decision of consumption which is of the same type as the one in Chap. 9 where we derived the Euler equation.

$$\begin{aligned} C_t^{-\gamma} &= \beta E_t \left\{ C_{t+1}^{-\gamma} R_{t+1} \right\} \\ &\Rightarrow 1 = E_t \left[ m_{t+1} R_{t+1} \right] \end{aligned} \tag{10.3}$$

$$m_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{10.4}$$

The second is the optimal decision of labor input which is

$$\frac{1}{\theta(1-N_t)} = \alpha \frac{A_t^{\alpha}}{C_t} \left( \frac{K_t}{N_t} \right)^{\alpha}, \tag{10.5}$$

where in equ.(10.3) $\mathrm{R}_{t+1}$ is the gross rate of return on investment in capital, which corresponds to the marginal product of capital in production plus undepreciated capital

$$R_{t+1} \equiv (1-\alpha) \left( \frac{A_{t+1} N_{t+1}}{K_{t+1}} \right)^{\alpha} + 1 - \delta. \tag{10.6}$$

At the steady state, technology, consumption, output and capital stock all grow at a common rate G, G$\equiv A_{t+1}/A_t$. Meanwhile, (10.4) becomes

$$G^{\gamma} = \beta R, \tag{10.7}$$

where R is the steady state of $\mathrm{R}_{t+1}$. Using lower case letters for the corresponding variables in logs, (10.7) can further be written as

$$\gamma g = \log(\beta) + r. \tag{10.8}$$

This indeed defines the relationship among g, r, $\beta$ and $\gamma$.

Note that there can be different ways to solve the above dynamic optimization problem. Here, we have used the log-linear approximation method which has also been used in King et al. (1988a, b), Campbell (1994) among others, for details see Gong and Semmler (2005). To apply this method, one first needs to detrend the variables so as to transform them into stationary forms. For a variable $X_t$ the detrended variable $x_t$ is assumed to take the form log $(X_t/\bar{X}_t)$, where $\bar{X}_t$ is the value of $X_t$ on its steady state path. One, therefore, can think of $x_t$ as the variable of zero-mean deviation from the steady state growth path of $X_t$. The advantage to

use this method of detrending is that one can drop the constant terms in the decision rules. Therefore, some structural parameters may not appear in the decision rule and hence one need not estimate them.

Campbell (1994) shows that the solution, using the log-linear approximation method, can be written as

$$c_t = \eta_{ck} k_t + \eta_{ca} a_t, \tag{10.9}$$

$$n_t = \eta_{nk} k_t + \eta_{na} a_t, \tag{10.10}$$

and the law of motion of capital is

$$k_t = \eta_{kk} k_{t-1} + \eta_{ka} a_t, \tag{10.11}$$

where $\eta_{ck}, \eta_{ca}, \eta_{nk}, \eta_{na}, \eta_{kk}$ and $\eta_{ka}$ are all complicated functions of the parameters $\alpha, \delta, r, g, \gamma$, and $\bar{N}$ ($\bar{N}$ is the steady state value of $N_t$). We shall remark that the parameters $\theta$, and $\beta$ do not appear in the different $\eta_{ij}$'s $(i, j = c, n, k, a)$. Therefore, one can not estimate them if one employs equations (10.9)-(10.11) as the moment restrictions of the estimation. However, one should also note that $\beta$ can be inferred from (10.8) for given g, $\gamma$ and $r$.

## 10.4 Asset Market Restrictions

Our asset market restrictions attempt to match the aforementioned stylized facts. We thus want to match the following risk-free rate

$$E\left[b_t - r_t^f\right] = 0 \tag{10.12}$$

Sharpe-ratio

$$SR = \gamma \eta_{ca} \sigma_\varepsilon \tag{10.13}$$

Premium on long term bond

$$LTBP = -\gamma^2 \beta \frac{\eta_{ck}\eta_{ka}}{1 - \beta\eta_{kk}} \eta_{ca}^2 \sigma_\varepsilon^2, \tag{10.14}$$

Premium on equity

$$LTEP = \left( \frac{\eta_{dk}\eta_{kk} - \eta_{da}\eta_{kk}}{1 - \beta\eta_{kk}} - \gamma\beta \frac{\eta_{ck}\eta_{kk}}{1 - \beta\eta_{kk}} \right) \gamma \eta_{ca}^2 \sigma_\varepsilon^2. \tag{10.15}$$

Above, $r_t^f$ is regarded to be the risk free interest rate, which is given by

$$r_t^f = \gamma \frac{\eta_{ck}\eta_{kk}}{1 - \eta_{ka} L} \varepsilon_t, \tag{10.16}$$

where L is the lag operator, $\varepsilon_t$ is the i.i.d. innovation, with the standard deviation of the shock as $\sigma_\varepsilon$.

In Lettau, Gong and Semmler (2001) we have obtained the following estimation results.

### 1. Parameter Estimates[119]

**Table 10.2.** Summary of Estimation Results

| | $\delta$ | $r$ | $\gamma$ | $\beta$ |
|---|---|---|---|---|
| variant 1 | 0.0189 | 0.0077 | 1 | 0.9972 |
| | (0.0144) | (0.0160) | | |
| variant 2 | 0.0220 | 0.0041 | 1 | 1.0001 |
| | (0.0132) | (0.0144) | | |
| variant 3 | 0.0344 | 0.0088 | 2.0633 | 1.0015 |
| | (0.0156) | (0.0185) | (0.4719) | |

variant 1 is the estimation without (10.12), variant 2 is with (10.12) and variant 3 includes the estimation of $\gamma$.

### 2. Asset Market Restrictions

**Table 10.3.** Asset Pricing Implications

| | SR | $\sigma_c$ | $\xi$ | LT Bprem | LT Eq Rem |
|---|---|---|---|---|---|
| variant 1 | 0.0065 | 0.0065 | 0.66 | 0.000 | -0.082 |
| variant 2 | 0.0065 | 0.0065 | 0.66 | -0.042 | -0.085 |
| variant 3 | 0.0180 | 0.0087 | 0.66 | -0.053 | -0.091 |

Note that the Sharpe-ratio is SR= $\gamma\sigma_c(\gamma)$ or SR= $\gamma\eta_{c\alpha}\sigma_\varepsilon$. Hereby $\sigma_c$ is the standard deviation in consumption which can be computed to $\eta_{c\alpha}\sigma_\varepsilon$, and $\xi$ is the leverage ratio (see the appendix of Lettau, Gong and Semmler 2001).

In contrast to the empirical Sharpe-ratio presented above which is about 0.27, given the parameters from our estimation, the computed Sharpe-ratio is off by a factor of 40 for variant 1 and 2. Due to the slight increase in $\gamma$ and $\sigma_c$, the computed Sharpe-ratio in variant 3 seems to be improved, but is still far away from the empirical Sharpe-ratio of 0.27.

### 3. Matching the SR

**Table 10.4.** Matching the Sharpe-Ratio

| | $\delta$ | $r$ | $\gamma$ |
|---|---|---|---|
| variant 4 | 1 | 0 | 50 (given $\gamma$=50) |
| variant 5 | 1 | 1 | 60 (matching SR) |

[119] The standard errors are in parentheses.
[119] The standard errors are in parentheses.

We first consider the exercise that $\gamma = 50$ and the remaining parameters, $\sigma$ and $r$, are estimated. Note that this variant, called variant 4, is different from variant 2 only in the way that $\gamma$ is pre-fixed to 50 rather than 1.

As an alternative exercise, we try to enforce the predicted Sharpe-ratio to be matched to the empirical one of 0.27 when we estimate the structural parameters $\delta$ and $r$. This can be done as follows. First, from (10.13), when SR=0.27, we obtain

$$\gamma = \frac{0.27}{\eta_{ca}(\gamma)\sigma}. \tag{10.17}$$

Thus, if we impose the restriction that the Sharpe-ratio of the model should be matched with the empirical Sharpe-ratio there does not exist a $\gamma$ that would help to match those two Sharpe-ratios.

## 10.5 Conclusions

We have discussed for the case of a power utility function a dynamic production based asset pricing model and shown that the stochastic growth model of a RBC type with power utility is not able to match asset price restrictions except for the risk-free rate. Boldrin, Christiano and Fisher (2001) take into account habit formation in the utility function and adjustment costs of capital in a two sector model. By doing so they are more successful in replicating financial statistics, such as the equity premium and the Sharpe-ratio in the context of a RBC model but the model then fails along some real dimensions. Recently in numerous contributions further generalizations of the above base line model are considered. This is undertaken in the papers by Den Haan and Marcet (1990), Duffie and McNelis (1997) and Wöhrmann, Semmler and Lettau (2001). In those papers numerical solutions of the Euler equation are explored and in the latter paper time varying asset price characteristics, in particular a time varying Sharpe-ratio, are studied. Finally we want to note that both the consumption and the production based dynamic asset pricing theory have used, by and large, the framework of a representative agent who has preferences over a consumption stream. Recently, researchers have departed from this approach by employing the framework of agent based and evolutionary modelling of asset markets, see Chap. 14 and by moving beyond and consumption based asset pricing models, see Chap. 16.

CHAPTER 11

# Balance Sheets and Financial Instability

## 11.1 Introduction

So far we have considered financial markets[120] such as the money and bond markets, the credit market and the stock market separately. Next, we go back to the macroeconomic perspective and consider, more properly, the interaction of those markets, their response to monetary, financial and exchange rate shocks as well as their interaction in propagating financial instability affecting output. To study those problems we will heavily rely on the balance sheets of the economic agents. Indeed, balance sheets of economic agents have been at the center of recent studies on the financial interaction, the financial sector and economic activity.[121] We will leave out, in a first step, the foreign exchange market which might in fact be very important for triggering and propagating financial instability. Dynamic models including the foreign exchange market will be developed in Chap. 12. In Chap. 13 then a portfolio model with international assets is introduced. The present chapter will prepare the ground work for the next two chapters.

Although the Keynesian oriented strand of financial modelling has emphasized the importance of the financial sector for economic activity, the Keynesian aggregate model has often focused only on money and neglected other financial assets. As Chap. 1 has shown, in the tradition of IS-LM models loans are lumped together with other forms of debt in the bond market, which is automatically cleared when the money market is in equilibrium. In these models money exerts its effect on the real side only through the monetary channel. In particular the balance sheets of the banks, households, firms, the state and the economy as a whole have not sufficiently been paid attention. As we have discussed in Chaps. 3-4, and as recent literature on financial crises has suggested, balance sheets of the economic agents are important in understanding the dynamics of financial crises and recessions. As has recently been maintained, moreover, it is the balance sheets of economic agents, that are important in understanding the effects of the transmission of monetary, financial and currency shocks to output. Those shocks are often transmitted through the balance sheets of firms, households and banks.

Indeed, a number of recent papers have considered more detailed transmission mechanisms. Considering a closed economy many authors have concentrated on the credit-output relation. One can replace the LM curve by a curve which represents

[120] The subsequent part is based on Franke and Semmler (1999).
[121] See Krugman (1999a,b) and Miller and Stiglitz (1999).

credit demand and supply. [122] Then, besides the bond rate, a second rate of interest, the interest rate on loans, has to be introduced. Incorporating the stock market, the credit channel is seen to operate through the impact of shocks on, first, the spread between the loan rate and the bond rate and, second, the equity price as discussed in Chap. 5. This would then be a macromodel with a fully developed financial market, building on the portfolio approach, with a transmission mechanism of real, monetary and financial shocks. Details of such a model can be found in Franke and Semmler (1999).

## 11.2 The Economy-Wide Balance Sheets

Building on the balance sheets of economic agents and the portfolio approach formulates the demand and supply of assets along the lines of Tobin (1969), Tobin and Buiter (1980), Franke and Semmler (1999) and Frankel (1995). Considering here first a closed economy such a version will be briefly sketched which allows, for the study of monetary policy and financial shocks. Exchange rate shocks for an open economy will be discussed in the next chapter. Empirical evidence on such a portfolio approach is provided in Frankel (1996). This type of portfolio approach, which builds on economy-wide balance sheets, can be used to describe the mechanism of financial destabilization.

Let us assume that firms finance investment both internally, by retaining earnings, and externally, by issuing equities and debt. We disregard commercial paper markets and postulate that credit is solely supplied by financial intermediaries, that is, by commercial banks. The asset side of the balance sheets of firms is composed of the capital stock, which *via* the equity market is evaluated at its equity price, and liquid assets, which are held as deposits in the banking system (possibly to be used for smoothing out revenue fluctuations). Regarding the public, the assets held by private households are equities, treasury bonds, and deposits. The latter might be assumed as non-interest-bearing for simplicity. The government sector sells treasury bonds on the bond market and issues high-powered money to the commercial banks.

The banks hold bank reserves and government bonds and supply loans to firms and banks supply deposits to households and firms. In specifying the demand and supply of these assets, we may also include the perceived bankruptcy risk of firms as an additional variable. This permits us to study the effects of a change in the lending practices of banks that may result from a change in the creditworthiness of their customers. The change of perceived bankruptcy risk usually plays an important role in financial crises and is usually preceded by large domestic or international borrowing. We will return to this topic in Chap. 12.

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[122] See in particular Bernanke and Blinder (1998). Early versions of this line of research are in Brainard and Tobin (1963, 1968), Brainard (1964), Backus *et al.* (1982). More recent views on how monetary or financial shocks affect real activities through the credit market are documented in Friedman (1986), Bernanke (1990), Bernanke and Blinder (1992), Kashyap, Lamont and Stein (1992), Friedman and Kuttner (1992), Kashyap, Stein and Wilcox (1993).

**Table 11.1.** Economy-Wide Balance Sheets

| Assets | | Liabilities |
|---|---|---|
| | Central Bank | |
| High-powered money | $M$ : $D_b$ | Deposits of commercial banks (interest-free bank reserves) |
| | Commercial Banks | |
| Bank reserves | $D_b$ : $D_h$ | Deposits from households (interest-free) |
| Loans to firms | $L : D_f$ | Deposits from firms (interest-free) |
| Government bonds | $B_b$ : | CD's |
| | Firms | |
| Capital stock (equity price) | $qpK : L$ | Loans from commercial banks |
| Liquid assets (held with commercial banks) | $D_f$ : $p_eE$ | Equity |
| | Households | |
| Deposits (with commercial banks) | $D_h$ : $V_h$ | Total wealth |
| Government bonds | $B_h$ | |
| Equity | $p_eE$ | |

## 11.3 Households' Holding of Financial Assets

Completing the portfolio approach, we may assume that households hold bonds, deposits and equity. The total wealth of households is, $V_h$, $q$ represents Tobin's $q$ and $p$ the price of capital goods.

$$V_h = qpK + M + B \qquad\qquad V_h = \text{ total wealth.}$$

The asset holding of households is determined by

$$B_h = f_bV_h = f_b(r + \rho, i - \pi, u, \pi, \rho, \dot{\rho})V_h \tag{11.1}$$
$$D_h = f_dV_h = f_d(r + \rho, i - \pi, u, \pi, \rho, \dot{\rho})V_h \tag{11.2}$$
$$p_eE = f_eV_h = f_e(r + \rho, i - \pi, u, \pi, \rho, \dot{\rho})V_h. \tag{11.3}$$

Naturally, the adding-up constraint is

$$f_b + f_d + f_e = 1 \tag{11.4}$$

where $\rho$ = state of confidence; $\pi$ = expected inflation rate; $r$ = rate of return on capital and $u = \frac{Y}{K}$, the utilization of capacity.

As the above balance sheets show there are all together four assets: equity, bonds, money and debt. They are assumed to be imperfectly substitutable. Since debt here is inside debt it cancels out. The corresponding market clearing variables are the price for equities, the interest rate on bonds, and the interest rate on loans.

The following signs of the partial derivatives are assumed.

$$f_{br} < 0 \qquad f_{bi} > 0 \qquad f_{bu} \leq 0 \qquad f_{b\pi} \leq 0 \qquad f_{bx} \leq 0$$

$$f_{dr} < 0 \qquad f_{di} < 0 \qquad f_{du} \geq 0 \qquad f_{d\pi} \leq 0 \qquad f_{dx} \leq 0$$

$$f_{er} > 0 \qquad f_{ei} < 0 \qquad f_{eu} \leq 0 \qquad f_{e\pi} \geq 0 \qquad f_{ex} \geq 0, \qquad x = \rho, \dot{\rho}$$

where, with respect to the indices $a = b, d, e$, $f_{ar} = \partial f_a / \partial (r + \rho). f_{ai} = \partial f_a / \partial (i - \pi)$, $f_{au} = \partial d_a / \partial u$, etc. It goes without saying that (11.4) implies $f_{bx} + f_{dx} + f_{ex} = 0 (x = r, i, u, \pi, \rho, \dot{\rho})$. The signs state that the three assets are (possibly weak) gross substitutes if this notion is extended to variables other than the direct own rates of return. The only exception is the rate of inflation, which is assumed to impact negatively on money as well as bond holding.

Our model is largely compatible with the recently developed credit view of macroeconomic activity. In a strict sense, the credit view maintains that (owing to the reserve requirement on deposits), monetary policy directly regulates the availability of bank credit (and thus the spending of bank-dependent customers). Taking into account that loans are quasi-contractual commitments then the stock of these is difficult to change quickly, and so we assume that the asset side of the banks has treasury bonds serving as the buffer. In this way banks are able to shield the loans from the impact of tight money by selling off bonds as opposed to contracting credit flows. As bonds are equal to buffers for banks, we highlight this point by employing the hypothesis that loans are actually predetermined in the short run and banks fully satisfy the loan demand by firms. Accordingly, bond holding is conceived of as a residual magnitude in the portfolio decisions of banks.

As indicated above, the stock of loans of firms, $L$, their liquid assets, $D_f$, and the number of shares $E$ are treated as predetermined variables. On the basis of equations (11.1)-(11.4) the temporary equilibrium conditions on the four asset markets for equities, bonds, loans and deposits can then be represented as follows (in that order).

$$f_e V_h - p_e E \overset{Pe}{=} 0 \tag{11.5}$$

$$f_b V_h + (1 - \mu)(D_f + f_d V_h) - L - B \overset{i}{=} 0 \tag{11.6}$$

$$L - f_l (1 - \mu)(D_f + f_d V_h) \overset{j}{=} 0 \tag{11.7}$$

$$D_f + f_d V - M/\mu = 0 \tag{11.8}$$

The last equation also takes into account that $M = D_b$ in table 11.1. In this formulation, the left-hand sides are, of course, the excess demands for the respective assets. The stock-market (11.5) is cleared by variations of the equity price $p_e$, the bond market (11.6) by the bond rate $i$, and the loan market (11.7) by the loan rate $j$. Walras' law applies and equilibrium in these three markets ensures equilibrium

**Table 11.2.** Qualitative Impact Effects on Temporary Equilibrium Variables

| Response in | $u$ | $\pi$ | $\rho$ | $\dot{\rho}$ | $\alpha_e$ | $\lambda$ | $m$ | $b$ | $d$ |
|---|---|---|---|---|---|---|---|---|---|
| $i$ | + | + | + | - | + | ? | ? | + | + |
| $j$ | + | + | + | - | - | + | - | + | + |
| $j-i$ | 0 | 0 | 0 | 0 | - | + | - | 0 | 0 |
| $q$ | + | - | + | + | + | ? | ? | ? | - |

on the remaining money market (11.8). As a matter of fact, bond and stock market equilibrium will already be sufficient for this.

Tobin's $q$ is defined in our context

$$q = (p_e E + L + D_f)/pK \tag{11.9}$$

The following normalization is used

$$b = B/pK,\ d = D_f/pK,\ m = M/pK,\ \lambda = L/pK$$

Though loans are predetermined, full financial equilibrium can still prevail if it is assumed that the loan rate is instantaneously adjusted to that level at which banks just wish to supply this amount of credit. In addition, however, we also consider lagged adjustments of the loan rate, which means that banks are temporarily off their loan supply curve. This modelling device is based on the notion of imperfect competition in the banking sector. In particular, banks have explicit or implicit credit line commitments to firms within the short period which they feel compelled to honour at the going interest rate. Competition then increases (decreases) the loan rate in the next period if the loans presently advanced exceed (fall short of) the amount of credit that banks wish to supply, but this adjustment is only partial.

## 11.4 Shocks and Financial Market Reactions

Franke and Semmler (1999) undertake a comparative static exercise for the above portfolio approach, based on economy-wide balance sheets as shown in table 11.1. This gives us information on the financial markets reaction to real, monetary and financial shocks. The following table reports qualitative results of those shocks.

Ceteris Paribus, increases in the variables listed in the first row results in the following changes to the bond rate, loan rate, interest spread and stock price.

The notation is: $i$ is the bond rate, $j$ the loan rate, $q$ Tobin's $q$. At the bond rate $i_{EB}$, the equity and bond market are clearing while the loan rate is held constant. $u$ denotes the output-capital ratio, $\pi$ the expected rate of inflation, $\rho$ the public's state of confidence, $\dot{\rho}$ its time derivative, $a_e$ banks' willingness to lend, $\lambda$ the debt-asset ratio of firms, $m$ the monetary base, $b$ bonds outstanding and, $d$ deposits of firms (the latter three stock variables are also in relation to the capital stock).

Table 11.2 reports interesting results of how shocks are translated into financial market reactions. The question mark represents an ambiguous reaction. Most of the signs in Table 11.2 represent results from our comparative static exercise that one would also expect from a study of empirical data. In particular we want to note that banks' willingness to lend, expressed in $\alpha_e$, produces the expected signs for the bond rate (rising), loan rate (falling) and equity price (rising). On the other hand, as well known from financial crises, the banks' actions to restrict loans (or recalling loans) – a falling $\alpha_e$ – will give rise, among other things, to rising interest rates and falling stock prices. This is a scenario that we will again find useful in Chap. 12.

## 11.5 Conclusions

To sum up, we have strived to sketch a model of the financial sector that is, so to speak, ready for use for studying the impact of financial shocks on the real side of the economy (for example in a small macro economic model). A related model that formulates the financial-real interaction in a consistent way can be found in Flaschel, Franke and Semmler (1997, Chap. 12). There, however, the financial market includes only money, credit and stock markets. Leaving out the bond market, the full interaction with the real side, the IS-side of the economy, is then easier to describe and to analyze. There we also show how the financial-real interaction with endogenized Keynesian long swings from the "state of confidence" can give rise to a strong impact on real activity. Since exchange rates are left aside, in the approach presented here fluctuations arise from monetary or financial shocks, propagated through the balance sheets of the economic agents to the real side of the economy. This approach suffices as a framework that will help to explain how external shocks to an economy, for example exchange rate shocks, may generate a financial crisis and large output loss. This is considered next.

# Part V

# Foreign Exchange Market, Financial Instability and Economic Activity

CHAPTER 12

# Exchange Rate Shocks, Financial Crisis and Output Loss

## 12.1 Introduction

This chapter is concerned with exchange rate volatility, balance sheets and the economic activity of economic agents and asset prices. Indeed, with the end of the Bretton Woods system in the 1970s and the financial market liberalization of the 1980s and 1990s, the international economy has experienced several financial crises in certain countries and regions which entailed in most cases, credit contraction, asset price depreciation, declining economic activity and large output losses. This occurred whether the exchange rates were pegged or flexible. There appear to be destabilizing mechanisms at work from which even flexible exchange rate regimes cannot escape. Subsequently, we review some of the stylized facts that appear to be common to such financial crises and survey some recent theories that attempt to model such exchange rate-caused financial and real crises.

With respect to exchange rates and financial and real crises, three views, in fact, three generations of models, have been presented in the literature. A first view maintains that news on macroeconomic fundamentals (differences in economic growth rates, productivity and price levels, short term interest rates as well as monetary policy actions) causes exchange rate movements. The second view maintains that speculative forces e.g., self-fulfilling expectations may be at work, which destabilize exchange rates without deterioration of fundamentals. Third, following the theory of imperfect capital markets, it has also been maintained that the dynamics of self-fulfilling expectations depend on some fundamentals, for example, the strength and weakness of the balance sheets of the economic units such as households, firms, banks and governments. From the latter point of view we can properly study the connection between the deterioration of fundamentals, exchange rate volatility, financial instability and declining economic activity. We in particular want to answer the question of why are large currency depreciations contractionary. Although, diverse micro-, as well as macro-economic theories to explain financially caused recessions have been proposed, we think that those models which are particularly relevant are those that exhibit non-linearities and multiple equilibria. Such models appear to be particularly suited to explaining recent financial crises which have affected asset prices and caused large output losses.

## 12.2 Stylized Facts

There have been three major episodes of international financial crisis for certain regions or countries entailing a large output loss. They were 1) the 1980s Latin American debt crisis, 2) the 1994-95 Tequila crisis (Mexico, Argentina), 3) the 1997-98 Asian financial crises (as well as the Russian financial crisis 1998). To study such crises we will look at the interplay of exchange rates, financial markets, the severe reversal of financial flows and large output losses.

Central in this context will be the balance sheets of firms, households, banks and governments. The weak balance sheets of these economic units mean that liabilities are not covered by assets. In particular, heavy external liabilities of economic units such as firms, banks or countries can cause a sudden reversal of capital flows initiating a currency crisis. Exchange rate risk and a sudden reversal of capital flows is often built up by a preceding increase of insolvency risk. The deterioration of balance sheets of households, firms and banks often have come about through preceding lending boom and an increase in risk taking. A currency crisis is likely to entail a rise in the interest rate, a stock market crash and a banking crisis. Yet, financial and exchange rate volatility does not always lead to an interest rate increase and a stock market crash. It is thus not necessary that financial instability be propagated. The major issue in fact is what the assets of the economic units represent. If economic units borrow against future income streams, they may use net worth as collateral. The wealth of the economic units, or of a country for that matter, are the discounted future income streams. Sufficient net wealth makes agents solvent, otherwise they are threatened by insolvency, which is equivalent to saying that liabilities outweigh assets. The question is only what are good proxies to measure insolvency, i.e. what is sustainable debt?[123] But of course, exchange rate volatility and currency crisis are relevant factors as well and are what determine exchange rate movements.

There are typical stylized facts to be observed before and after the financial crises which have been studied in numerous papers [124]. Empirical literature on financial crisis episodes may allow us to summarize the following stylized facts:[125]

- there is a deterioration of the balance sheets of economic units (households, firms, banks, the government and the country)
- before the crisis the current account deficit to GDP ratio rises
- preceding the currency crisis the external debt to reserve ratio rises
- there is a sudden reversal of capital flows and unexpected depreciation of the currency

[123] In Chaps. 3.2 a procedure is proposed of how to determine and estimate sustainable debt. A sketch of this model and estimations are undertaken in Chap. 4.4. For debt dynamics in a macro model, see Chiarella et al. (2000, Chap. 3).

[124] See for example Mishkin (1998), Milesi-Ferretti and Razin (1996, 1998), Kamin (1999), Aghion et al. (2000), Corsetti et al. (1998), Cho and Kaso (2002) and Kato and Semmler (2005). In the latter two papers currency crises are considered in the context of a monetary policy model.

[125] For a summary of the following stylized facts, see Kamin (1999).

- domestic interest rates jump up (partly initiated by central bank policy)
- subsequently stock prices fall
- a banking crisis occurs with large loan losses by banks and subsequent contraction of credit (sometimes moderated by a bail out of failing banks by the government)
- the financial crisis entails a large output loss due to large scale bankruptcies of firms and financial institutions
- during and after the crisis the current account recovers (partly due to the fact that imports declines)

Since most of the recent financial crises were indeed triggered by a sudden reversal of capital flows and an unexpected depreciation of the currency (partly caused by deteriorating fundamentals, such as the balance sheets of agents, the current account deficit, rising foreign debt and a declining short term debt to reserve ratio) we will first consider the traditional exchange rate model to study whether it helps us to understand the above financial crisis mechanism.

## 12.3 The Standard Exchange Rate Overshooting Model

In earlier work, starting with Dornbusch's seminal paper on open economy dynamics (1976) and in following contributions by other authors, the economy is stylized in a very simple way through an asset market and a product market. The asset market, represented by the money market, is always at a temporary equilibrium which clears by the fast adjustment of the nominal interest rate. In the product market, prices are postulated to adjust in a Walrasian fashion. In flex-price models the temporary equilibrium in the product market is established through the fast adjustment of prices. Alternatively, it is often assumed that prices are sticky or prices move only sluggishly. In the next section we consider the case when output is fixed and prices move to clear the market. In section 4 we study a model of the IS-type where prices are sticky and output moves.

Dornbusch's original version belongs to the first variant of flexible prices. His model, as well as subsequent papers employ a differential equation approach to formulate the exchange rate and the price dynamics. With the assumption of perfect foresight, the change of the expected exchange rate is then equated with the right hand derivative of the actual exchange rate. This assumption is related to the interest rate parity theory. The same is proposed, where taken up, for the expected price change.

A number of variations of this general approach can be found in the literature. For details of such models and their critical evaluation, see Flaschel, Franke and Semmler (1997).

The dynamics of perfect foresight rational expectations models are characterized by saddle path stability. Small displacements from the equilibrium path will give rise to unstable dynamics. In these models it is then postulated that the variable in question – the exchange rate or price level – will always jump back to the stable path,

in more technical terms, to the stable manifold which secures that the transversality condition holds. What the observer would thus see is some jump or overshooting of exchange rates when there is some news concerning fundamentals observed. Due to this overshooting, the exchange rate (or other asset prices, if they are in the model) may fluctuate or even be volatile.

Let us study the basic exchange rate overshooting model more formally. Dornbusch (1976) and Gray and Turnovsky (1979) have provided us with basic models of exchange rate volatility. Here, only simple domestic foreign assets are considered. Moreover, borrowing and lending and the credit markets are left aside as well. There is only domestic and foreign currency.

As mentioned above the traditional exchange rate model results in saddle path stability under perfect foresight using interest parity theory. To explain this model we use the following notation:
$i$ = domestic interest rate; $i^*$ = foreign interest rate; $x$= expected rate of exchange rate depreciation; $e$ = current exchange rate; $M$ = log of domestic money supply; $p$ = log of price level; Y = log of output

$$i = i^* + x \tag{12.1}$$
$$x = \dot{e} \qquad \text{(perfect foresight)} \tag{12.2}$$
$$M - p = \alpha_1 Y + \alpha_2 i \qquad \alpha_1 > 0, \alpha_2 < 0 \tag{12.3}$$
$$\dot{p} = \rho\left[\beta_0 + (\beta_1 - 1)\, Y + \beta_2 i + \beta_3 (e - p)\right] \tag{12.4}$$
$$0 < \beta_1 < 1;\ \beta_2 < 0;\ \beta_3 > 0,\ \rho > 0.$$

The equilibrium

$$\bar{i} = i^*$$
$$\bar{x} = 0$$
$$\bar{M} - \bar{p} = \alpha_1 Y + \alpha_2 \bar{i}$$
$$\beta_0 + (\beta_1 - 1)Y + \beta_2 \bar{i} + \beta_3(\bar{e} - \bar{p}) = 0.$$

Thus

$$d\bar{p} = d\bar{e} = d\bar{M}$$

We obtain the following dynamics
From (12.1) and (12.2) we get

$$\dot{e} = i(M, p, Y) - i^* \tag{12.5}$$

and from (12.3) we obtain

$$i(M, p, Y) = \frac{M - p - \alpha_1, Y}{\alpha_2}. \tag{12.6}$$

Therefore, we have, as differential equations, (12.5) and the following (12.7)

$$\dot{p} = \rho\left[\beta_0 + (\beta_1 - 1)\, Y + \beta_2 i + \beta_3 (e - p)\right]. \tag{12.7}$$

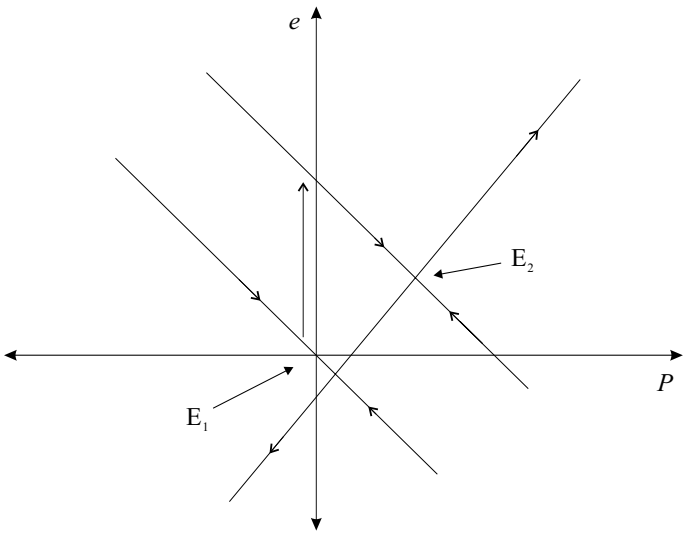



**Fig. 12.1.** Illustration of the Jump Variable Technique

Equs. (12.5) and (12.7) are our two differential equations which exhibit saddle path stability (for details, see Gray and Turnovsky, 1979).

$$\begin{pmatrix} \dot{e} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & -1/\alpha_2 \\ \rho\beta_3 & -\rho(\beta_3 + \beta_2/\alpha_2) \end{pmatrix} \begin{pmatrix} e \\ p \end{pmatrix} + \underbrace{\begin{pmatrix} 1/\alpha_2 \\ \rho\beta_2/\alpha_2 \end{pmatrix} m(t)}_{\text{drift term (can be neglected for the local dynamics)}} \tag{12.8}$$

Here, however, $e$ is free to jump instantaneously to the stable branch of the saddle paths. Thus, the usual jump variable technique is applied.

"This frees $e$ to jump at time zero, thereby rendering the predetermined value $e_0$ irrelevant for the future evolution of the system" (Gray and Turnovsky, 1979: 649) "...we find that an important role in the solution procedure is played by the transversality conditions... The effect of imposing these conditions is typically to force the system on to the stable arm of the saddle, thereby ensuring stability of the resulting dynamic system" (p. 650).

We want to note that first an increase in the money supply makes $e$ jump up and then slowly move down to $E_2$ (with prices then increasing).

Note also that the product market is in disequilibrium, but the price movement equilibrates it. Yet, we could also assume output changes, if prices are sticky. This is a model to be considered in the next section.

Let us now consider a financial crisis in the context of an open economy with a flexible exchange rate system. We start with the following modification of the overshooting model, again leaving aside other assets and the credit markets.

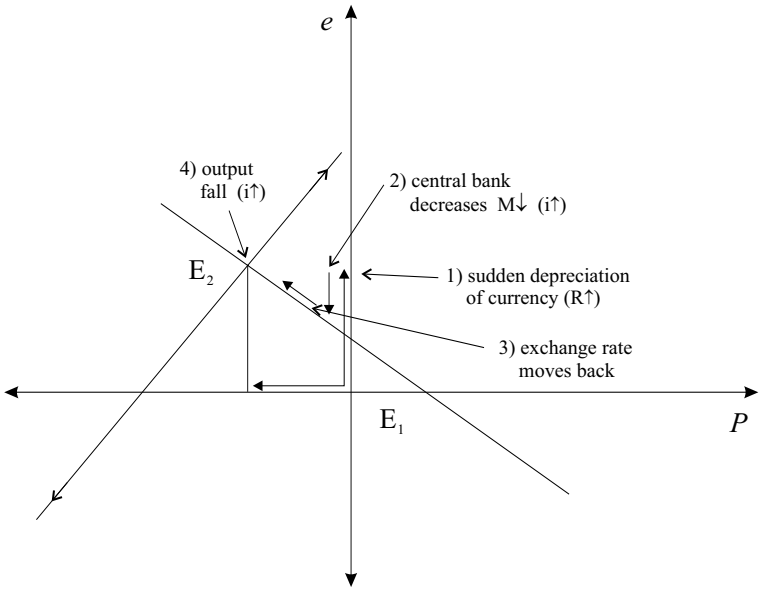



**Fig. 12.2.** Financial Crisis in an Overshooting Model

The financial crisis in the framework of the overshooting model could then look like:

We have posited the following sequence:

1. sudden depreciation of the currency due to an increase of risk, $R$, to be included in equ. (12.1), (12.2).
2. the central bank decreases the money supply (increase the interest rate).
3. exchange rate has been overshooting but jumps back to the stable branch and moves to $E_2$.

Therefore, given equ.

$$\dot{p} = \rho \left[\beta_0 + (\beta_1 - 1)\, Y + \beta_2 i + \beta_3 (e - p)\right] \tag{12.9}$$

demand will contract (because $i$ increases) and prices will fall. On the other hand the increase in $e$ has only a small effect on the increase in demand (a depreciation will only slowly increase demand).

Such treatment of exchange rates– through perfect foresight rational expectations models– have been called into question[126]. A variable's jump to the stable manifold requires a lot of information for the agents. Stiglitz has always argued that there are no conceivable market adjustment processes that could allow for such a fast adjustment to the stable branch. In addition, there is an absence of convincing empirical evidence in support of such jumps. In light of these shortcomings, recently economists prefer to employ adaptive learning procedures to explain the convergence to the

[126] See Flaschel, Franke and Semmler (1997).

stable branch. Such mechanisms are then supposed to explain whether and how a rational expectations path is reached. The development of the econometrics of ARMA processes has strongly strengthened this direction of research. Small-scale macro dynamic models in which the right hand derivative of the price level is replaced by one or multi period forecasts of the endogenous variables (i.e. learning mechanism) have already been studied. In Adelzadeh and Semmler (1996) a model is constructed and an econometric learning procedure is utilized for the forecast of the exchange rate which avoids the difficulties of the perfect foresight versions of rational expectations models. The procedure does not require the variable under study to be always on the stable manifold (or to get back to it through jumps). The recursive procedure iteratively allows for the adaptive learning of forecasted endogenous variables. One can fruitfully use those learning procedures to understand exchange rate dynamics in open economies.

Yet note that the overshooting model has in place of an IS equation an equation for price dynamics, see equ. (12.4). In equ. (12.4) output is fixed. This and the missing asset markets may not be very realistic features of the model and will be relaxed in the next model.

## 12.4 Exchange Rate Shocks and Balance Sheets

The work by Krugman has been particularly useful in modelling exchange rate volatility, financial instability and financially caused recessions in IS-LM type of models. Krugman has been involved in elaborating on the three generations of models that were mentioned in the introduction. Yet, recently Krugman (1999a, 1999b) presented some further work and developed extensions of the IS-LM model that include exchange rates, foreign debt dynamics and output dynamics. In his recent papers he has particularly stressed the importance of the balance sheets of economic agents (banks, households, and firms) and the effect of balance sheets on investment. As in Mishkin (1998), with sound balance sheets of banks, firms and households, exchange rate or financial shocks do not translate into deep financially caused recessions. On the other hand, weak balance sheets are vulnerable to shocks and can be translated into large output losses.[127] In Krugman this result is obtained in a model of multiple equilibria. Central to the Krugman models is the debt that is denominated in foreign currency as a fraction of total debt. Firms need collateral for borrowing. With low collateral they are likely to receive less credit. When an exchange rate shock occurs the debt denominated in foreign currency rises, the debt service obligation of firms, households and banks rise and — due to the loss of collateral — firms and households receive less credit. Formally the Krugman (1999a, 1999b) balance sheet model suggests a modification of the traditional IS-model. The traditional IS-model reads

[127] For an important paper along similar lines, see Aghion et al. (2000).

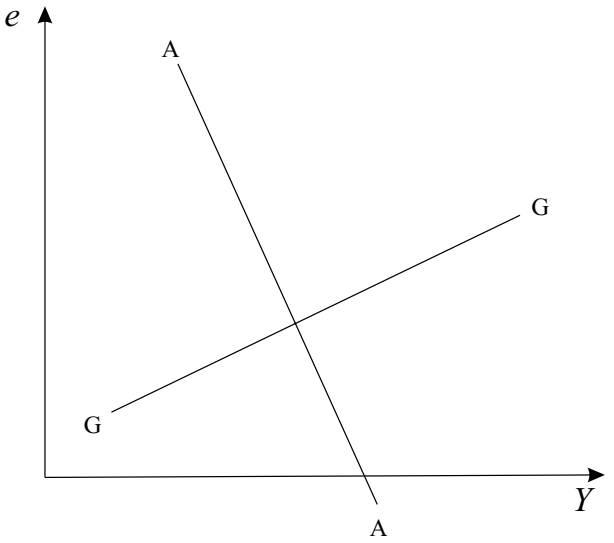



**Fig. 12.3.** The IS-LM Model

$$Y = D(Y, i) + NX(eP^*/P, Y) \tag{12.10}$$

$$\frac{M}{P} = L(Y, i) \tag{12.11}$$

$$i = i^* \tag{12.12}$$

with NX, net exports, $eP^*/P$, real exchange rate and (12.12) the arbitrage equation. Figures 12.3 and 12.4 represent the dynamics of two different model variants.

The line A-A represents in both figures all the points at which, given (12.11), the domestic and foreign interest rates are equal, a lower interest rate and thus a depreciation of the currency is asociated with lower output and investment. The line G-G in figure 12.3 shows that output is positively influenced by a rising $e$ (depreciation of currency). For details of its construction, see Krugman and Obstfeld (2003, Chap. 16).

We will give, before building up a model in the next section, a brief sketch of the importance of the balance sheets effects. A modified IS-model variant with foreign debt reads as follows. With a large fraction of debt (foreign debt) denominated in foreign currency, the net worth effect becomes important with the devaluation of the currency. So we can write (12.10) as

$$Y = D(Y, i, eP^*/P) + NX(eP^*/P, Y) \tag{12.13}$$

There is a nonlinear feedback effect from exchange rates to the net worth of the balance sheets and demand. This may give rise to the fact that the economy goes through a low level IS-equilibrium entailing a large output loss. It is thus not a quick convergence to a steady state that makes a financially caused downturn a transitory

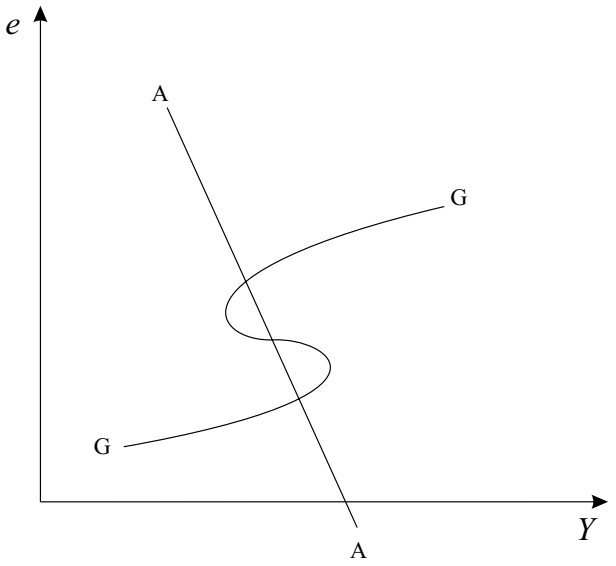



**Fig. 12.4.** The IS-LM Model with Multiple Equilibria

phenomenon but it is rather the effect of the currency shock on the balance sheets that makes the economy switch from high to low level IS-equilibria that seems to cause a protracted crisis.

Thus, if the economy is close to the middle point of the A-A and G-G curve in figure 12.4 (and to the left of A-A), the economy is likely to contract with a sudden depreciation of the currency and a high exposure to debt denominated in foreign currency. The decline of net worth and thus collateral will, through the credit channel, reduce economic activity and lead to low output. The main feature of such a currency crisis model is based, in a very simple way, on the financial accelerator concept first introduced by Bernanke, Gertler, and Gilchrist (1994). Focusing especially on the balance sheets, it is assumed that domestic firms can only finance their investment projects through loans denominated in foreign currency.[128] A more elaborate macroeconomic model in which a currency crisis, through the balance sheet effects, can trigger a large output loss is presented next.

## 12.5 Exchange Rate Shocks, Balance Sheets and Economic Contraction

The main issue in the different versions of the Krugman model is the effect of exchange rate shocks, via balance sheets, on investment. A basic dynamic model

[128] Hereby it is irrelevant if the creditors are foreign or domestic financial institutions. The main point here is the currency denomination of the loan, not its origin.

of exchange rate shocks, investment and output loss is developed in Flaschel and Semmler (2005) and Proano et al (2005).

There it is shown how the decline of investment is triggered through balance sheet effects. As above, the net worth of a firm is defined as the difference between its assets and its liabilities (both expressed here in domestic currency).[129] In that model the nominal exchange rate is the sole variable which can influence balance sheets and thus the net worth of firms. We assume that banks (domestic and foreign) evaluate creditworthiness based on the actual net worth of domestic firms, or on the dollarized debt-to-capital ratio $\tilde{q} = e\bar{F}_f/\bar{p}\bar{K} = \tilde{q}(e)$.[130] The supply of credit, $Cr^S$, is equal to the change of foreign currency bonds by domestic firms accepted by the credit institutions, that is

$$Cr^S = e\dot{F}_f(\tilde{q}), \quad \frac{\partial Cr^S}{\partial \tilde{q}} < 0 \quad \text{if} \quad \frac{d\dot{F}_f}{\dot{F}_f} \Big/ \frac{de}{e} < -1 \tag{12.14}$$

since $\hat{q} = \hat{e}$. A glance at the firms' balance sheets can clarify why a depreciation of the domestic currency has a negative effect on credit by banks: a rise of the nominal exchange rate leads to an increase in the nominal (and here also real) value of the firms' liabilities and therefore to a decrease in its net worth.

Under the assumption that investment solely depends on the creditworthiness of the borrowing firms — which in turn depends on the firms' balance sheets — a sharp devaluation of the currency can lead to a radical investment contraction and thus to a severe economic slowdown.

Let us define the aggregate investment function as

$$I = min(I^d, Cr^S) \tag{12.15}$$

which shows clearly that the credit constraint determines the actual investment level, under the assumption that $I^d > Cr^S$ holds.

The shape of the following aggregate investment function may represent Krugman's ideas as discussed in Chap. 12.4. The elasticity of the investment function now with respect to changes of $\tilde{q}$ is assumed to be state-dependent: for high values of $\tilde{q}$, where the nominal value of the liabilities denominated in foreign currency is significantly higher than the nominal replacement costs of capital, the investment reaction is assumed to be inelastic. In such a situation the firms' balance sheets are in such a bad state that the firms either cannot afford to invest in projects or cannot get any bank loans, so that a further deterioration of their financial situation only has a minimal effect on their investment projects. Therefore, for $e \to \infty$ ($\tilde{q} \to \infty$) the existence of a (still positive) minimal gross investment level or "investment floor" $\underline{I}$ is postulated. Some positive level of gross investment therefore remains even in

[129] Note that no intertemporal considerations are taken into account in this definition, as the firms' net worth is defined by means of actual stocks and prices solely.

[130] We assume here $F_f < 0$, indicating a negative foreign currency stock of bonds held by domestic firms, or in other words, that firms are indebted.

the worst scenario, since not all investment projects will be canceled, because of replacement investment and high scrapping costs.[131]

In the opposite case, where $\tilde{q}$ is low, we may assume that the firms' foreign currency liabilities are low relative to their assets and therefore firms do not face any constraints in the credit markets. In such a benign situation, the investment spending is at its maximum. Because of the existence of supply-side bottlenecks, the investment function is assumed to be again very inelastic in such a situation, i.e. for $e \to 0$ ($\tilde{q} \to 0$) aggregate investment is at its maximum level defined as the "investment ceiling" $\overline{I}$.

For intermediate values of $\tilde{q}$, by contrast, we presume that the gross investment function is very elastic with respect to changes in the debt to capital ratio, reflecting the activation of credit constraints.[132]

Even though the gross investment function is of a very simple nature, it incorporates the financial accelerator concept as discussed in Chap. 12.4 and, more generally, the basic implications of the theory of imperfect capital markets, leading to the possibility of multiple equilibria as Krugman has suggested and, therefore, to the existence of "normal" and "crisis" steady states respectively.

Similarly to the investment function we also may assume for the export function the existence of some kind of "export floor" and "export ceiling". The reason for this assumption may be that there are foreign demand saturation effects and that, even in the case of a very strong real appreciation and a subsequent loss of competitiveness, there might be some domestic products which still will be demanded from abroad because of their uniqueness, for example.

Following Chap. 12.4, and more specifically, in view of the above type of investment and export behavior, the goods market equilibrium in the analyzed small open economy can be written as

$$Y = C_1(Y - \delta\bar{K} - \bar{T}) + I(e) + \delta\bar{K} + \bar{G} + X(Y^{n*}, e) \tag{12.16}$$

where $\bar{G}$ represents government expenditures (which for simplicity are also assumed to be composed of domestic goods solely).[133]

The export function $X(Y^{n*}, e)$ furthermore is supposed to depend in a standard way positively on foreign (normal) output and the nominal exchange rate

$$X_{Y^{n*}} > 0, \qquad X_e > 0.$$

The following simple dynamic adjustment process in the goods markets, a traditional type of dynamic multiplier process, is now assumed:

$$\dot{Y} = \beta_y(Y^d - Y) = \beta_y \left[C_1(Y^D) + I(e) + \delta\bar{K} + \bar{G} + X(Y^{n*}, e) - Y\right]. \tag{12.17}$$

[131] See Flaschel and Semmler (2003: 7).

[132] An nonlinear investment function with a similar shape, though there in the (Y,K) phase space, can be found in Kaldor's 1940 business cycle.

[133] Note that we have removed here from explicit consideration all imported consumption goods $C_f$ and thus have reduced the representation of aggregate demand to include only domestic consumption goods $C = C - eC_f$. In view of this only exports $X$ have therefore to be considered from now on.

Using the Implicit Function Theorem, it follows for the displacement of the IS-Curve with respect to currency shocks

$$\left.\frac{\partial Y}{\partial e}\right|_{\dot{Y}=0} = -\frac{I_e + X_e}{C_Y - 1} \gtreqless 0.$$

Here, one of the essential points of this model is as follows. The effect of a devaluation of the domestic currency on economic activity depends on the relative strength of the export reaction as compared to the reaction of aggregate investment.[134] In the "normal case", where firms are not wealth constrained, the exchange rate effect on investment is supposed to be very weak and thus dominated by the exports effect. Then we have

$$X_e > |I_e| \Longrightarrow \left|\frac{\partial Y}{\partial e}\right| > 0.$$

In the "fragile case", i.e. in the above discussed middle range for the exchange rate, the balance-sheet effect of a devaluation of the domestic currency is assumed to be large so that it overcomes the positive exports effect:

$$|I_e| > X_e \Longrightarrow \left|\frac{\partial Y}{\partial e}\right| < 0.$$

The financial sector is also an important feature of this basic currency crisis model. Following Roedseth (2000), a portfolio approach of Tobin type, which allows different rates of return on domestic and foreign bonds, is chosen for the modelling of the financial markets. The defining financial market equations are:

$$W_p = M_0 + B_{p0} + eF_{p0} \tag{12.18}$$

$$\xi = i - \bar{i}^* - \epsilon \tag{12.19}$$

$$eF_p = f(\xi, W_p, \alpha), \tag{12.20}$$

$$M = m(Y, i), \qquad m_Y > 0, \quad m_i < 0 \tag{12.21}$$

$$B_p = W_p - m(Y, i) - f(\xi, W_p) \tag{12.22}$$

$$F_p + F_c + \bar{F}^* = 0 \qquad \text{or} \qquad F_p + F_c = -\bar{F}^*. \tag{12.23}$$

Equation (12.18) describes the initial financial wealth of the private sector, expressed in domestic currency, consisting of domestic money $M_o$, bonds in domestic currency $B_{po}$ and bonds in foreign currency $F_{po}$. Domestic and foreign-currency bonds are assumed to be imperfect substitutes, which means that the Uncovered Interest Rate Parity does not hold. The expected rate of return differential between the two interest

[134] The denominator is assumed to be unambiguously negative, so that the sign of the numerator is decisive for the slope of the IS-Curve.

bearing financial assets, with $\epsilon$ denoting the expected rate of currency depreciation, is referred to as *risk premium*, see equation (12.19).[135]

Equation (12.20) stands for the foreign currency bond market equilibrium. The demand for foreign currency denominated bonds is assumed to depend negatively on the risk premium, positively on private financial wealth and positively on a parameter $\alpha$. This parameter is supposed to represent other foreign exchange market pressures like the propensity for a speculative attack on the domestic currency, political instability, etc.

Equation (12.21) represents the domestic money market equilibrium with the usual reactions of the money demand to changes in interest rates and output. The domestic bond market (equation (12.22)) is then in equilibrium via Walras' law of stocks, if this holds for the bonds denominated in foreign currency.

The last equation describes the equilibrium condition for the foreign exchange market. It states that the aggregate demands of the three sectors — domestic private sector, the monetary authority and foreign sector — sum up to zero.[136] On the assumption that the supply of foreign-currency bonds from the foreign sector is constant ($-\bar{F}^*$), the additional amount of foreign-currency bonds available to the private sector (besides its own stocks) is solely controlled by the monetary authorities.[137] The prevailing exchange rate regime thus depends on the disposition of the central bank to supply the private sector with foreign-currency bonds.

The mechanism for expected exchange rate fluctuations is described by the following equation:

$$\varepsilon = \beta_\varepsilon \left( \frac{e_0}{e} - 1 \right), \qquad \varepsilon_e \leq 0. \tag{12.24}$$

It is obvious that we have for the steady state exchange rate $e_o$ that $\varepsilon(e_0) = 0$ holds. Note that the exchange rate devaluation expectations can be perceived as purely forward looking and in this respect asymptotically rational, by assuming that economic agents have perfect knowledge of the future steady state exchange rate level $e_o$ with respect to which the actual exchange rate is expected to converge in a monotone fashion after each shock that hits the economy.

By inserting the money market equilibrium interest rate (the inverse function of equation (12.21)) in equation (12.20) the Financial Markets Equilibrium- or AA-Curve can be derived

$$eF_p = f\left( i(Y, M_o) - \bar{i}^* - \beta_\varepsilon \left( \frac{e_0}{e} - 1 \right), M_o + B_{po} + eF_{po} \right). \tag{12.25}$$

---

[135] See Roedseth (2000, p. 17). Even though financial capital markets throughout the world were liberalized during the last decades, it still seems to be a very unrealistic assumption to suppose that international capital mobility is perfect. Significantly high spreads (and thus risk premia) between domestic and international interest rates (for example to the U.S. 3-month T-Bill) are observable, especially in emerging market economies.

[136] See Roedseth (2000, p. 18).

[137] This assumption can be justified by assuming as in Roedseth (2000) that domestic bonds cannot be traded internationally.

This equilibrium equation can be interpreted as a representation of the $\dot{e}$=0−isocline. Under the assumption that the exchange rate does not adjust automatically to foreign exchange market disequilibria, one may postulate as exchange rate dynamics:

$$\dot{e} = \beta_e \left[ f\left( i(Y, M) - \bar{i}^* - \beta_\varepsilon \left( \frac{e_0}{e} - 1 \right), M + B_p + eF_p, \alpha \right) - eF_p \right]. \quad (12.26)$$

The slope of the $\dot{e}$=0−isocline is determined by the Implicit Function Theorem in the following way:

$$\left. \frac{\partial e}{\partial Y} \right|_{\dot{e}=0} = -\frac{f_\xi i_Y}{f_\xi \epsilon_e + \left( f_{W_p} - 1 \right) F_{p0}} < 0.$$

Looking at system (12.17) and (12.26) by local stability analysis[138] we can identify three steady states, as shown in figure 12.5. Steady state $E_1$ represents the "normal" steady state, where the economy's output is high as well as the domestic investment activity. In this steady state, the standard case $|I_e| < X_e$ holds. There is a second steady state where the IS and AA-curve intersect (see the middle intersection). This steady state represents the fragile case with $|I_e| > X_e$: Because a slight deviation of the output level from this steady state level can lead the economy to a short-run investment boom or to a decline in the economic activity, this equilibrium point is unstable. The upper intersection of the IS and AA curve, the third steady state, constitutes the "crisis equilibrium". At this equilibrium point the investment activity is highly depressed due to the high value of $e$. Nevertheless, the slope of the $\dot{Y}$−isocline is again positive because of $|I_e| < X_e$ describing the dominance of exports over (the remaining) investment demand in the considered situation.

Assume the economy is initially at steady state $E1$ in figure 12.5 and presume that the prevailing exchange rate system is a currency peg which is fully backed by the domestic central bank. Now suppose that the demand for foreign-currency bonds increases due to, say, a rise in the 'capital flight' parameter $\alpha$. As long as the central bank is disposed to defend the prevailing currency peg by selling foreign currency bonds or alternatively through interest rate increases, there are no real effects on the domestic economic activity.[139]

In the case that the domestic monetary authority runs out of currency reserves and decides to give in to the speculative pressures and to let the exchange rate float, the AA-Curve becomes the binding curve in the model. The exchange rate then jumps from the initial equilibrium $E1$ to the corresponding point at the AA-Curve (with a still unchanged output level). The sharp devaluation of the domestic currency leads to a severe deterioration of the balance-sheets and thus to an investment contraction. Because the economic agents consider the exchange rate level at the steady state $E2$ as the long-run level, the actual exchange rate depreciates further, reducing investment even further, and thereby aggregate demand and output which

[138] For further details the reader is referred here Proano et al. (2005).

[139] Note that this assertion follows from the assumption that interest rates do not directly affect the real side of the economy.

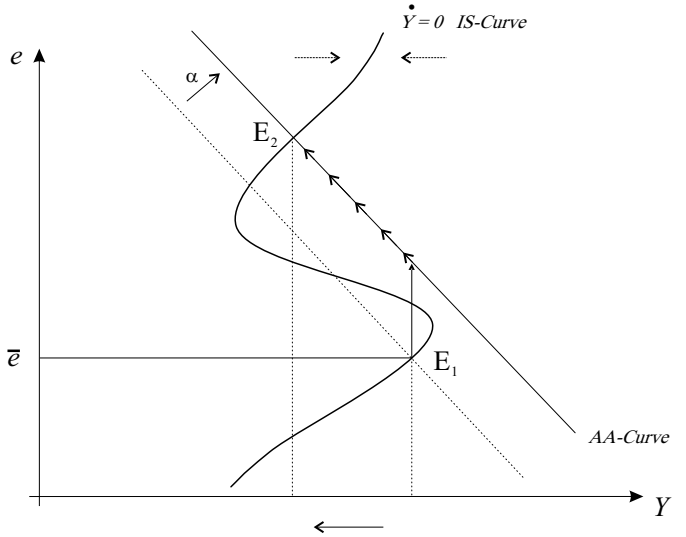



**Fig. 12.5.** The Macroeconomic Effects of a Currency and Financial Crisis

follows the large decline in aggregate demand with some time delay until a new goods market equilibrium with a large output loss is reached.

The adjustment process after the currency crisis comes to a rest when the economy arrives at its new equilibrium $E2$. Because of the assumption of fixed domestic prices and wages the economy exhibits no endogenously determined mechanism to return to the initial high output level.[140]

## 12.6 Exchange Rate Shocks, Credit Rationing and Economic Contractions

Implicit in Krugman's theory and in the model presented in Chap. 12.5 is the assumption of imperfect capital markets. In the context of this theory the major issue is credit rationing. A proper theory of credit rationing has been made possible by the economics of information which provided a theoretical foundation of credit market imperfections.

Here, the main concepts are asymmetric information, moral hazard and adverse selection. Asymmetries of information refers to the borrower-lender relationships. For lenders it is costly to acquire information about the opportunities, characteristics, or actions of borrowers. Financial contracts have to take account of information costs

[140] For describtion of a further medium run dynamics with wage and price movements, see Proano et al. (2005).

which increases borrowing costs. Risk in credit markets increases the real cost of extending credit. It therefore, reduces the efficiency of the process of matching lenders and potential borrowers. All this may have extensive real effects. The literature on the economics of imperfect information has made an attempt to rationalize several characteristics of credit markets such as the form of financial contract, the existence of financial intermediaries, the form of bankruptcy and the existence of credit rationing and borrowing cost depending on collateral.

Following Jaffee and Stiglitz (1990) we first summarize some elements of the theory of imperfect capital markets. A credit contract involves the relationship between a creditor and a borrower. The first important element in this relationship is asymmetric information. The borrower knows for what purpose the loan will be used, but the lender is less informed about the use of the loan. The borrower promises to pay back the loan with interest. The lender faces heterogeneous agents and each borrower's promise is different. The risk of not getting the loan back depends on the borrower's ability to pay back the loan. A risk for the lender may, however, also arise if the borrower has some incentives not to pay. This concerns the willingness to pay by the borrower.[141] In recent credit market theories this has been discussed under the topic of incentive compatible debt contracts.

The essential features of imperfect capital markets are best presented by using a zero horizon or two period model as in Chap. 3. Let us give a short summary. The problem of the ability to pay for the one period zero horizon case is as follows. Let there be two possible outcomes for the project of the borrower $x^a$ and $x^b$ whereby $x^a > x^b$ and $x^a =$ good result; $x^b =$ bad result. Let $p^a, p^b$ the probability of the occurrence of $x^a, x^b$; with $p^a + p^b = 1$. Then we have the expectations: $x^e = p^a x^a + p^b x^b$. Thus let us describe the second important element in modern debt contracts. This is the limited liability of the borrower which we have discussed in Chap. 3.

Note that limited liability refers to the bad outcome where the borrower is not liable for the loss. The creditor would thus be inclined to require collateral so as to cover any potential loss. The collateral of the borrower, promised to be transferred to the creditor in case of a loss, could be liquid assets, financial assets, property or physical capital. Yet, note that in most cases the value of the collateral is uncertain and may be subject to shocks. On the other hand, the creditor may grant credit but charge different types of borrowers at different interest rate because different borrowers have different idiosyncratic risk characteristics. These interest rates may in particular depend on the size of the collateral that each borrower is willing to offer. So one would expect endogenous credit costs depending on each agent's value of collateral.

A third important element in modern credit markets is rationing of loans that we also have discussed in Chap. 3. Pure rationing of credit might occur only for few borrowers, although all potential borrowers are assumed to be equal. Mostly,

[141] Consider for example the case of a sovereign borrower whose value of the debt is $D$ and $M$ is the value of the access to the capital market. Then if sovereign debt $D > M$ the debtor might not be willing to pay.

credit rationing is connected to the collateral that borrowers can provide. Usually it is assumed that credit is granted up to a certain fraction of the offered collateral.

For the literature on imperfect capital markets and macroeconomic activity, as was shown in Chap. 3, it holds that, although the diverse models in the literature differ in their basic features and predictions, three basic results emerge, providing the basis for the study of macroeconomic financial crises. First, external finance is more expensive than internal finance. The agency cost of lending, possibly depending on the agent's idiosyncratic risk characteristics is the reason for the higher cost of external finance. Second, given the amount of finance required, the premium on external finance depends inversely on the borrower's net worth as collateral. Third, a decrease in the borrower's net worth (value of collateral) causing a rise in the premium on external finance reduces spending and investment of the borrower. This result provides the key to the financial crisis. Since adverse shocks to the economy reduce the net worth of borrowers (or through positive shocks net worth increases), the spending and production effects of the initial shock will be propagated and amplified.

Important recent work on imperfect credit markets and macroeconomics can be found in Kiyotaki and Moore (1995). In their basic two period framework entrepreneurs operate a technology that uses an input in period 0 to produce output in period 1. There are two types of inputs – a fixed factor $K$ (already in place) and a variable input $x_1$. The fixed factor could be an input such as land, for example. The variable input could be any kind of input such as raw materials, labor or firm-specific capital. Finally, at the end of period 1, the entrepreneur can sell the fixed factor at the market price, $q_1$, per unit. The variable input depreciates fully in use and its price is normalized to one. Output in period 1 is $\alpha_1 f(x_1)$, whereby $\alpha_1$ is a technology parameter and $f(\cdot)$ is increasing and concave. Given the cash flow, $\alpha_0 f(x_0)$, and a debt obligation inherited from the past, $r_0 b_0$, where $b_0$ is past borrowing and $r_0$ is the gross real interest rate, the link between the entrepreneur purchases of the variable input $x_1$ and the borrowing $b_1$ is given by

$$x_1 = \alpha_0 f(x_0) + b_1 - r_0 b_0 \tag{12.27}$$

The entrepreneur chooses $x_1$ and $b_1$ to maximize period 1 output net of debt repayment. Moreover, there exists an incentive problem, since it is costly for the lender to seize the entrepreneur's output in case of default. In case the borrower does not pay his obligation the ownership of the fixed factor is transferred to the lender. According to the above considerations the fixed factor serves as collateral. With credit rationing the funds provided by the lender will be limited by the discounted market value of the fixed factor:

$$b_1 \leq (q_1/r_1)K \tag{12.28}$$

where $r_1$ is the new real interest rate on funds. Thus, there is a collateral-in-advance constraint for spending on the variable input. Unsecured lending is not feasible in this model and thus credit is rationed. By taking together equation (12.27) and (12.28) the incentive constraint is obtained from

$$x_1 \leq \alpha_0 f(x_0) + (q_1/r_1)K - r_0 b_0 \tag{12.29}$$

where the right hand side of the above equation represents entrepreneur's net worth as collateral. The above equation tells us that spending on the variable input cannot exceed the entrepreneur's net worth. This is equal to the sum of cash flow $\alpha_0 f(x_0)$ and net discounted assets, $(q_1/r_1)K - r_0 b_0$. The constraint (12.29) binds if the entrepreneur's net worth is less than the unconstrained optimal value of $x_1$.

This simple framework illustrates the results on imperfect capital markets discussed earlier. When the incentive constraint (12.29) binds, the shadow value on an additional unit of internal funds exceeds the gross real interest rate, $r_1$, prevailing in external capital markets. This difference reflects the agency cost of lending. Moreover, the decrease in the entrepreneur's net worth[142], and thus the fall in the collateral value increases the agency premium, and reduces the borrowers spending (for the intermediate input) and production. The financially caused recession can be explained by a shock to the borrower's networth – or the interest rate– leading to a downturn of the real economy and large output loss.

This incentive constraint (12.29) shows the different factors impacting the borrower's net worth, the borrower's spending and the level of production. A decline in cash flow, $\alpha_0 f(x_0)$, a fall in asset prices $q_1$, a rise in $r_1$ or an increase in initial debt obligations $b_0$ reduces net worth. All of them make the constraint binding sooner. Given a binding collateral constraint an increase in $r_1$ reduces the borrower's spending by a corresponding decrease in asset values, i.e by the borrower's networth. An increase in the interest rate on previous debt, $r_0$, also reduces the borrower's spending since it reduces the cash flow net of current interest payments $(\alpha_0 f(x_0) - r_0 b_0)$.[143]

Miller and Stiglitz (1999) follow the approach by Kiyotaki and Moore (1995) by including exchange rates and debt denominated in foreign currency in a model of imperfect capital markets. This variation of the model gives then again rise to multiple equilibria. The Miller and Stiglitz paper concentrates on the balance sheet effects arising from an unexpected devaluation of the currency and the impact on highly-leveraged, fully collateralized firms which have borrowed in foreign currency. According to their theory, a fall in the currency triggers margin calls and consequently a "fire-sale" of collateralized assets; the economy may then collapse to a low level equilibrium and a large output loss.

Formally we can write the Miller and Stiglitz model as

$$q_t(k_t - k_{t-1}) + Rb_{t-1} = \alpha k_{t-1} + b_t \tag{12.30}$$

with $q$, asset price, $b$, debt, $\alpha k$, income and $R = 1 + r$. with $r$ the interest rate. From the above we get

$$b_t = (1 + r)b_{t-1} - (\alpha k_{t-1} - q_t(k_t - k_{t-1}))\,. \tag{12.31}$$

---

[142] This decrease may result from a decline in cash flow or a lower value of the collateralizable asset.

[143] However, some models have modified the above framework allowing for unsecured lending and the possibility of default. This is the case in models of "costly state verification" where the probability of costly auditing by the lender adds to costs for the borrower. This additional mechanism make unsecured lending feasible although defaults may occur with some positive probability. Some models of this type were discussed in Chap. 3.

With $x$ the loss arising from the unexpected devaluation of the foreign currency loans we have

$$b_t = (1+r)b_{t-1} - (\alpha k_{t-1} - q_t(k_t - k_{t-1}) - x) \tag{12.32}$$

Without the shock $x$ we have: $b \leq \frac{\alpha k}{r}$. Here again, as in Kiyotaki and Moore, the debt should be smaller than discounted present values of the income stream $\alpha k$ serving as collateral.[144]

However, with a shock $x$ we may have: $b > \left(\frac{\alpha k - x}{r}\right)$. The latter case arises from a collateral shock (triggered by unexpected devaluations of the currency) possibly leading to a "fire-sale" of collateralized assets and a fall of $q$ whereby the economy is likely to end up in a low level equilibrium and a large output loss. Note that, here again not all shocks will drive the economy to a low level equilibrium. Only large shocks accelerated by bad balance sheets will lead to macro-caused financial and real crises. Miller and Stiglitz (1999) estimate the thresholds for those shocks to be a thirty to forty percent unexpected devaluation of the currency to generate such a systemic crisis.

## 12.7 Exchange Rate Shocks, Default Premia and Economic Contractions

In the Miller and Stiglitz model the interest rate and the credit costs, per unit of currency borrowed, is fixed. Yet, one of the major issues in modern credit market theory is that credit costs are state dependent. Each agent is likely to face his or her own credit cost. Thus, another way of modeling the transmission mechanism of a currency shock to output may be the default premium channel.[145] While the main features of the Miller and Stiglitz model are preserved this additional aspect is modeled next.

Credit market imperfections suggest that credit cost is state dependent. In a first view interest rates are perceived indeed as being convex in the agents debt. This has been discussed in Bhandary, Haque and Turnovsky (1995). Work on endogenous credit cost can also be found in Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1998) and Grüne, Semmler and Sieveking (2004). In those models credit cost depends on net worth of the agent (households, firms, countries). Net worth in their conception is the difference between the agent's own assets and his or her liabilities. We follow a similar idea and make the agents credit cost dependent on assets as well as liabilities (debt). The agents liability may depend on the debt denominated in foreign currency and thus on the exchange rate. In addition in our model there is an adjustment cost of capital which prevents capital from being costlessly reallocated. Due to those additional assumptions, a credit market model with imperfect

[144] Note that we presume here that there is an equilibrium $\alpha k$ that holds forever.

[145] Immediately after the Asian financial crisis 1997-98 it was quite visible that the default premia of many emerging markets went up considerably. Also the financial crisis in Argentinia in 2001 was triggered by a large jump in default premia. An alternative measure for the increase of risk are credit default swaps (see Neftci, 2004).

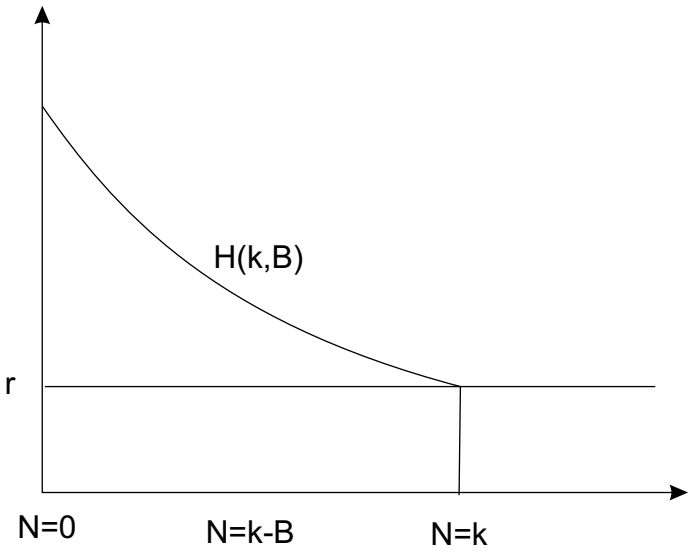



**Fig. 12.6.** Endogenous Credit Cost

capital markets can have multiple equilibria. Thus for income shocks or changes in the credit cost function there can be different domains of attraction and the economy can, due to shocks, move down from a high to a low level equilibria exhibiting a large output loss.

Our model starts from the Miller and Stiglitz (1999) model. In the Miller and Stiglitz case there is a discrete time debt accumulation equation

$$b_t = (1+r)b_{t-1} - (\alpha k_{t-1} - q_t(k_t - k_{t-1}) - x) \tag{12.33}$$

where $b_t$ is debt, $\alpha k_{t-1}$, the income, $q_t$ the price of the investment good (in their case land) and $k_t - k_{t-1}$ the investment (land), $x$ the income loss due to unexpected devaluation of the currency and $r$ the net real interest rate.

In our proposed model there are two changes as compared to Stiglitz and Miller: first, there is endogenous credit cost. Thus we posit a credit cost $H(k, B)$ instead of $rb$, above, and second we take as net income

$$\alpha k_{t-1} - q_t(k_t - k_{t-1}) = f(k, j) = k^\alpha - j - \gamma^\beta k^{-\gamma} \tag{12.34}$$

where $\gamma, \alpha, \beta > 0$. The right hand side of (12.34) represents income generated from a production function minus investment (including an adjustment cost for capital). More specifically, our model reads as follows. We consider a continuous time model and for net income $f_t = \alpha k_{t-1} - q_t(k_t - k_{t-1})$ we take

$$f(k, j) = k^\alpha - j - j^\beta k^{-\gamma} \tag{12.35}$$

with the evolution of capital stock given by

$$\dot{k} = j - \sigma k, \quad k(0) = k. \tag{12.36}$$

With endogenous credit cost $H(k, B)$ we have the evolution of debt

$$\dot{B} = H\left(k, B\right) - f\left(k, j\right) \tag{12.37}$$

where $H\left(k, B\right)$ is the above mentioned endogenous credit cost.The endogenous credit cost can be defined as

$$H(k, B) = \frac{\alpha_1}{(\alpha_2 + \frac{N}{k})^2} r.$$

Figure 12.6 shows the graph of the credit function where $N$ is net worth.

We define creditworthiness, $B^*(k)$, the maximum amount that the economic agent (household, firm, government or country) can borrow given the initial conditions $k(0) = k_0$, $B(0) = B_0$.

Note that if the interest rate $r = H(k, B)$ is constant, as in the Miller and Stiglitz case, then, as is easy to see, $B^*(k)$ is the present value of the income stream generated by $k$ (subtracting the initial debt $B(0)$):

$$B^*(k) = \underset{j}{Max} \int_0^\infty e^{-rt} f\left(k, j\right) dt - B(0) \tag{12.38}$$

s.t.

$$\dot{k} = j - \sigma k, \qquad k = k(0) \tag{12.39}$$

$$\dot{B} = rB - f\left(k, j\right), \; B(0) = B. \tag{12.40}$$

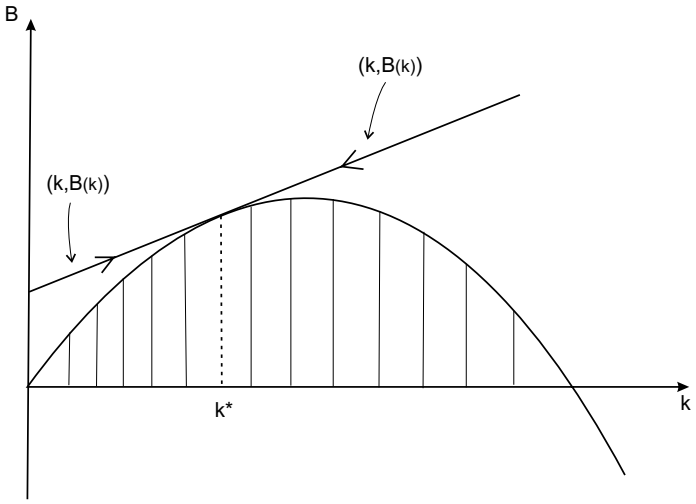



**Fig. 12.7.** Model with Endogenous Credit Cost and Unique Equilibrium

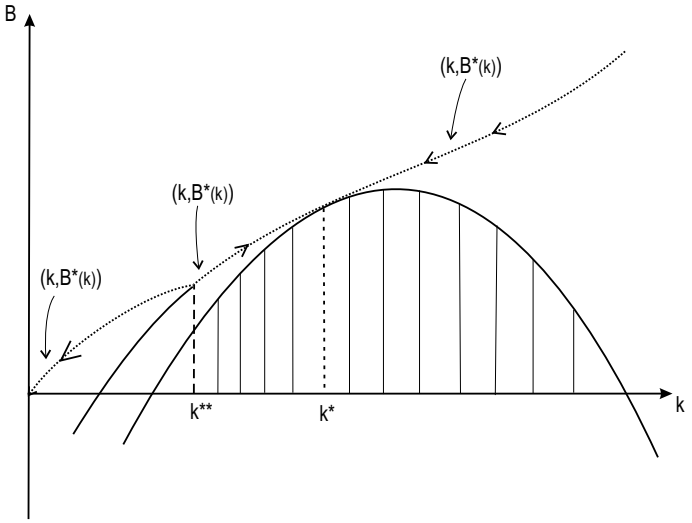



**Fig. 12.8.** Model with Endogenous Credit Cost and Multiple Equilibria

In Semmler and Sieveking (1998, 1999) and Grüne, Semmler and Sieveking (2004) the more general case where $r$ is not a constant is considered. Then, not only the relationship of the present value to creditworthiness but also the notion of present value itself becomes difficult to treat. Note that the endogenous credit cost $H(k, B)$ is determined by creditworthiness $B^*(k)$ implying a default premium due to leverage. Yet, on the other hand, the maximum amount an agent can borrow depends on the credit cost. This is the reason why commonly used present value computations (through the Hamiltonian) are not feasible. Grüne, Semmler and Sieveking (2004) develop a special technique to solve this problem.

Moreover, exchange rate shocks (depreciation of the currency) may decrease net income and possibly increase $H(k, B)$. Due to the assumed nonlinear relationship in the model (nonlinear cost of capital adjustment and the nonlinear credit cost function) there can be multiple steady states. The possibility of a unique steady state is illustrated in figure 12.7.

Below the line $(k, B^*(k))$, moving from both sides into the steady state $k^*$, the agent is creditworthy because the value of debt is lower than the present value from the agent's action. Above that line the agent will be bankrupt.

Figure 12.8 shows the case when there are multiple steady state equilibria. Again, below the dotted line the agent will be solvent and above that line bankruptcy will arise. Note that the slope $(k, B^*(k))$ of the line depends on $H(k, B)$, the credit cost function. A large shock to the net income function, a large shock to the exchange rate, an increase to the initial debt, or a change of the credit cost function $H(k, B)$ which makes credit cost rise, will either render the agent – in our case the country – insolvent or make the low level equilibrium (the one with large output loss) an

attractor. In the latter case, $k^{**}$ may act as a tipping point or a threshold, where either an expansion or contraction is triggered. Numerical examples of those outcomes and further discussions are provided in Semmler and Sieveking (1998) and Grüne, Semmler and Sieveking (2004).[146]

## 12.8 Conclusions

This chapter studied stylized facts and the basic mechanisms of exchange-rate caused financial and real crises. As we have shown it is likely to be the connection of weak balance sheets (of households, firms, financial intermediaries, governments and countries) and large exchange rate shocks that lead to positive feedback mechanisms and thus to credit contraction, declining asset prices and economic activity, real crisis and large output loss. This in particular appears to be a basic mechanism if there exists in the country large debt denominated in foreign currency. Moreover, as we have shown, credit rationing and state dependent default premia may entail destabilizing mechanisms, possibly leading to low level equilibria.[147] The insight of how financial and real risk can be enlarged by large currency shocks and to what extent an international portfolio might be able to hedge this risk is studied further in Chap. 13.

[146] For a stochastic version of the above model, see Grüne and Semmler (2004c).

[147] For a model that studies to what extent the adverse effects of currency shocks can be reduced by currency hedging, see Roethig et al. (2005).

CHAPTER 13

# International Portfolio and the Diversification of Risk

## 13.1 Introduction

This chapter continues our study from Chap. 8 and Chaps. 11-12. We want to explore the particular question: to what extent one can diversify risk through an international portfolio of assets. There are two types of risks. The first type of risk is exchange rate risk. The second type of risk is asset specific risk in different countries. There are earlier studies on an international CAPM. Important mile-stones include work by Grubel (1966) who pursued studies on international equity markets in order to explore potential gains for U.S. investors from an international portfolio due to low correlations between equity indices of national markets. Here dividends are not included in the returns and only small samples were explored. Grubel's work indicated a significant reduction in risk through international diversification. The work was pursued on the basis of the mean-variance framework as introduced in Chap. 8. Furthermore, Solnik (1973, 2000) extensively computed international portfolios and compared them to national portfolios. He also computed efficient frontiers of international portfolios. In recent times in particular Levich's (2001) work has been concerned with international equity as well as bond portfolios. Here, as above mentioned, one of the major issues is the volatility of exchange rates. We thus first explore exchange rate risk arising from volatility of exchange rates.

## 13.2 Risk from Exchange Rate Volatility

We first present the standard theory of exchange rate determination which is based on uncovered interest rate parity (UIP). Since we are interested in the short run movements of exchange rates, we do not discuss the purchasing power parity (PPP) theory of exchange rate determination, which pertains more to the long run. We first discuss spot rate, forward rate and interest parity from the point of view of a home country, e.g., U.S. The exchange rate of, for example, the US dollar and the euro is

$$e = \frac{dollar}{euro}$$

The spot rate may be 1.5325 dollar per euro. The forward rate (to avoid exchange rate risk through a forward contract) is 1.5308 dollar to purchase one euro, for example, 90 days in the future.[148] Next we define interest parity. Let the spot rate be $S = \frac{dollar}{euro}$

[148] We are neglecting discounting here.

or $\frac{1}{S} = \frac{euro}{dollar}$ and let the forward rate be $F_t$. In the U.S. one dollar investment from $t$ to $t+1$ delivers $(1+R_t)$ as return. For the same time period there is an alternative investment in euro, which delivers $\frac{1}{S_t}(1+R_t^*)$ with $R_t^*$ the return in Europe.

Thus, for the time period $t$ to $t+1$ one can obtain $\frac{1F_t}{S_t}(1+R_t^*)$ due to arbitrage (except transaction cost).

With riskless gain the arbitrage condition reads

$$1 + R_t = \frac{1F_t}{S_t}(1 + R_t^*) \tag{13.1}$$

Use from (13.1), $\frac{F_t}{S_t} = \left(\frac{1+R_t}{1+R_t^*}\right)$, and then take logs,

$$\begin{aligned} logF_t - logS_t &= \underbrace{log(1+R_t) - log(1+R_t^*)}_{\text{or} \approx R_t - R_t^*} \end{aligned} \tag{13.2}$$

The covered interest parity theory with $f_t = \log F_t$ and $s_t = \log S_t$ gives us

$$\begin{aligned} \underbrace{f_t - s_t}_{1 + \text{forward premium}} &= R_t - R_t^* \end{aligned} \tag{13.3}$$

A forward premium on the foreign currency is $(F_t - S_t)/S_t$, $S_{t+1}$ is the spot rate at $t+1$.

Take

$$E_t(S_{t+1}) = S_{t+1} + \varepsilon_{t+1}$$

and

$$\frac{E_t(S_{t+1})}{S_t} = \left(\frac{1+R_t}{1+R_t^*}\right)$$

Take logs, then the Uncovered Interest Parity (UIP), can be written as

$$s_{t+1}^e - s_t = R_t - R_t^* \tag{13.4}$$

or as

$$R_t = R_t^* + \underbrace{(s_{t+1}^e - s_t)}_{\dot{e}_t}$$

with $\dot{e}$ the corresponding time continuous change of the exchange rate. This means, the domestic interest rate is equal to the foreign rate plus the expected change of the exchange rate.
Thus, one could postulate

$$s_{t+1}^e - s_t = \alpha + \beta(f_t - s_t)$$

Take $s_{t+1} = s^e_{t+1}$, which should hold under rational expectations, then we would have

$$s_{t+1} - s_t = \alpha + \beta(f_t - s_t) \tag{13.5}$$

with $\alpha = 0$, and $\beta = 1$.

McCallum, (1996, Chap. 9) reports the following empirical estimates on the UIP across countries.

**Table 13.1.** UIP-Estimates

| | Estimates (std. errors) | | |
|---|---|---|---|
| United States relative to | $\alpha$ | $\beta$ | $R^s$ |
| Germany | -0.0156 | -4.201 | 0.040 |
| | (0.006) | (1.70) | |
| Japan | 0.0153 | -3.326 | 0.051 |
| | (0.0052) | (1.17) | |
| United Kingdom | -0.0078 | -4.740 | 0.111 |
| | (0.0032) | (1.09) | |

As can be observed from the table, neither the predicted $\alpha$ nor $\beta$ are obtained, thus indicating the empirical failure of the UIP.

Next we introduce an exchange rate with a risk premium. As above noted the empirical evidence on the covered interest parity theory we have

$$s_{t+1} - s_t = \alpha + \beta(f_t - s_t)$$

with $\alpha \neq 0$ and $\beta \neq 1$. Empirically, we can observe a poor fit for $\alpha$ and $\beta$, possibly because of non-rational expectation formation.[149] Another way to explain the failure of the above equation is to presume a time varying risk premium driving a wedge between domestic and world interest rates. We can take into account a time varying risk premium and write

$$R_t = R^*_t + (s^e_{t+1} - s_t) + \rho_t \tag{13.6}$$

The risk premium, $\rho_t$, could be seen to be positively correlated to interest differences, $(R_t - R^*_t)$, and negatively correlated to central bank's foreign reserves (see Kato and Semmler, 2005). In the case of the existence of a (time varying) risk premium $\rho_t$ there will be lower expected net return on assets and there might also be a risk structure of returns, arising from $\rho_t$, which may be different for different time periods and countries. Empirical evidence of such risk premia are reported in Hallwood and MacDonald (1999, Chap. 3.4).

[149] This line of research is pursued by Frankel and Froot (1990) and, Chiarella and He (2001) using heterogeneous agent models.

## 13.3 Portfolio Choice and Diversification of Risk

Following up on our description of portfolio theory in Chap.8 we again presume[150] the mean portfolio returns to be

$$\begin{aligned} R_{p,t+1} &= \alpha_t R_{t+1} + (1-\alpha_t) R_{f,t+1} \\ &= R_{f,t+1} + \alpha_t (R_{t+1} - R_{f,t+1}) \end{aligned}$$

with $R_{f,t+1}$ the risk-free rate and $R_{t+1}$ the risky return.
The expected mean returns are

$$E_t R_{p,t+1} = R_{f,t+1} + \alpha_t (E_t R_{t+1} - R_{f,t+1}) \tag{13.7}$$

and the variance is

$$\sigma_{p,t}^2 = \alpha_t^2 \sigma_t^2 \tag{13.8}$$

The preference of the investor can be written as

$$\max_{\alpha_t} \left( E_t R_{p,t+1} - \frac{k}{2} \sigma_{p,t}^2 \right) \tag{13.9}$$

with $k$ the aversion to variance.

Substituting (13.7) and (13.8) into (13.9), leaving aside $R_{f,t+1}$, gives

$$\max_{\alpha_t} \left[ \alpha_t \left( E_t R_{t+1} - R_{f,t+1} \right) + R_{f,t+1} - \frac{k}{2} \alpha_t^2 \sigma_t^2 \right] \tag{13.10}$$

maximizing (13.10) with respect to $\alpha_t$ gives the share of risky assets:

$$\alpha_t = \frac{E_t R_{t+1} - R_{f,t+1}}{k \sigma_t^2}$$

As shown in Chap. 8 the portfolio share in risky assets is equal to the expected excess return divided by the conditional variance times $k$ (the aversion to variance).
Since the Sharpe-ratio is:

$$SR_t = \frac{E R_{t+1} - R_{f,t+1}}{\sigma_t}$$

The portfolio share of risky assets can be written as

$$\alpha_t = \frac{SR_t}{k \sigma_t}$$

The following basic assumptions are usually made for standard portfolio choice: First, investors differ only with respect to cash and risky assets. Second, investors care only about mean and variance. Therefore everybody will hold the same portfolio of risky

[150] See also Campbell and Viceira (2002, Chap. 2).

assets (mutual fund theorem). Personal risk characteristics are not considered. Third, investors with different investment horizons (short and long ones) are disregarded.

As shown in Chap. 8, in general, portfolio mean returns are

$$E(R_p) = \gamma_1 E(R_1) + \gamma_2 E(R_2). \tag{13.11}$$

The portfolio variance ($\sigma^2_{R_p}$) for two assets is

$$\sigma^2_{R_p} = \gamma_1^2 \sigma^2_{R_1} + \gamma_2^2 \sigma^2_{R_2} + 2\gamma_1\gamma_2\sigma_{R_1}\sigma_{R_2} corr(R_1, R_2). \tag{13.12}$$

The portfolio variance is the sum of the weighted variances of the two assets plus the weighted correlation between the two assets.

In general, we have

$$\sigma^2_{R_p} = \sum_{g=1}^{G} \gamma_g^2 \sigma^2_{R_g} + \sum_{g=1}^{G}\sum_{b=1}^{G} \gamma_g\gamma_b cov(R_g, R_b).$$

Markowitz efficient portfolios are defined by using the mean-variance methodology. In Chap. 8 we distinguished between the one fund and two fund theory, the former consisting only of risky assets and the latter consisting of a risk free and a risky asset. Next, we will study an international portfolio of assets.

## 13.4 International Bond Portfolio

If we follow portfolio theory, we might think that investor's hold broadly diversified international portfolios[151], especially investors from smaller countries, because the national market is more incompletely diversified. However the stylized facts of international investing disagree with such a prediction. Private investors tend to overweight their portfolios with domestic financial assets (home country bias). Transaction costs, taxes, information problems and, language barriers in dealing with foreign securities, etc. are reasons for a home bias in the investor's portfolio. As we argued in Chap. 12.1, other types of risk besides asset risk, namely currency risk and and spillovers to the financial market may also play an important role for restrained international asset holdings.

Let us first assume that an investor's base currency is US $ and we consider an investment in international bonds. The return on a foreign bond, which is measured in US $ has three components: (1) interest income earned or accrued, (2) the capital gain or loss on the bond, resulting from the inverse relationship between interest rates and bond prices and (3) the foreign exchange gain or loss, applied to the above two items.

The initial purchase price of a bond in foreign currency terms is $B_t$, $S_t$ is the spot exchange rate and $B_t S_t$ is the US $ purchase price of the foreign bond. $\overline{B}_{t+1}$, the

[151] For details see Levich (2001), Chap. 14.

value of the bond after one month, is given by the initial bond price plus the price change ($\overline{\Delta}_{t+1}$) plus a cash flow from accrued interest ($\tilde{C}_{t+1}$). Therefore

$$\overline{B}_{t+1} \equiv B_t + \overline{\Delta}_{t+1} + \tilde{C}_{t+1}$$

When the bond price goes up, the interest rate will go down and vice versa. A foreign bond measured in US \$, and on an unhedged basis, has the following rate of return:

$$\overline{R}_{\$,US} = ln(\overline{B}_{t+1}\overline{S}_{t+1}/(B_t S_t) = ln(\overline{B}_{t+1}/B_t) + ln(\overline{S}_{t+1}/S_t) = \overline{B}_{FC} + \tilde{S}_{US\$,FC}$$

An investor has to deal with the uncertainty of a possible capital gain or loss on the bond and, additionally, a foreign exchange gain or loss. In analogy to the variance problem of equ. (13.12) we here find the following variance problem

$$\sigma^2(R_{\$,US}) = \sigma^2(\tilde{B}_{FC}) + \sigma^2(S_{US\$,FC}) + 2cov(\tilde{B}_{FC}; \tilde{S}_{US\$,FC})$$

**Table 13.2.** Currency and Bond Markets

| | | Currency Market Return[152] | | |
|---|---|---|---|---|
| | | Negative | Positive | |
| | Negative | FC interest rates ↑ | FC interest rates ↑ | |
| Bond | | Spot FX ↓ (A) | Spot FX ↑ | (C) |
| Market | | | | |
| Returns | Positive | FC interest rates ↓ | FC interest rates ↓ | |
| | | Spot FX ↓ (D) | Spot FX ↑ | (B) |

Table 14.2[153] nicely summarizes the forces affecting the return and risk of an international bond portfolio. (A) and (B) show a positive covariance between currency and bond market returns. Currency and bond position entail either both a loss or a profit. A negative covariance between currency and bond market returns are given in (C) and (D); (C) shows that interest rates have been rising, but foreign capital is attracted and therefore the exchange rate appreciates; (D) results in a low interest return.

Next, assume than an investor agrees on a one-month forward currency contract, whose price is $F_t$, for the estimated value of next month's bond with accrued interest $\hat{B}_{t+1}$, and then

$$\hat{B}_{t+1} \equiv B_t + \Delta_{t+1} + C_{t+1}$$

A perfect hedge is made if $\overline{B}_{t+1} = \hat{B}_{t+1}F_t$ is the value of the foreign bond in US \$. Since the investor cannot predict the exact future price, we have to consider a prediction error $\overline{\varepsilon}_{t+1}$. If the hedge amount was too small, that means $\overline{\varepsilon}_{t+1} > 0$, the

[152] For the table see Levich (2001, Ch. 14).

[153] For more details of such an interpretation, see Levich (2001, Chap. 14).

unexpected excess value of the bond is valued at $S_{t+1}$. If the hedge amount was too large, $\tilde{\varepsilon}_{t+1} < 0$, we must buy unexpected additional funds in the market at $S_{t+1}$.

The variability in foreign bond prices and the variability in the foreign exchange rate, which strongly influences the overall global risk of a foreign bond investment, have to be compared to the returns of unhedged foreign bonds. In order to reduce the riskiness of foreign bond returns, we have to apply currency hedging. We receive a similar result for the return on currency-hedged and unhedged portfolios, if the forward rate and the future spot exchange rate average turn out to be the same. A "perfect" currency hedge is the base for this calculation. The amount $B_{t+1}$ must be sold forward at price $F_t$. Presuming, for example, that every month this hedge is done.

We also have to distinguish between two passive strategies, unhedged and hedged currency. The first method never hedges currency risk, while the second always hedges currency risk. A strategy that accepts currency risk at some times, but hedges it at others is called an active strategy.

We often observe that investments in foreign bonds and foreign currency are related. However, one should separate investments in these two categories. This is because currency future contracts only deal with foreign exchange risk, whereas foreign bonds on a currency-hedged basis deal only with foreign interest rate risk.

One way to speculate on a currency is to take a position in foreign exchange spot, forwards or future contracts. The speculation will be profitable, if the currency forecast is correct, Nevertheless an investor can also ignore currency risk when he or she makes international investments. In this case he or she speculates that the currency effects cancel out in the long run. Next, we discuss an international equity portfolio.

## 13.5 International Equity Portfolio

Equity investment[154] is often evaluated by its expected return and risk. Here too we have to consider two sources of international risk, the risky asset return and the exchange rate risk.

Expected value gains could occur if foreign equity prices do not reflect all available information. Another reason is that foreign equity markets may be segmented from other capital markets, meaning that investors in the foreign market receive a different compensation for bearing equity risk than in other markets.

Diversification gains are usually considered a positive aspect of international investment in assets. Risk can be reduced because new investments, for which the returns are imperfectly correlated with the original portfolio, are included in the existing portfolio. Normally, the correlation of return across countries is low. Even if there is integration between foreign and domestic markets, diversification gains can be made. Superior "sharing" of international equity risk offers risk averse investors the possibility to make welfare gains.

[154] For details, see Levich (2001), Chap. 15.

Investors have a direct way to trade in foreign equity shares through the foreign stock market. Usually, however only large institutional investors are able to purchase securities in the foreign stock market.[155] Mutual funds are a possibility for small investors who wish to overcome barriers and to invest in foreign equity shares. One has to distinguish between the following categories: (1) gobal: investing in US and non-US shares, (2) international: investing in non-US shares only, (3) regional: investing in a geographic area, (4) country: investing in a single country, (5) speciality: international investments in an industry group such as information technology, or special themes such as newly privatized firms. Next, we again illustrate how to calculate unhedged returns. We denote $D$, dividend, and $\Delta$, capital gain (price change)

$$\tilde{E}_{t+1} = E_t + \tilde{\Delta}_{t+1} + D_{t+1}$$

The return is measured in US $ as

$$\begin{aligned}\tilde{R}_{\$,U} = ln\Big(\frac{\tilde{E}_{t+1}\tilde{S}_{t+1}}{E_t S_t}\Big) &= ln\Big(\frac{\tilde{E}_{t+1}}{E_t}\Big) + ln\Big(\frac{\tilde{S}_{t+1}}{S_t}\Big) \\ &= \tilde{E}_{FC} + \tilde{S}_{US\$,FC}\end{aligned}$$

We thus separate the two effects of an international portfolio, now for stocks.

**Table 13.3.** Currency Market Return

| | | Negative | | Positive | |
|---|---|---|---|---|---|
| Stock Market Returns | Negative | Stock market prices ↓<br>Spot FX ↓ | (A) | Stock market prices ↓<br>Spot FX ↑ | (C) |
| | Positive | Stock market prices ↑<br>Spot FX ↓ | (D) | Stock market prices ↑<br>Spot FX ↑ | (B) |

Table 15.3[156] illustrates the different sources affecting the return and variance of an international asset portfolio, here an equity portfolio.

If one holds both foreign and domestic stocks one accepts risk in both the foreign portfolio (due to asset price and exchange rate volatility) and the domestic portfolio. In principle, risk can be reduced if an investor invests in a broadly diversified portfolio of many securities. $\overline{R}_A$ and $\overline{R}_B$ are the returns of securities from the foreign asset A, and from the domestic asset B, and $\sigma(\overline{R}_A), \sigma(\overline{R}_B)$ are our risk measures. The portfolio return and risk are:

$$\overline{R}_P = \gamma_A \overline{R}_A + \gamma_B \overline{R}_B$$

$$\sigma^2(\overline{R}_P) = \gamma_A^2 \sigma^2(\overline{R}_A) + \gamma_B^2 \sigma^2(\overline{R}_B) + 2\gamma_A \gamma_B Cov(\overline{R}_A \overline{R}_B)$$

[155] For details on laws and regulations, see Levich (2001, Chap. 15).
[156] For details of such a table see Levich (2001, Chap. 15).

$$\sigma^2(\overline{R}_P) = \gamma_A^2\sigma^2(\overline{R}_A) + \gamma_B^2\sigma^2(\overline{R}_B) + 2\gamma_A\gamma_B Corr(\overline{R}_A\overline{R}_B)\sigma(\overline{R}_A)\sigma(\overline{R}_B)$$

$\gamma_A$ and $\gamma_B$ are the investment weights in securities A and B that sum up to one. The above equation can be used as a basis for hedging risk. In order to hedge the risk in one asset, we have to find another asset that varies as the first does and is perfectly negatively correlated with the first asset.

## 13.6 Efficient Frontier of an International Portfolio

As demonstrated in Chap. 8 a computer program, written in the computer language Gauss, can be used to compute the efficient frontier of an international portfolio of assets. As concerning an international equity portfolio, which we want to study here, usually only stock market time series data are available where the stock prices and returns are denominated in the home countries' currency. Morgan Stanley[157] has, however, made available a time series of the stock market indices of numerous countries that is already converted into a dollar price index. The data series is available for monthly stock market data. The index thus captures the risk and return of both the equity and the exchange rate of the respective countries.

In the subsequent portfolio we have put together a portfolio of international assets for Japan, France, Germany, UK, U.S. and Mexico for monthly data, for a sample period 1990.01 to 2004.12. This is a rather highly volatile period of stock prices and returns and it may be compared with the risk return trade-off of the stock prices and returns in Chap. 8 of solely domestic US companies, also for monthly data. Yet there the study is undertaken for the time period 1990.1 to 1999.12.

Figure 13.1 shows the Markowitz efficient frontier of a U.S. portfolio composed of the stock market index of Japan, France, Germany, UK, U.S., and Mexico. Again the horizontal axis represents the standard deviation of the stock returns and the vertical axis the expected returns from a portfolio of equities of the aforementioned countries. The straight line represents the line for the risk free interest rate, here again chosen as monthly rate of 0.03/12. As one can observe from the comparison of figure 13.1 and figure 8.4, in the 1990's a domestic stock portfolio would have been preferable to an international portfolio of equity assets.

## 13.7 Conclusions

The movement of countries to flexible exchange rates and the globalization of finance has created new sources of risk. On the other hand it has also created new opportunities for investors to diversify portfolios and to move into new frontiers of risk-return trade-offs. Individuals as well as companies are exposed to new risk and international portfolios can be used to hedge the risk and to secure returns. We have

[157] The data series can be downloaded from the following web-site: www.msci.com/equity/index2/html.

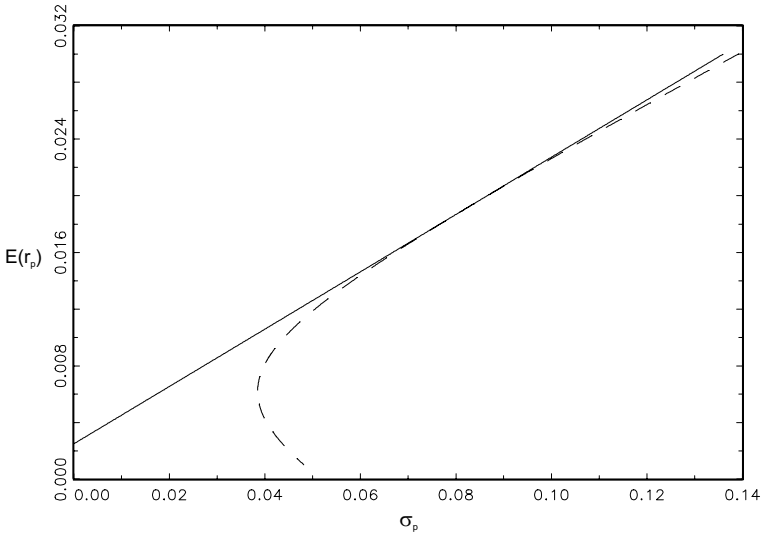



**Fig. 13.1.** Risk-Return Trade Off for an International Portfolio of Assets

sketched in this chapter what issues are involved in international portfolios and how a Markowitz efficient frontier for an international portfolio can be computed. Yet, as elaborated at the end of Chap. 8, static portfolio decisions might not always be appropriate, in particular when a new environment of investment opportunities arises. Models of dynamic portfolio decisions, which may also have to consider a time path of consumption, are needed. We will turn to this topic in part VI of the book.

CHAPTER 14

# Agent Based and Evolutionary Modeling of Asset Markets

## 14.1 Introduction

The next chapters deal with some advanced topics in the dynamic modeling of asset markets. The current chapter represents a brief introduction to recent developments in financial market studies that build on heterogeneous agent and evolutionary models of asset market dynamics. We have already briefly discussed, in Chap. 5, heterogeneous agent models which borrow from evolutionary theory. Here we want to go into more details of recent heterogeneous agent and evolutionary models. We will only discuss two interesting prototype models. In Chap. 15 then we will study further approaches that have also moved away from expected utility maximizing models.

## 14.2 Heterogeneous Agent Models

Let us first discuss a heterogeneous agents model. The model we present here originates in the work by LeBaron.[158] It is based on two assets, a stock paying a stochastic dividend and a risk-free asset paying a fixed interest rate. Agents are represented by preferences with a common discount factor. There are two types of traders: traders with short-term and traders with long-term perspectives. Each group of agents evolve over time. The study examines how the diverse population evolves and whether long-horizon agents can eventually drive out agents with short-horizon. The agents can choose a trading-rule from a common set of rules. In each period the agent will change his or her current rule by choosing the one that has done best over certain time horizon in the past. The rules are embedded in a neural network which is a flexible nonlinear function, mapping past information into current portfolio weights. Rules evolve over time and new rules are generated by a kind of genetic algorithm.

Agents' income consists of dividends and capital gains from purchases and sales of equity. The entire income is used for building up wealth and current consumption. An agent is endowed by a power utility preferences of logarithmic form,

$$u_{i,t} = E_t \sum_{s=0}^{\infty} \beta^s log c_{i,t+s} \tag{14.1}$$

[158] See LeBaron (2001 a,b; 2002; 2004).

subject to the intertemporal budget constraint,

$$w_{i,t} = p_t s_{i,t} + b_{i,t} + c_t = (p_t + d_t)s_{i,t-1} + (1 + r_f)b_{i,t-1} \tag{14.2}$$

Here $s_{i,t}$ denotes risky asset holdings, $b_{i,t}$ the risk free asset holdings and $w_{i,t}$ the wealth. The risk free rate of return is $r_f$, $r_t$ is the risky asset return at time $t$, $p_t$ is the price of the security, $d_t$ the dividend payment and $c_t$ is consumption. The above preferences are used for tractability in LeBaron's work. In case of a logarithmic utility the agent's optimal consumption choice can be separated from the portfolio choice. For this utility function, consumption is a constant fraction of wealth[159]

$$c_{i,t} = (1 - \beta)w_{i,t} \tag{14.3}$$

To generate some additional diversity of the agent an idiosyncratic noise term can be added to the consumption choice as in

$$c_{i,t} = (1 + \gamma_i)(1 - \beta)w_{i,t} \tag{14.4}$$

where $\gamma_i$ is the noise term, independent across agents and time. The discount factor $\beta$, can be set to $\frac{1}{1+\rho}$ for $\rho > 0$. We thus have

$$\rho = \frac{1}{\beta} - 1 \tag{14.5}$$

Note that $\rho$ is different from $r_t$ and $r_f$.
The portfolio decision is myopic in that agents maximize the logarithm of next period's portfolio return. Agents direct their learning efforts to this optimal portfolio decision. The objective is to find a rule that will maximize the expected logarithm of the portfolio return. Thus we have as problem

$$\max_{\alpha_j} E_t log[1 + \alpha_j r_{t+1} + (1 - \alpha_j)r_f] \tag{14.6}$$

for the set of all existing rules and information up to time period $t$. It is presumed that it is not possible for agents to compute this optimization each period. The impossibility results from the above expectations. The expectations depend on the state of all other agents in the market, along with the state of the dividend payments. The portfolio decision, therefore, is replaced by a simple rule which will be continuously tested against other alternative rules. This testing represents a key part of the learning procedure in the financial market.

Agent $i$ will consider the following portfolio objective, where the index $j$ corresponds to the different possible trading rules.

$$\hat{E}(r_p) = \frac{1}{T_i}\sum_{k=1}^{T_i} log\{1 + \alpha(z_{t-k}; \omega_j)r_{t-k+1} + [1 - \alpha(z_{t-k}; \omega_j)]r_f\} \tag{14.7}$$

[159] See Cochrane (2001, Chap. 9.1) and LeBaron (2002).

Hereby $z_{t-k}$ is the time $t-k$ information and $\omega_j$ the parameters specific to rule $j$. Agents choose a rule based on averaging past performance over a horizon of length $t_i$, but they have a memory span $T_i$ which entails heterogeneity across agents' decisions. A further element of heterogeneity of the decision rule is a random element. Agents look over the set of rules and randomly choose one from the top 25% of all rules over the past horizon $T_i$. If the rule is better than the current strategy, it will replace the older one. If not, the agents continue to use the same rule. However, the agents will only change the rule if the new one exceeds the current one by a fixed percentage. Both, rules and agents are allowed to evolve over time.

The agents trading strategies are based on a simple information structure. These information structures are input into a neural network, and are used to generate the trading strategies, $\alpha(z_t; w_j)$. The choice of the information set $z_t$ is very important. The information set includes past dividends, returns, the price dividend ratio and trend following technical trading indicators. The exponential moving averages are used as types of technical rules. The moving averages are mapped as:

$$m_{k,t} = \rho m_{k,t-1} + (1-\rho)p_t \tag{14.8}$$

with $\rho$ a fixed parameter. As trading begins at time $t$, all $t-1$ and earlier information is known. The dividends at time $t$ have been revealed and paid. This means that $\alpha_j$ can be written as a function of $p_t$ and information that is known at time $t$,

$$\alpha_j = \alpha_j(p_t; I_t) \tag{14.9}$$

Before trading begins in period $t$, all variables are known and $p_t$ will then be determined endogenously to clear the market.

Trading is executed by finding the aggregate demand for shares and setting it equal to the fixed aggregate supply of one share. Each agents demand for shares, $s_{i,t}$ at time $t$ can be written as

$$s_{t,i}(p_t) = \frac{\alpha_i(p_t; I_t)\beta w_{i,t}}{p_t} \tag{14.10}$$

$$w_{i,t} = (p_t + d_t)s_{i,t-1} + (1+r_f)b_{i,t-1} \tag{14.11}$$

Hereby $w_{i,t}$ is the total wealth of agent $i$ and $b_{i,t-1}$ are the bond holdings from the previous period. The aggregate demand function is

$$D(p_t) = \sum_{i=1}^{I} s_{i,t}(p_t) \tag{14.12}$$

If one sets $D(p_t) = 1$, one finds the equilibrium price $p_t$. Yet, there may not be only one price at time $t$ in the equilibrium. If the agent wants to change the trading rule and switch to another rule, then the equilibrium is only a temporary one.

The process of creating rules, is an evolutionary dynamic process that follows adaption and learning and is finally solved numerically. The rules are evolving by

using a genetic algorithm, which is a widely used technique in computational learning. This method takes useful rules and either modifies them (mutation) or combines them with parts of other rules (crossover).

There is an evolutionary process in which the first step is to identify the set of rules. The algorithm chooses between three methods with equal probability: (1) Mutation – choosing the first rule from the parent and then adding a uniform random variable distributed uniformly to one of the network weights. (2) New weight – choose one rule from the parent set, choose one weight at random, and replace it with a new value chosen uniformly from [-1,1]. (3) Crossover – take two "parents" at random from the set of good rules. Afterwards take all weights from one "parent", and replace one set of weights corresponding to one input with the weights from the other "parent".

The evolution of a rule is characterized by evaluating its past performance. This produces new and interesting strategies that must then survive the competition with the other rules in terms of properly forecasting returns. This is studied in detail in the papers by LeBaron (2001 a,b; 2002; 2004) where the returns, trading volume and the consumption paths of the agents with long and short memory are tracked. This approach then comes close to replicating many features of actual financial markets such as excess volatility, and excess kurtosis as well as persistence in trading volume. Yet it cannot track the dichotomizing volatility of returns and consumption. Presumably other models are needed to track the latter feature of financial markets (see Chaps. 6 and 15). To what extent agent-based models are able to replicate asset market characteristics is extensively surveyed in LeBaron (2004).

## 14.3 Evolutionary Models

Evolutionary models are Darwinian oriented and focus on strategies, market selection and mutation. A recently developed prototypical evolutionary approach is the one by Hens and Schenk-Hoppe (2004a,b). It follows a Darwinian approach where there are two forces working: one reducing the variety of species and one increasing it. A Darwinian theory of portfolio selection is confronted with the problem of portfolio rules that may be operative in the market and it wants to determine which portfolio rule is evolutionary stable in the sense that it cannot be undercut by any other strategy in managing and building up wealth. The approach thus addresses the selection of the best strategies.

As it turns out there is one specific strategy, namely the strategy that allocates funds in proportion to relative dividends, that seems to be evolutionarily superior as compared with other strategies such as for example, the mean-variance rule, the growth optimal rule, the CAPM rule, a naive diversification rule and prospect theory based rules. Although as shown in Hens et al (2004a,b) every strategy that differs from the relative dividend's rule can be driven out by some other strategy, this does not hold for the relative dividend's rule. The model presumed here is an exchange economy where dividends are randomely generated and the dividends pay off a

perishable consumption good as in the paper by Lucas (1978). A brief sketch of the model is appropriate.

We follow Hens and Schenk-Hoppe (2004a,b) and consider a financial market with $K \geq 1$ long-lived assets $k = 1, ...K$ in unit supply, each paying random dividend $D_t^k \geq 0$ at any period in time $t = 0, 1....$ We can normalize the price of the consumption good to one. An investor's wealth in terms of the numeraire is then given by

$$w_{t+1}^i = \sum_{k=1}^{K} (D_{t+1}^k + p_{t+1}^k)\theta_{t,k}^i. \tag{14.13}$$

Hereby, for time period $t$, $(\theta_{t,1}^i, ...., \theta_{t,K}^i)$ denotes investor $i$'s portfolio and $p_t^k$ is asset $k$'s price. They are determined by

$$\theta_{t,k}^i = \frac{\lambda_{t,k}^i w_t^i}{p_t^k} \text{ and } p_t^k = \sum_{i=1}^{I} \lambda_{t,k}^i w_t^i = \lambda_{t,k} w_t \tag{14.14}$$

Hereby $\lambda_{t,k}^i$ is investor $i$'s fraction of the budget that purchases asset $k$. Prices are given by equating each asset's market value with the investment in that asset (supply is normalized to 1).

Concerning consumption it is assumed that all investors always consume the same fraction of their wealth. Denoting the budget share used up by consumption by $\lambda_0 > 0$, one has

$$D_t = \sum_{k=1}^{K} D_t^k = \lambda_0 \sum_{i=1}^{I} w_t^i = \lambda_0 W_t \tag{14.15}$$

Then equ. (14.13) can give us an equation for investors' market shares $\alpha_t^i = w_t^i / W_t$:

$$\alpha_{t+1}^i = \sum_{k=1}^{K} \left( \lambda_0 d_{t+1}^k + \sum_{j=1}^{I} \lambda_{t+1,k}^j \alpha_{t+1}^j \right) \frac{\lambda_{t,k}^i \alpha_t^i}{\sum_{j=1}^{I} \lambda_{t,k}^j \alpha_t^j} \tag{14.16}$$

where $d_{t+1}^k = D_{t+1}^k / D_{t+1}$ denotes asset $k$'s relative dividend pay-off. It is assumed that at least one asset pays a dividend, $D_{t+1} > 0$. The equation (14.16) is linear in $\alpha_{t+1} = (\alpha_{t+1}^1, ...\alpha_{t1}^I)$ and its solution is given by

$$\alpha_{t+1} = \lambda_0 \left( Id - \left[ \frac{\lambda_{t,k}^i \alpha_t^i}{\lambda_{t,k} \alpha_t} \right]_i^k \Lambda_{t+1} \right)^{-1} \left[ \sum_{k=1}^{K} d_{t+1}^k \frac{\lambda_{t,k}^i \alpha_t^i}{\lambda_{t,k} \alpha_t} \right]_i \tag{14.17}$$

with $\Lambda_{t+1}^T = (\lambda_{t+1,1}^T, ...\lambda_{t+1,K}^T) \in \mathbb{R}^{I \times K}$ denoting the matrix of budget shares in period $t+1$ and $Id$ the identity matrix.

Overall, equ. (14.17) gives us the evolution of market shares for given trading strategies of investors. The authors refer to this as the market selection process.

There is randomness of dividend pay-offs. Dividend pay-offs are given by the states of nature revealed up to and including time $t+1$, whereas the state of nature

$\omega_t \in S$ (where $S$ is a finite set) is given by a stationary stochastic process. The relative dividend $d_t^k = d_t^k(\omega^t)$, with the observed history of states denoted by $\omega^t = (\omega_0, ..., \omega_t)$.

Let us define the trading strategies. A trading strategy is a sequence of budget shares $\lambda_t^i = (\lambda_0, \lambda_{t,1}^i, ..., \lambda_{t,K}^i)$ with $\lambda_0 + \sum_{k=1}^K \lambda_{t,k}^i = 1$. Hereby $\lambda_t^i$ depends on all past observations. Yet it does not depend on current market-clearing prices nor on other investors' current strategies.

The evolution of market shares is well-defined if no bankruptcy occurs and markets always clear. If short sales are allowed, bankruptcy could occur in the absence of short selling. Equ. (14.17) is well-defined and the following conditions hold. Assume that for all $t, \lambda_{t,K}^i \geq 0$ (for all $i, k$) and that there is an investor with $\alpha_t^j > 0$ such that $\lambda_{t,K}^j > 0$ for all $k$. Then equ. (13.17) is a well-defined map on the simplex $\Delta^I = \{\alpha \in \mathbb{R}^I \mid \alpha^i \geq 0, \sum_i \alpha^I = 1\}$.[160] Equ. (14.17) generates a (non-autonomous) random dynamical system on $\Delta^I$. For any initial distribution of wealth $w_0 \in \mathbb{R}_+^I$, equ. (14.17) defines the path of market shares on the event tree with branches $\omega^t$. The initial distribution of market shares is given by $(\alpha_0^i)_i = (w_0^i/W_0)_i$.

Moreover, the wealth of a strategy $i$, in any time period, can be derived from the market share and the aggregate wealth, defined by equ. (14.15), as

$$w_t^i = \frac{D_{t+1}(\omega^{t+1})}{\lambda_0} \alpha_{t+1}^i \tag{14.18}$$

In order to present a simple analysis of the evolutionary model as a decision model for holding financial assets the authors confine it to a simple strategy: $\lambda^i \in \Delta^{K+1}$ for all $i = 1, ..., I$ and $\lambda_0^i = \lambda_0$; i.i.d. dividend payments $d_t^k(\omega^t) = d^k(\omega_t)$, for all $k = 1, ..., K$ and the state of nature $\omega_t$ follows an i.i.d. process.

Furthermore, the authors define an evolutionary investment rule $\lambda^*$ which allocates funds in proportion of dividends as a portfolio rule. They claim that it is in fact the only candidate for a rule that can attract the entire market wealth. They give an interpretation of the evolutionary investment rule $\lambda^*$ in terms of the well-known growth optimal portfolio rule which dominates all other rules.[161] Then furthermore in simulation experiments the authors show that this Darwinian model of portfolio strategy which allocates funds in proportion to relative dividends is evolutionary superior to strategies following the mean-variance rule, a value at Risk (VaR) strategy, the growth optimal rule, a naive diversification rule (that allocates equal budget shares to all assets and which can itself dominate other rules)[162] and a prospect theory based rule.

Overall, the authors claim that the evolutionary portfolio rule that is put forward by the authors, where the portfolio weight should be proportional to the expected relative dividends to the assets, dominate all other portfolio strategies. It can be

[160] For a further elaboration on this, see Schenk-Hoppe (2004a).

[161] See Hens and Schenk-Hoppe (2004a).

[162] Hens and Schenk-Hoppe (2004a) show that a static optimal strategies such as the mean variance rule can be inferior to a naive rule, since the latter underdiversifies in the market selection process.

interpreted as some kind of CAPM rule which first fixes budget shares according to the expected market capitalization and then rebalances as prices fluctuate. They claim that this is particularly important for sufficiently patient investors, such as pension funds or insurance companies. In Chap. 15 we will consider further dynamic portfolio models that allow not only re-balancing of the portfolio as new investment opportunities arise but also include dynamic consumption decisions over time which in the above model is set to a fixed fraction of the agent's wealth.

## 14.4 Conclusions

We have introduced and discussed two prototype models that go beyond consumption based approaches. Consumption is present in both agent based and evolutionary modeling of asset markets. Yet, at the forefront of those studies are the dynamics of asset returns and volatility. An interesting feature of evolutionary models is the idea of the replicator dynamics, borrowed from mathematical biology and evolutionary game theory according to which both the issue of asset market dynamics as well as wealth distribution in the long run can be addressed. Chap. 15 will consider those issues from the perspective of intertemporal dynamic asset pricing theory, including dynamic consumption decisions, but going beyond the consumption based asset pricing model.

# Part VI

# Advanced Modeling of Asset Markets

CHAPTER 15

# Behavioral Models of Dynamic Asset Pricing

## 15.1 Introduction

As mentioned in Chaps. 9-10, extensions of consumption based asset pricing models can be found in the work that employs non-separable utility functions, for example models with habit formation and recursive preferences. The latter can be found in Zin and Epstein (1989, 1991). Recent development in modeling asset pricing is characterized by moving away from the paradigm of the rational expected utility maximizing economic agent by emphasizing behavioral features in the agent's decision making. Since the beginning of the 1990's, although still grounded in the consumption based asset pricing tradition, models have been developed that stress the role of habits in economic decision making. These are called habit formation models. Another direction was pursued by Thaler et al. (1997), Benartzki and Thaler, and Barberis et al. (2001) who design models in the tradition of behavioral finance. A firm foundation of those behavioral finance models was given by the prospect theory developed by Kahneman and Tversky (1979). This has led to a further development of asset pricing models that take the precautionary behavior of economic agents and their attitude toward risk taking seriously. This approach can more realistically model economic behavior and is designed to give a better account of the risk-free interest rate, equity premium, and Sharpe ratio.

## 15.2 Dynamic Habit Formation Models

Concerning habit formation, we can write a one period utility function as $U(C_t, X_t)$ where $X_t$ is the time-varying habit also called subsistence level. We first have to think about the functional form for $U(\cdot)$. Abel (1990, 1996) presumes that $U(\cdot)$ should be a power function of the ratio $C_t/X_t$ while Constantinides (1990) and Campbell and Cochrane (1999), have used a power function of the difference $C_t - X_t$.

We first follow Abel (1990, 1996), and assume that an agent's utility can be written as a power function of the ratio $C_t/X_t$,

$$U_t = \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j}/X_{t+j})^{1-\gamma} - 1}{1-\gamma} \tag{15.1}$$

hereby $X_t$ indicates the impact of past consumption levels on today's utility and $X_t$ can be specified as an internal habit or as an external habit. To simplify one can use a 1-period lag in consumption to define the agent's internal habit. We thus may write

$$X_1 = C^*_{1-t}$$

The external-habit specification where the aggregate past consumption is important, can be written as

$$X_t = \overline{C}^*_{t-1}$$

The two different formulations of habit formation yield different Euler equations.[163] As aforementioned, another development is represented by difference models. Consider a model in which the utility function is

$$U_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1-\gamma} \right]. \tag{15.2}$$

Here too, for simplicity, we treat the level $X_t$ as external. This model differs from the ratio model in two ways. First, in the difference model the agent's risk aversion varies with the level of consumption relative to habit, whereas risk aversion is constant in the ratio model. Second, in the difference model, consumption must always be above habit for utility to be well defined, whereas this is not required in the ratio model. In the difference model one gets

$$S_t \equiv \frac{C_t - X_t}{C_t} \tag{15.3}$$

$$\frac{-CU_{CC}}{u_C} = \frac{\gamma}{S_t} \tag{15.4}$$

The measure of equ. (15.4) shows that risk aversion rises, as the surplus consumption ratio $S_t$ declines, that is, as consumption declines toward habit. Time varying surplus consumption will give rise to a non-constant risk-aversion, which is in the power utility function only a constant parameter, $\gamma$.

When habit is given by the difference form the marginal utility of consumption is $u'(C_t) = (C_t - X_t)^{-\gamma} = S_t^{-\gamma} C_t^{-\gamma}$. The stochastic discount factor is then

$$m_{t+1} \equiv \beta \frac{u'(C_{t+1})}{u'(C_t)} = \beta \left( \frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\gamma} \tag{15.5}$$

In the standard power utility model we have $S_t = 1$, so the discount factor is just consumption growth raised to the power $-\gamma$. To get a more volatile stochastic discount factor one needs a large value of $\gamma$. This might, however, lead to a too unrealistic $\gamma$ and, moreover, this usually leads to a to risk free interest rate that is too volatile as compared to the data.

In a habit formation model one can instead get a volatile stochastic discount factor from a volatile surplus consumption ratio $S_t$ as equs. (15.4) and (15.5) show. Accordingly, the risk free interest rate is:

$$(1 + R^f_{t+1}) = 1/E_t(m_{t+1})$$

[163] For further details on the Euler equation and stochastic discount factor, see Campbell et al. (1997, Chap. 8.3).

This basic construction gives us a time variation in risk aversion. When consumption falls relative to habit, the resulting increase in risk aversion drives up the risk premium in risky assets such as stocks and the reverse holds for a rise in surplus consumption.

In order to further explore the role of habit formation in asset pricing we want to include it in a model with production. To match the asset price characteristics of the model to the data, economic research has extended the baseline stochastic growth model as presented in Chap. 10 to include habit formation, adjustment costs of investment, idiosyncratic technology shocks to firms and the effect of leverage on firm value.[164] In this chapter we will focus on a model with habit formation and adjustment costs of investment, but we presume an inelastic labor supply. Thus, there will be no choice of labor effort.

Since, as aforementioned, time separable preferences fail to match financial market characteristics, an enormous effort has been invested into models with time non-separable preferences, such as habit formation models. If one chooses a habit formation model, risk aversion is then, as above discussed, generally time varying.

There is a long tradition in economic theory where it is assumed that habits are formed through past consumption.[165] Habit persistence is nowadays used to understand a wide range of issues in growth theory (Carrol et al. 1997, 2000, Alvarez-Cuadrado et al. 2004) macroeconomics (Fuhrer, 2002), and business cycle theory (Boldrin et al, 2001). In all of those models of habit persistence, a high level of consumption in the past depresses current welfare and a high current consumption depresses future welfare. This can be written as ratios of current over past consumption (Abel 1990, 1999) or in difference form as $(1-b)C_t + b(C_t - C_{t-1})$ with $C_t$ current, $C_{t-1}$ past consumption and $b$ a respective weight. The difference form of habit formation, which allows for a time varying risk aversion, will be chosen below.

This type of habit specification gives rise to time non-separable preferences where risk aversion and intertemporal elasticity of substitution are separated and, as discussed above, a time variation of risk aversion will arise. If we define surplus consumption as in equ. (15.3) with $X_t$, the habit, and $\gamma$, the risk aversion parameter, then the time variation of risk-aversion is as defined in equ. (15.4): the risk aversion falls with rising surplus consumption and the reverse holds for falling surplus consumption. Then, as indicated in equ. (15.5) a high volatility of the surplus consumption will lead to a high volatility of the growth of marginal utility and thus to a high volatility of the stochastic discount factor.

In asset pricing, the idea of habit persistence has been introduced by Constantinides (1990) in order to account for high equity premia. Asset pricing models along this line have been further explored by Campbell and Cochrane (1999), Jerman (1998), and Boldrin et al. (2001). Yet, asset pricing with habit persistence in stochastic models with production may just produce smoother consumption. But with

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[164] For further detailed studies of those extensions see, for example, Campbell and Cochrane (1999), Jerman (1998), Boldrin, Christiano and Fisher (2001) and Cochrane (2001, Chap. 21). For the effect of leverage, see Grüne and Semmler (2005a).

[165] First descriptions of habit formation can be found in Marshall, Veblen and Duesenberry. For a first use of habit persistence in a dynamic decision model see Ryder and Heal (1973).

income different from consumption, for example, due to shocks, habit formation amplifies investment and demand for capital goods. Yet, Boldrin et al. (2001) have argued if there is, however, a perfectly elastic supply of capital there is no effect on the volatility of the return on equity. As the literature has demonstrated (Jerman 1998, and Boldrin et al. 2001) one also needs adjustment costs of investment to minimize the elasticity of the supply of capital. It seems to be both habit persistence and adjustment costs for investment which are needed to generate higher equity premia. Following Jerman (1998) by choosing such a model we will not, as in the model of Chap. 10, allow for elastic labor supply, but rather employ a model with fixed labor supply, since the latter, as shown in Lettau and Uhlig (2000), provides the most favorable case for matching the model with the financial market characteristics.

Since the accuracy of the solution method is an intricate issue for models with more complicated decision structure, we first have to have sufficient confidence in the accuracy of the stochastic dynamic programming method that we will use. In Grüne and Semmler (2004a,b) a stochastic dynamic programming algorithm with flexible grid size has been tested for the most basic stochastic growth model as based on Brock and Mirman (1972) and Brock (1979, 1982). This model can be analytically solved for the sequence of optimal consumption. Asset prices, the risk-free interest rate, the equity premium and the Sharpe-ratio, can, once the model is solved analytically for the sequence of optimal consumption, easily be solved numerically and those solutions can be compared to the numerical solutions obtained from a numerical procedure.

One can apply the numerical procedure of Grüne and Semmler (2004b) to the model given by

$$\begin{aligned} k_{t+1} &= \varphi_1(k_t, z_t, C_t, \varepsilon_t) = k_t + \frac{k_t}{1-\varphi}\left[\left(\frac{I_t}{k_t}\right)^{1-\varphi} - 1\right] \\ ln\ z_t &= \varphi_2(k_t, z_t, C_t, \varepsilon_t) = \rho\ ln\ z_t + \varepsilon_t, \end{aligned}$$

with $I_t = z_t A k_t^{\alpha} - C_t$, where in our numerical computations we used the variable $y_t = \ln z_t$ instead of $z_t$ as the second variable.

The utility function is given by the difference model

$$u(C_t, X_t) = \frac{(C_t - bX_t)^{1-\gamma} - 1}{1-\gamma}$$

for $\gamma \neq 1$ and by

$$u(C_t, X_t) = \ln(C_t - bX_t)$$

with $\gamma = 1$. Hereby the parameter $b > 0$ is assumed. Since, in our case, we are working with internal habit we have $X_t = C_{t-1}$.

For a numerical study of the above habit formation model Grüne and Semmler (2004b) employ the values

$$A = 5,\ \ \alpha = 0.34,\ \ \rho = 0.9,\ \ \beta = 0.95,\ \ b = 0.5$$

and $\varepsilon_t$ was chosen as a Gaussian distributed random variable with standard deviation $\sigma = 0.008$, which we restricted to the interval $[-0.032, 0.032]$. With this choice of

parameters it is easily seen that the interval $[-0.32, 0.32]$ is invariant for the second variable $y_t$.[166]

It is interesting to compare the numerical results that we have obtained this way to previous quantitative studies undertaken for habit formation, but using other solution techniques. We can restrict ourselves to a comparison with the results obtained by Boldrin et al. (2001) and Jerman (1998).

Whereas Boldrin et al. use a model with log utility for internal habit, but endogenous labor supply in the household's preferences, Jerman studies the asset price implication of a stochastic growth model, also with internal habit formation but, as in Grüne and Semmler (2004b), labor effort is not a choice variable. All three papers use adjustment costs of investment in the model with habit formation. The first two studies claim that habit formation models with adjustment costs can match the financial characteristics of the data. Yet, both studies have chosen parameters that appear to be conducive to results which replicate better the financial characteristics such as the risk free rate, equity premium and the Sharpe ratio.

In comparison to the first two papers Grüne and Semmler (2004c) have chosen parameters that have commonly been used for stochastic growth models[167] and that seem to describe the first and second moments of the data well. Table 15.1 reports the parameters and the results.

Both, the study by Boldrin et al. (2001) and Jerman (1998) have chosen a parameter, $\varphi = 4.05$, in the adjustment costs of investment, a very high value which is at the very upper bound found in the data.[168] Since the parameter $\varphi$ smoothes the fluctuation of the capital stock and makes the supply of capital very inelastic, Grüne and Semmler (2004b) have rather worked with a $\varphi = 0.8$ in order to avoid such strong volatility of returns generated by high $\varphi$. Moreover, the first two papers use a higher parameter for past consumption, $b$. Both papers have also selected a higher standard deviation of the technology shock. Boldrin et al. take $\sigma = 0.018$, and Jerman takes a $\sigma = 0.01$, whereas Grüne and Semmler (2004c) use $\sigma = 0.008$ which has been employed in many models.[169] Those parameters increase the volatility of the stochastic discount factor, a crucial ingredient to raise the equity premium and the Sharpe ratio.

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[166] However, the habit persistence implies that for a given habit $X_t$ only those value $C_t$ are admissible for which $C_t - bX_t > 0$ holds, which defines a constraint from below on $C_t$ depending on the habit $X_t$. On the other hand, the condition that investment should be $I_t \geq 0$ defines a constraint from above on $C_t$ depending on $k_t$ and $y_t = \ln z_t$. As a consequence, there exist states for which the set of admissible control values $C_t$ is empty, i.e., for which the problem is not feasible.

[167] See Prescott (1985) and Santos and Vigo-Aguiar (1998).

[168] See, for example, Kim (2002) for a summary of the empirical results reported on $\varphi$ in empirical studies.

[169] This value of $\sigma$ has also been used by Prescott (1985) and Santos and Vigo-Aguiar (1998).

**Table 15.1.** Habit Formation Models

| Boldrin et al.[a)] | Jerman[b)] | Grüne[c)] and Semmler | US Data[d)] (1954-1990) |
|---|---|---|---|
| $b$= 0.73-0.9<br>$\varphi$=4.05<br>$\sigma$= 0.018<br>$\rho$= 0.9<br>$\beta$= 0.99999<br>$\gamma$= 1 | $b$= 0.83<br>$\varphi$=4.05<br>$\sigma$= 0.01<br>$\rho$= 0.99<br>$\beta$= 0.99<br>$\gamma$= 5 | $b$= 0-0.5<br>$\varphi$= 0-0.8<br>$\sigma$= 0.008<br>$\rho$= 0.9<br>$\beta$= 0.95<br>$\gamma$= 1-3 | |
| $r^f = 1.2$ | $r^f = 0.81$ | $r^f = 5.1 - 8.5$ | $r^f = 0.8$ |
| $E(r) - r^f$ =6.63 | $E(r) - r^f$ =6.2 | $E(r) - r^f$ =1.32 | $E(r) - r^f$ =6.18 |
| $SR$= 0.36 | $SR$= 0.33 | $SR$=0.11 | $SR$=0.35 |

a) Boldrin et al.(2001) use a model with endogenous labor supply, log utility for habit formation and adjustment costs, $\sigma$ quarterly, return data and Sharpe ratio are, in percentage terms, annualized.

b) Jerman (1998) uses a model with a exogenous labor supply, habit formation with coefficient of $RRA$ of 5, and adjustment costs, $\sigma$ quarterly, return data and Sharpe ratio are annualized.

c) See Grüne and Semmler (2004b). The return data and the Sharpe ratio are annualized. Note that a $\beta$ is chosen that represents an annual subjective discount factor. Yet, one can think of the time unit for the standard deviation of the shock as a quarter.

d) The following financial characteristics of the data are reported in Jerman (1998), for annualized returns and Sharpe ratio in percentage terms.

Jerman, in addition, takes a very high parameter of relative risk aversion, $\gamma = 5$, which also increases the volatility of the discount factor and increases the equity premium when used for the pricing of assets. Jerman also presumes a much higher persistence parameter for the technology shocks, $\rho = 0.99$, from which one knows that it will make the stochastic discount factor more volatile too. All in all, Boldrin et al. and Jerman have chosen parameters which are known to bias the results toward the empirically found financial characteristics.

We want to note, since the risk aversion, $\frac{\gamma}{S_t}$, for power utility rises with the consumption surplus ratio, given by

$$S_t = \frac{C_t - bX_t}{C_t},$$

the habit persistence model predicts a rising risk aversion (rising Sharpe ratio) in recessions and falling risk aversion (falling Sharpe ratio) in booms, for details see Cochrane (2001, p. 471). Thus, the risk aversion and the Sharpe ratio move over time. What is depicted in table 15.1 are averages along the optimal trajectories in the neighborhood of the steady state.

A further remark is needed concerning the high risk free rate, in table 15.1 computed as $r^f$ for quarterly data, annualized in percentage terms. As table 15.1

shows the risk free rate is very high for the high $\gamma$. In the Grüne and Semmler model a $\beta$ is chosen that may in fact represent an annual subjective discount factor. So one does not have to scale it up by a factor of four. In the basic model, with $\gamma = 1$, no habit persistence and no adjustment costs, $r^f$ is about 5.1 percent. This is, of course, still too high as compared to empirical data. As has been pointed out in the literature[170] the equity premium and Sharpe ratio are favorably be impacted by a higher $\gamma$, and by habit persistence but they also produce a higher risk free rate which increases then to 8.5 percent. We are thus successful to increase the equity premium and Sharpe ratio in our model, but the risk free rate moves in the wrong direction, namely it rises too.[171]

Overall, one is inclined to conclude that previous studies because of the specific parameter choice have not satisfactorily solved the dynamics of asset prices and the equity premium puzzle. As can be observed from table 15.1 our results show that even if habit formation is jointly used with adjustment costs of investment there are still puzzles remaining for the consumption-based asset pricing models. Finally, we want to note that in our study we have chosen a model variant with no endogenous labor supply, which, as Lettau and Uhlig (2000) show, is the most favorable model for asset pricing in a production economy. This is because since including labor supply as a choice variable, would even reduce the equity premium and the Sharpe ratio.

## 15.3 Moving Beyond Consumption Based Asset Pricing Models

Next we want to study asset pricing models that move further beyond the consumption based asset pricing approach. Psychologists and experimental economists have found that in experimental settings, people make choices that differ, in several respects, from the standard model of expected utility. In response to these findings unorthodox psychological models with new preferences have been suggested, and some recent research has begun to apply these models to asset pricing.[172]

The psychological approach can be contrasted to the standard time-separable utility function. In the latter case an investor maximizes

$$U_t = E_t \left[ \sum_{j=0}^{\infty} \beta^t U(C_{t+j}) \right]$$

A well-known psychological model of decision-making is, as above noted, based on the prospect theory of Kahneman and Tversky (1979) and Tversky and Kahneman

[170] See Cochrane (2001, Chap. 21) and Campbell, Lo and MacKinley (1997, Chap. 8.2).

[171] Yet, we want to note that a high risk free interest rate can be reduced again by a higher $\beta$, see Cochrane (2001, Chap. 21) and Campbell, Lo and MacKinley (1997, Chap. 8.2.). Hornstein and Uhlig (2000, p. 58) point out that the risk free rate also declines with higher adjustment costs. Boldrin et al. and Jerman obtain a low risk free rate because they use a very high $\beta$ and a very high parameter, $\varphi$, in the adjustment cost function.

[172] An important reference is Kreps (1988).

(1992).[173] This theory was originally formulated in a static context, and not for a dynamic decision problem with discounting. The basic idea is as follows. Given that $X$ represents a loss or gain then we might have

$$v(X) = \begin{cases} \frac{X^{1-\gamma_1}-1}{1-\gamma_1} & \text{if } X \geq 0 \\ \lambda \frac{X^{1-\gamma_2}-1}{1-\gamma_2} & \text{if } X < 0 \end{cases}$$

In general here $\gamma_1$ and $\gamma_2$ are curvature parameters for gains and losses, and $\lambda > 1$ measures the loss aversion. Hereby a greater weight is given to losses than to gains.

Another important development is that experimental evidence suggests not geometric discounting but hyperbolic discounting: The discount factor for horizon $K$ is not $\delta^K$. It is rather a function of the form $(1+\delta_1 K)^{-\delta_2/\delta_1}$, where both $\delta_1$ and $\delta_2$ are positive. Thus a lower discount rate is used for future periods. Laibson's work (1996) suggests that hyperbolic discounting can be approximated by a utility specification such as

$$U(C_t) + \beta E_t \left[ \sum_{j=1}^{\infty} \beta^t U(C_{t+j}) \right]$$

Here the additional parameter $\beta < 1$ implies greater discounting over the next period than for periods further in the future.

In particular, the loss aversion theory has been applied to asset pricing. As discussed above, the basic problem in matching the asset market features to data using a consumption based model, is that, empirically, there is a lack of correlation of consumption growth and asset returns. Thus, consumption based asset pricing models have not been successful in capturing the historical average return and volatility in stock returns. Since even a power utility function with a large coefficient of relative risk aversion fails to match the consumption based asset pricing model to the data, researchers have used more sophisticated utility functions. One might think to improve on the equity premium and Sharpe ratio puzzles by building models that increase consumption volatility through increasing the parameter of risk aversion as in power utility models or time varying risk aversion as in habit formation models. Yet since, empirically, the correlation of consumption growth with asset returns is low, this might be a misleading approach towards improving the equity premium and Sharpe ratio.

Current research on loss aversion moves away from consumption based models. The new strategy is to look for the impact of the fluctuation of wealth on the households' welfare, so that the stochastic decision on a portfolio is impacted by both preferences over a consumption stream as well as by changes in financial wealth. In the preferences there will be thus an extra term representing the change of wealth. Furthermore, as prospect theory has taught us, an investor may be much more sensitive to losses than to gains, known as loss aversion, this, in particular, seems to hold if there have been prior losses already. By extending the asset pricing model in this direction one does not need to raise the variance of consumption growth and increase

[173] For further details, see Campbell, Lo and MacKinlay (1997, Chap. 8).

the correlation of consumption growth with asset returns, a feature not to be found in the data anyway.[174]

A low variance of consumption growth but a high mean and volatility of asset returns, with a low correlation with consumption growth, might be achieved by a time varying risk aversion arising from the fluctuation of wealth. The idea is that after an asset price boom the agents may become less risk averse because the gains may dominate any fear of losses. On the other hand, after an asset price fall the agent become more cautions and more risk averse. This way the variation of risk aversion would allow the asset returns to be more volatile than the underlying pay offs, the dividend payments, a property that Shiller (1991) has studied extensively. Generous dividend payments and an asset price boom makes the investor less risk averse and drives the asset price still higher. The reverse can be predicted to happen if large losses occur. This may give rise to some waves of optimism and pessimism. Whereas habit formation models attempt to increase the equity premium and Sharpe ratio by constructing a varying risk aversion. This occurs as current consumption moves closer to (or further away) from an (external) habit level for consumption. Risk aversion, in models with loss aversion, is varying not through surplus consumption as in the habit formation model, but rather through the fluctuation in financial wealth. Hereby, the risk aversion is affected by prior investment experiences. This is likely to produce a substantial equity premium and Sharpe ratio, high volatility of returns, yet lower the variance of the growth rate of consumption, actually found in the data.

Whereas the risk aversion in the habit formation model is finally driven by consumption, this is not so in the loss aversion model, where the changes of risk aversion are driven by changes in the value of assets. In the consumption based asset pricing model assets are only risky because they co-vary with consumption, see equ. (15.4). In the loss aversion model changes of risk aversion arise from the fluctuation of asset prices regardless of whether those fluctuations are correlated with consumption growth or not.

Yet, the above is the most interesting feature of the loss aversion model; the feedback effect of asset value – and changes of wealth – on preferences, on the one hand, and the choice of consumption path on asset value, on the other, creates a complicated stochastic dynamic optimization problem.

As aforementioned the idea of loss aversion has been developed in the so-called prospect theory which goes back to Kahneman and Tversky (1979). It has been further developed for applications in asset pricing by Benartzi and Thaler (1995), although there it is in the context of a single period portfolio decision model. Yet, without the asymmetry in gains and losses, with prior losses playing an important role, the risk aversion will be constant over time and the theory cannot contribute to the explanation of the equity premium and Sharpe ratio.

[174] See Chap. 9.

## 15.4 The Asset Pricing Model with Loss Aversion

In order to formalize the newest idea on asset pricing we may follow Barberis et al. (2001) and Grüne and Semmler (2005b) who specify the following preference

$$E_t\left[\sum_{t=0}^{\infty}\left(\beta^t\frac{C_t^{1-\gamma}}{1-\gamma}+b_t\beta^{t+1}\nu(X_{t+1},S_t,z_t)\right)\right] \tag{15.6}$$

The first term in equ. (15.6) represents, as usual, the utility over consumption, using power utility, $\beta$ is the discount factor and $\gamma$, the parameter of relative risk aversion. The second term captures the effect of the change of wealth on the agent's welfare. Here $X_{t+1}$ is the change of wealth, $S_t$, the value of the agent's risky assets. Finally, we want to note that $z_t$ is a variable, measuríng the agent's gains or losses prior to period $t$ attraction of $S_t$. The variables $S_t$ and $z_t$ express the way the agent experienced gains or losses in the past thus affecting his or her willingness to take risks.

In particular, it is presumed that

$$X_{t+1}=S_tR_t-S_tR_f \tag{15.7}$$

This means that the gain or loss $S_tR_t$, with $R_t$ the risky return, $R_f$ the risk free return, is measured relative to a return $S_tR_f$ from a risk-free asset. The variable $z_t$ can be greater, equal or smaller than one, with

$$\nu(X_{t+1},S_t,1)=\begin{cases}X_{t+1} & \text{for} \quad X_{t+1}\geq 0\\ \lambda X_{t+1} & \text{for} \quad X_{t+1}<0\end{cases} \tag{15.8}$$

and $\lambda>1$ as defined by

$$\lambda(z_t)=\lambda+k(z_t-1) \tag{15.9}$$

expressing the fact that a loss is more severe than a gain with $k>0$, and

$$z_{t+1}=z_t\frac{\overline{R}}{R_{t+1}}+(1-\mu)$$

with $0<\mu<1$ and $\overline{R}$ a fixed parameter which is chosen to be the long term average of the risk free interest rate. Moreover, let us presume a model that works with some aggregate consumption $\tilde{C}$. Then we can write

$$b_t=b_0\tilde{C}_t^{-\gamma} \tag{15.10}$$

with $\tilde{C}_t^{-\gamma}$, a scaling factor, and $\tilde{C}_t$ some aggregate consumption. This way, as Barberis et al (201) show the price-dividend ratio and the risky asset premium remain stationary. Here $b_0$ is an important parameter indicating the relevance that financial wealth has in utility gains or losses relative to consumption. In case $b_0=0$, we recover the consumption based asset pricing model with power utility.

Barberis et al. (2001) apply the above model of loss aversion and asset pricing to two stochastic variants of an endowment economy without production. In the

first model variant there is only one stochastic pay-off for the asset holder, a stochastic dividend, and whereby dividend pay-offs are always equal to consumption. In their other model variants dividends and consumption follow different stochastic processes.

From the agent's Euler equation for optimality of the equilibrium Barberis et al. (2001) obtain a risk free rate

$$1 = \rho R_f E_t \left[ (\tilde{C}_{t+1}/\tilde{C}_t)^{-\gamma} \right] \tag{15.11}$$

and a stochastic discount factor

$$1 = \rho E_t / R_{t+1} (\tilde{C}_{t+1}/\tilde{C}_t)^{-\gamma} + b_0 \rho E_t \left[ \tilde{\nu}(R_{t+1}, z_t) \right] \tag{15.12}$$

As compared to the usual stochastic discount factor, the equ. (15.12) has two terms. The first term represents the usual one obtained from consumption based asset pricing. The second term expresses the fact that if the agent consumes less today and invests in risky assets the agent is exposed to the risk of greater losses, a risk that is represented by the state variable $z_t$. The term $\tilde{\nu}(R_{t+1}, z_t)$ is given by:

$$\hat{\nu}(R_{t+1}, z_t) = \begin{cases} R_{t+1} - R_{f,t}, & R_{t+1} \geq z_t R_{f,t} \text{ and } z_t \leq 1 \\ (z_t - 1) R_{f,t} + \lambda (R_{t+1} - z_t R_{f,t}), & R_{t+1} < z_t R_{f,t} \text{ and } z_t \leq 1 \\ R_{t+1} - R_{f,t}, & R_{t+1} \geq R_{f,t} \text{ and } z_t > 1 \\ \lambda(z_t)(R_{t+1} - R_{f,t}), & R_{t+1} < R_{f,t} \text{ and } z_t > 1 \end{cases} \tag{15.13}$$

From (15.11) we obtain the stochastic discount factor for the risk-free rate, $R_f$

$$m_{f,t+1} = (\tilde{C}_{t+1}/\tilde{C}_t)^{-\gamma},$$

which coincides with the stochastic discount factor for the consumption based model, see Cochrane (2001, sect. 1.2).

As compared to (15.11), the stochastic discount factor for risky assets, equ. (15.12) has two terms. The first term represents the usual one, obtained from consumption based asset pricing. The second term expresses the fact that if the agent consumes less today and invests in risky assets the agent is exposed to the risk of greater losses, a risk that is represented by the state variable $z_t$. If we consider the cases in (15.13) seperately, one sees that for each single case the right hand side of (15.13) is affinely linear in $R_{t+1}$.

Using the equation

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

for the risky return, with $P_t$ denoting the asset price and $D_t$ the dividend, which one can choose equal to $\tilde{C}_t$, Grüne and Semmler (2005b) show that the following discount factor $m_t$ for risky assets arises.

$$P_t = E_t\left[\underbrace{\frac{\rho\left(\tilde{C}_{t+1}/\tilde{C}_t\right)^{-\gamma} + \rho b_0 \alpha_1}{1 + \rho b_0 E_t(\alpha_2) R_{f,t}}}_{=m_{t+1}} (\tilde{C}_{t+1} + P_{t+1})\right] \tag{15.14}$$

with $\alpha_1$ and $\alpha_2$ given by

$$\begin{array}{llll} \alpha_1 = 1, & \alpha_2 = 1 & \text{for } R_{t+1} \geq z_t R_{f,t} & \text{and } z_t \leq 1 \\ \alpha_1 = \lambda, & \alpha_2 = (\lambda - 1) z_t + 1 & \text{for } R_{t+1} < z_t R_{f,t} & \text{and } z_t \leq 1 \\ \alpha_1 = 1, & \alpha_2 = 1 & \text{for } R_{t+1} \geq R_{f,t} & \text{and } z_t > 1 \\ \alpha_1 = \lambda(z_t), & \alpha_2 = \lambda(z_t) & \text{for } R_{t+1} < R_{f,t} & \text{and } z_t > 1 \end{array} \tag{15.15}$$

Note, again, that for $b_0 = 0$ this equation coincides with the stochastic discount factor for the consumption based model, see Cochrane (2001, sect. 1.2). Note, however, that for $b_0 \neq 0$ in contrast to the consumption based case the stochastic discount factor depends on $R_{t+1}$, which in turn depends on $P_{t+1}$, thus the right hand side of (15.14) becomes nonlinear and even discontinuous in $P_{t+1}$.

In order to generate the consumption $\tilde{C}_t$, one can use the aforementioned basic growth model by Brock and Mirman (1972). This amounts to choosing $\tilde{C}_t$ to be the optimal control of the problem

$$\max_{\tilde{C}_t} E\left(\sum_{t=0}^{\infty} \rho^t \frac{\tilde{C}_t^{1-\gamma}}{1-\gamma}\right) \tag{15.16}$$

subject to the dynamics

$$k_{t+1} = y_t A k_t^{\alpha} - \tilde{C}_t \tag{15.17}$$

$$\ln y_{t+1} = \sigma \ln y_t + \varepsilon_t \tag{15.18}$$

with $\varepsilon_t$ being i.i.d. random variables. Here $\gamma$ is the same as in (15.6) and as there we replace the utility function by log–utility $\ln \tilde{C}_t$ for $\gamma = 1$. In this case, i.e. for log–utility, the optimal consumption policy is known and is given by

$$\tilde{C}(k_t, y_t) = (1 - \alpha\rho) A y_t k_t^{\alpha}.$$

For $\gamma \neq 1$ we compute $\tilde{C}_t$ numerically.

For this model we want to compute a number of financial measures: The risk free interest rate $R_{f,t}$, the equity return $R_{t+1}$, the stochastic discount factors $m_{t+1}$ and $m_{f,t+1}$, all of which are specified above. In addition we will compute the Sharpe Ratio given by[175]

$$SR = \left|\frac{E(R_{t+1}) - R_{f,t}}{\sigma(R_{t+1})}\right| = \frac{-R_{f,t}\mathrm{Cov}\left(m_{t+1}, R_{t+1}\right)}{\sigma(R_{t+1})}. \tag{15.19}$$

[175] See Cochrane (2001).

**Table 15.2.** Loss Aversion Models

| Barberis et al[a]) | Grüne and Semmler[b]) | US Data[c]) (1954-1990) |
|---|---|---|
| $\lambda = 2.25$<br>$b_0 = 2.0$<br>$k = 3.0$<br>$\gamma = 1.0$<br>$\rho = 0.98$ | $\lambda = 2.25(10)$<br>$b_0 = 1.0$<br>$k = 3.0$<br>$\gamma = 1.0$<br>$\rho = 0.95(0.98)$ | |
| $R_f = 3.79$ | $R_f = 5.3(2.1)$ | $R_f = 0.8$ |
| $E(R) - R_f$ | $E(R) - R_f$ | $E(R) - R_f = 6.18$ |
| $SR = 0.17$ | $SR = 0.176(0.47)$ | $SR = 0.35$ |
| – | $Cov(m_f, R) = -0.00007$ | $Cov(\Delta c, R)$=0.0027 |

a) Barberis et al. (2001) use a loss aversion variant with exogenous dividends (equal consumption).
b) Note that Grüne and Semmler (2005b) use a model for a production economy with endogenous consumption.
c) Data sources, see table 7, $Cov(\Delta c, R)$ is from Campbell (1999).

As above discussed, concerning habit formation models one is inclined to state that previous studies on consumption based asset pricing have not satisfactorily solved the dynamics of asset prices and the equity premium puzzle. There are still puzzles remaining for the consumption-based asset pricing model. At the heart of the consumption based asset pricing model is the co-variance of consumption growth with asset return, which needs to be improved to get a higher equity premium and Sharpe ratio. Yet as the empirical data show, see table 15.2, column 3, this co-variance is very low.

On the other hand the models recently developed in behavioral finance, using loss aversion, do not have to match consumption growth data with asset returns. Indeed, as table 15.2 shows, see column 3, the co-variance of consumption growth with asset returns is empirically very low and thus the (negative) co-variance with the growth rate of marginal utility would be low too. Consumption based models attempt to improve this co-variance by employing other preferences (such as power utility with a very large parameter of relative risk aversion, habit formation and recursive utility) but the co-variance does not need to be improved in the loss aversion version of an asset pricing model.

In fact, as Grüne and Semmler (2005b) show, in the loss aversion model one has $Cov(m_f, R) = -0.00007$ which is very small. As can be seen from table 15.2 the proposed loss aversion model, where gains and losses of wealth also appear in the preferences, produce a time varying risk aversion, a low risk free rate (with low volatility), a high equity premium (with high volatility) and a reasonably high Sharpe ratio. For $b_0 = 1$ and $\lambda = 2.25(10)$ one obtains a Sharpe ratio of 0.176 (0.47). Of course, a higher $b_0$ makes also the Sharpe ratio rising. Moreover, for a discount factor of $\rho = 0.98$ one obtains a risk-free interest rate of 2.1 percent.

If one presumes that the latter is roughly half of the annual risk-free rate then one can also convert the Sharpe ratio for the same time period. One can use a conversion formula developed by Lo (2002)[176] with $SR(q) = \sqrt{q}SR$ with $q$ the period return. Then one has approximately an annual Sharpe ratio of 0.21 even for the parameters $\lambda = 2.25,\ b_0 = 1$ and $\gamma = 1$. Finally we want to note that the habit formation model in Grüne and Semmler (2004b), see also the above table 15.1, column 3, is the same, in terms of its basic structures and parameters, as the loss aversion model reported in table 15.2, column 2. Overall, one can therefore be quite confident that the loss aversion model produces quantitatively important contributions to the equity premium and Sharpe ratio puzzles. Moreover, the risk-free interest rate of the model moves also more in line with the actual data.

## 15.5 Conclusions

The recently developed asset pricing models with habit formation and loss aversion seem to go a long way to explain the risk-free interest rate, equity premium and Sharpe ratio in a more plausible way than the earlier consumption based asset pricing models. In particular, the asset pricing model with loss aversion has great potentials not only to match the dynamics of equity prices but other markets with volatile price movements and risky returns as well.[177] This new approach moves beyond the consumption based asset pricing model and allows to de-link consumption and asset returns. It also nicely explains the time varying risk aversion by referring to the actual gains and losses of financial wealth. This view of gains and losses giving rise to a time varying risk aversion, is not only relevant for the individual investor but in particular seems to be very important for institutional investors such as pension funds (that had guaranteed a certain return), universities (that have large operating costs) and foundations (that grant fellowships). For those institutions painful adjustment processes have to be enacted, once losses have occurred and thus a time varying risk aversion can easily predicted.

[176] This is developed for IID returns.

[177] For an application to exchange rates and foreign currency reserves, see Aizenman and Morion (2003).

CHAPTER 16

# Dynamic Portfolio Choice Models

## 16.1 Introduction

In this chapter we want to study portfolio strategies that both allow for dynamical adjustment of the asset allocation as new investment opportunities come up as well as to permit changes in consumption over time so as to maximize some welfare over a longer time horizon. This type of dynamic portfolio choice models which goes back to Merton (1973, 1990) has, in recent times, been suggested and studied by Campbell and Viceira (2002).[178] As in their study, in the subsequent models we also revert back to traditional preferences. The portfolio models presented here are based on log utility and power utility. Dynamic portfolio choice models are complex enough so that the models studied in this chapter are based on simple preferences. We also consider the case when wealth instead of consumption is in the preferences.

## 16.2 Wealth Accumulation and Portfolio Decisions

As previous chapters have already shown, instead of the mean-variance preference of the investor one frequently assumes a preference of the investor that defines utility over wealth. A similar static portfolio strategy, as discussed in Chaps. 8 and 13, depending on some parameter of risk aversion, will arise. For further details of the subsequent summary of the static approach and the issues involved, see Campbell and Viceira (2002, Chap. 2). We are using here their exposition as an introduction to dynamic portfolio choice. We want to study to what extent and under what conditions the results of the static portfolio theory carry over to dynamic portfolio choice.

Consider again first the static portfolio choice problem for an investor facing two assets. One is a riskless, asset with return, and the other is a risky one. The risky asset generates a return $R_{t+1}$ with conditional mean $E_t(R_{t+1})$ and variance $\sigma^2$. If the investor chooses a share $\alpha_t$ of his or her portfolio as the risky asset, then the portfolio return in the notation of Campbell and Viceira (2002, Chap. 2) can be expressed as

$$R_{p,t+1} = \alpha_t R_{t+1} + (1-\alpha_t)R_{f,t+1} = R_{f,t+1} + \alpha_t(R_{t+1} - R_{f,t+1}).$$

The expected mean portfolio return is

$$E_t R_{p,t+1} = R_{f,t+1} + \alpha_t(E_t R_{t+1} - R_{f,t+1}). \tag{16.1}$$

[178] For an excellent recent survey on static and dynamic portfolio choice problems, see Brandt (2004).

The variance of the portfolio return could be time varying, defined by $\sigma_{pt}^2 = \alpha_t^2 \sigma_t^2$.

The investor aims at maximizing a combination of mean and variance, such as

$$\max_{\alpha_t} \left( E_t R_{p,t+1} - \frac{k}{2} \sigma_{pt}^2 \right). \tag{16.2}$$

Using in (16.1) in (16.2) and leaving aside $R_{f,t+1}$ in

$$\max_{\alpha_t} \left[ \alpha_t (E_t R_{t+1} - R_{f,t+1}) - \frac{k}{2} \alpha_t^2 \sigma_t^2 \right] \tag{16.3}$$

then the solution is

$$\alpha_t = \frac{E_t R_{t+1} - R_{f,t+1}}{k \sigma_t^2}. \tag{16.4}$$

The coefficient $k$ represents aversion to variance. The Sharpe ratio for the model is

$$SR_t = \frac{E_t R_{t+1} - R_{f,t+1}}{\sigma_t}. \tag{16.5}$$

We then can write

$$\alpha_t = \frac{SR_t}{k \sigma_t}. \tag{16.6}$$

According to the static portfolio theory, all portfolios will have the same Sharpe ratio since they represent the same risky assets.

For several risky assets we may follow the notation of Campbell and Viceira (2002, Chap. 2). With $R_{t+1}$, a vector of risky returns with $N$ elements, vector of means $E_t(R_{t+1})$, a variance-covariance matrix $\Sigma_t$, and $\alpha_t$ a vector of proportions of risky assets, the maximization problem (16.3) is

$$\max_{\alpha_t} \left[ \alpha_t' (E_t R_{t+1} - R_{f,t+1} \mathbf{1}) - \frac{k}{2} \alpha_t' \Sigma_t \alpha_t \right]. \tag{16.7}$$

Hereby $\mathbf{1}$ is a vector of ones and $(E_t R_{t+1} - R_{f,t+1} \mathbf{1}$ the vector of excess returns. The variance of the portfolio return is given by $\alpha_t' \Sigma_t \alpha_t$.

The solution is

$$\alpha_t = \frac{1}{k} \Sigma_t^{-1} (E_t R_{t+1} - R_{f,t+1} \mathbf{1}). \tag{16.8}$$

with $\Sigma_t^{-1}$, the inverse of the variance-covariance matrix of returns.

Here again, investors differ only in their fractions of riskless and risky assets, not in their optimal choice of the risky assets. Conservative investors will choose more of the riskless asset, yet they do not change the relative proportions of their risky assets. Here again this represents the mutual fund theorem of Tobin and Samuelson.

Overall, we thus can point out that in the mean-variance approach: First, investors differ only with respect to cash and risky assets. Second, investors care only about mean and variance. Therefore, everybody will hold the same portfolio of risky assets (personal risk characteristics are not considered). Third, investors with different investment horizon (short and long) are disregarded. In the subsequent dynamic

models we will overcome those limitations. Those problems will be addressed in Chaps. 16.3-16.5. Yet, in Chap. 16.3 we will still get the same solution as in a static model.

Pursuing further the current myopic approach we want to note that frequently in the literature preferences over wealth are used instead of the above mean-variance formulation. With a utility function over wealth, see Campbell and Viceira (2002, Chap. 2), we can define the following

$$\max E_t U(W_{t+1}) \tag{16.9}$$

subject to

$$W_{t+1} = (1 + R_{p,t+1})W_t. \tag{16.10}$$

with $U(W_{t+1})$ a concave utility function.

As in utility over consumption, see Chap. 9, the steepness of the utility function is given by the magnitude of the investor's risk aversion. The curvature measured by the coefficient of absolute risk aversion is

$$ARA = -\frac{U''(W)}{U'(W)''}. \tag{16.11}$$

The coefficient of relative risk aversion is obtained by

$$RRA = -\frac{WU''(W)}{U'(W)}. \tag{16.12}$$

The inverse of these measures is called absolute and relative risk tolerance. Campbell and Viceira (2002, Chap. 2) consider three forms of simple utility functions over wealth

1. Investors exhibit quadratic utility over wealth. Here we have $U(W_{t+1}) = aW_{t+1} - bW_{t+1}^2$. The assumption of quadratic utility entails that absolute risk aversion and relative risk aversion are increasing in wealth.
2. Investors exhibit exponential utility, $U(W_{t+1}) = -exp(-\theta W_{t+1})$ and asset returns are normally distributed. Exponential utility entails that absolute risk aversion is represented by a constant, $\theta$, and relative risk aversion increases when wealth increases.
3. Investors exhibit power utility, $U(W_{t+1}) = (W_{t+1}^{1-\gamma} - 1)/(1 - \gamma)$, and asset returns are log-normally distributed. In power utility absolute risk aversion is decreasing in wealth, and relative risk aversion is constant, $\gamma$. Here again, as in preferences over consumption, if the limit $\gamma$ is approached, one obtains log utility $U(W_{t+1}) = log(W_{t+1})$.

Campbell and Viceira (2002, Chap. 2) show that the exponential and power utility imply distributional assumptions on returns. Exponential utility produces simple results if asset returns are normally distributed. Power utility produces simple results if asset returns are log-normal.

## 16.3 Discrete Time Dynamic Portfolio Choice under Log-Normality

Next, again following Campbell and Viceira (2002, Chap. 2), one can show that a dynamic model with longer horizon can, under certain assumptions, lead to the same result as the myopic static model. To derive this Campbell and Viceira (2002, Chap. 2) use a key result about the expectation of a log-normal random variable $X$ which is:

$$logE_tX_{t+1} = E_t logX_{t+1} + \frac{1}{2}Var_t logX_{t+1} = E_t x_{t+1} + \frac{1}{2}\sigma^2_{xt}. \tag{16.13}$$

Let us consider the solution to the myopic model of equs. (16.9)-(16.10). For power utility over wealth one can write equ. (16.9)

$$\max \frac{E_t W^{1-\gamma}_{t+1}}{(1-\gamma)} \tag{16.14}$$

Presuming that the next-period wealth is log-normal, one can then apply (16.13) to rewrite the objective function as

$$\max logE_t W^{1-\gamma}_{t+1} = (1-\gamma)E_t w_{t+1} + \frac{1}{2}(1-\gamma)^2\sigma^2_{wt}. \tag{16.15}$$

Writing the budget constraint (16.10) in log-form we have

$$w_{t+1} = r_{p,t+1} + w_t, \tag{16.16}$$

with $r_{p,t+1} = log(1 + R_{p,t})$. Restating equ. (16.15) by dividing it by $(1-\gamma)$ and employing (16.16) one obtains

$$\max E_t r_{p,t+1} + \frac{1}{2}(1-\gamma)\sigma^2_{pt}, \tag{16.17}$$

Since the portfolio return is presumed to be log-normal, we have

$$E_t r_{p,t+1} + \sigma^2_{pt}/2 = logE_t(1 + R_{p,t+1}). \tag{16.18}$$

Therefore, (16.17) can be rewritten as

$$\max logE_t(1 + R_{p,t+1}) - \frac{\gamma}{2}\sigma^2_{pt}. \tag{16.19}$$

The result is similar to that obtained by the mean-variance analysis of Chap. 16.2 of the risky and risk-free assets. Here, $\gamma$ plays the role of $k$. If $\gamma = 1$, the investor has log utility and selects a portfolio with the highest available log return. If $\gamma > 1$, than the investor seeks a safer portfolio. If $\gamma < 1$, then the investor seeks a riskier portfolio. The case $\gamma = 1$ is the boundary case.

Campbell and Viceira (2002, Chap. 2) state the following problem concerning the long-term portfolio choice. They correctly mention that it is a fallacy to argue

that there is a single best long-term portfolio for all long-term investors and that their preferences do not matter. It is preferable for an investor with log utility. Investors with greater risk aversion should choose less risky portfolios.

In the literature it is frequently argued that investors are concerned not with the level of wealth, but with the standard of living that their wealth delivers. Standard theory argues that the investor derives utility from consumption rather than wealth. In portfolio models with long time horizon one can let time go to infinity and work with a simple infinite-horizon model. Moreover, one can vary the effective investment horizon by varying the discount factor. Also, under special conditions, to be stated below, one can obtain for the dynamic portfolio choice the same result as for the static portfolio decision.

In order to demonstrate those points, we define a dynamic portfolio decision problem in discrete time. As in Chap. 9, we first assume that investors have time-separable power utility, defined over consumption:

$$\max E_t \sum_{i=0}^{\infty} \beta^t U(C_{t+i}) = E_t \sum_{i=0}^{\infty} \beta^t \frac{C^{1-\gamma} - 1}{1-\gamma}. \tag{16.20}$$

Hereby $\beta$ is the discount factor. The evolution of wealth is defined by the intertemporal budget constraint that wealth next period equals the portfolio return times reinvested wealth (after consumption):

$$W_{t+1} = (1 + R_{p,t+1})(W_t - C_t). \tag{16.21}$$

Following Chap. 9 we can obtain the first-order condition, or Euler equation, for the optimal consumption choice:

$$U'(C_t) = E_t[\beta U'(C_{t+1})(1 + R_{i,t+1})], \tag{16.22}$$

where $(1 + R_{i,t+1})$ denotes any available return, for example the riskless return $(1 + R_{f,t+1})$, the risky return $(1 + R_{t+1})$, or the portfolio return $(1 + R_{p,t+1})$.

As in Chap. 9 one can write (16.22) by using power utility with

$$U'(C_t) = C_t^{-\gamma}. \tag{16.23}$$

Therefore we have

$$1 = E_t\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}((1 + R_{i,t+1})\Big] \tag{16.24}$$

When the return is riskless, $R_{i,t+1} = R_{f,t+1}$, it can be brought outside the expectations operator; we thus have

$$\frac{1}{(1 + R_{f,t+1})} = E_t\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\gamma}\Big] \tag{16.25}$$

The term

$$\beta(C_{t+1}/C_t)^{-\gamma} \tag{16.26}$$

is the stochastic discount factor (SDF) as in Chap. 9. It can be used to discount expected pay-offs on any assets to find the asset prices. We can write the SDF as $m_{t+1}$, whereby (16.24) and (16.25) become

$$1 = E_t[m_{t+1}(1 + R_{i,t+1})] \tag{16.27}$$

and

$$\frac{1}{(1 + R_{f,t+1})} = E_t[m_{t+1}] \tag{16.28}$$

With power utility we have $m_{t+1} = \beta(C_{t+1}/C_t)^{-\gamma}$.

Next we turn to a discrete time dynamic portfolio choice under log-normality. Campbell and Viceira (2002, Chap. 2) show that the log form of the Euler equation (16.24) of the riskless-rate can be written as

$$E_t[\Delta c_{t+1}] = \frac{log\beta}{\gamma} + \frac{1}{\gamma} r_{f,t+1} + \frac{\gamma}{2}\sigma_{ct}^2 \tag{16.29}$$

where $\Delta$ is the first-difference operator and $c_{t+1} \equiv log(C_{t+1})$. Therefore $\Delta c_{t+1}$ is consumption growth. The log form of the general Euler equation (16.24), can be used to obtain

$$E_t r_{t+1} - r_{f,t+1} + \frac{\sigma_t^2}{2} = \gamma cov_t(r_{t+1}, \Delta c_{t+1}), \tag{16.30}$$

It is therefore clear that it is the size of $\gamma$ and the $cov(r_{t+1}, \Delta c_{t+1})$ that predominantly determine the equity premium.

In general, as Campbell and Viceira point out, a difficulty with the log-normal consumption-based model is that the budget constraint (16.21) is not generally log-linear. This is because consumption is subtracted from wealth before being multiplied by the portfolio return creating a complicated nonlinearity. Yet, presuming a constant consumption-wealth ratio one can write[179]

$$\frac{C_t}{W_t} = b, \tag{16.31}$$

The constraint (16.21) can then be written in log-form as

$$\begin{aligned} \Delta w_{t+1} &= r_{p,t+1} + log(1 - b) \\ &= r_{f,t+1} + \alpha_t(r_{t+1} - r_{f,t+1}) + \frac{1}{2}\alpha_t(1 - \alpha_t)\sigma_t^2 \\ &\quad + log(1 - b), \end{aligned} \tag{16.32}$$

The second equality is derived in Campbell and Viceira (2002, Chap. 2). Equ. (16.31) presumes that the growth rate of consumption is equal to the growth rate of wealth.

[179] Note that in general the subsequent assumption may hold only if one is close enough to some steady state that is constant in the long run. A constant consumption to wealth ratio is directly obtained for log-utility and linear state equation, see Chap. 14.2, and Cochrane (2001).

The terms referring to consumption in (16.29) and (16.30) can, therefore, be rewritten in terms of wealth. Then we have

$$E_t r_{t+1} - r_{f,t+1} + \sigma_t^2/2 = \gamma Cov_t(r_{t+1}, \Delta w_{t+1}) = \gamma \alpha_t \sigma_t^2, \tag{16.33}$$

where the second equality results from (16.32). Solving this equation for $\alpha_t$, we again obtain a static solution for our portfolio problem:

$$\alpha_t = \frac{E_t r_{t+1} - r_{f,t+1} + \sigma_t^2/2}{\gamma \sigma_t^2} \tag{16.34}$$

Campbell and Viceira (2002, Chap. 2) then show that the static solution for multiple risky assets[180] holds for a long-term investor with a constant consumption-wealth ratio. Overall, this is a convenient simplification since in this case one can by-pass the complicated intertemporal portfolio decision problem using advanced analytical or numerical methods. In the next section we turn to the use of those methods, although for a simple case, namely for continuous time models, and we also study under what conditions the static portfolio choice can be recovered.

## 16.4 Continuous Time Deterministic Dynamic Portfolio Choice

We next study the dynamic choice problem, in continuous time, where we have at most two assets and returns to those assets. We here employ first a deterministic framework and, thereafter, we introduce a stochastic setting. The objective of the investor will here be to maximize his or her welfare given by a power utility function over consumption. When we deal with the case of two assets they will be presumed to be a bond and equity or alternatively a short bond and a long bond. Our first example, however, refers to one asset only.

### 16.4.1 Constant Risk-Free Return – One Asset

Our first example[181] of a continuous time version of the dynamic asset allocation problem is a rather simple one. We study a choice problem that contains only one asset which generates a constant risk-free return. It could be thought of as a bond with a risk-free constant return. There is no choice between assets to be made, but only a choice of the optimal consumption path. We presume preferences over consumption of power utility type

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \tag{16.35}$$

[180] Note that the equ. (16.34) differs slightly from equ.(16.4), since in (16.32) there is involved a Taylor approximation to a nonlinear function, see Campbell and Viceira (2002, Chap. 2).

[181] This first example was worked out by C.Y. Hsiao. We want to thank her for her effort.

There is only one asset, $W$, with a risk-free constant return $r$.

We presume that the agent maximizes the intertemporal discounted utility

$$\max_{C} \int_0^{\infty} e^{-\beta t} U(C_t) dt. \tag{16.36}$$

The wealth dynamic is given by

$$\dot{W} = rW - C \tag{16.37}$$

Using a dynamic programming approach (DP) leads to the following formulation

$$J = \max_{C_t} \int_0^{\infty} e^{-\beta t} U(C_t) dt \tag{16.38}$$

here

$$W(0) = W_0$$

The problem is to find the path $C_t, t \geq 0$, such that the objective function (16.36) obtains its optimal value. $J$ is called the optimal value function, given the initial condition $W(0) = W_0$.

The Hamilton-Jacobi-Bellman (HJB) equation for the DP problem (16.38) is (see Kamien and Schwartz 1997: 260)

$$-J_t(t, W) = \max_{C} \{e^{-\beta t} U(C) + J_W(t, W)(rW - C)\} \tag{16.39}$$

The first order condition for (16.39) is

$$e^{-\beta t} U'(C) - J_W(t, W) = 0 \tag{16.40}$$

Then using (16.35) for $U$, we get

$$J_W(t, W) = e^{-\beta t} C^{-\gamma}$$

Thus,

$$C = (J_W e^{\beta t})^{-\frac{1}{\gamma}} \tag{16.41}$$

Replacing $C$ in (16.39) we obtain

$$\begin{aligned} 0 &= +J_t + e^{-\beta t} \frac{1}{1-\gamma} \left(J_W e^{\beta t}\right)^{-\frac{1}{\gamma}(1-\gamma)} + J_W (rW - \left(J_W e^{\beta t}\right)^{-\frac{1}{\gamma}} \\ &= J_t + J_W rW + \frac{1}{1-\gamma} e^{\beta t (1-\frac{1}{\gamma}+1)} J_W^{1-\frac{1}{\gamma}} - J_W^{1-\frac{1}{\gamma}} e^{-\frac{\beta t}{\gamma}} \\ &= J_t + J_W rW + \frac{\gamma}{1-\gamma} (e^{-\frac{\beta t}{\gamma}} J_W^{1-\frac{1}{\gamma}}) = 0 \end{aligned} \tag{16.42}$$

Our guess for the value function is

$$J(t, W) = R(t) e^{-\beta t} U(W) = R e^{-\beta t} \frac{W^{1-\gamma}}{1-\gamma}$$

Then we obtain

$$J_t = -\beta J \tag{16.43}$$

$$J_W = \frac{1-\gamma}{W} J \tag{16.44}$$

$$\begin{aligned} e^{-\frac{\beta t}{\gamma}} J_W^{\frac{\gamma-1}{\gamma}} &= e^{-\frac{\beta t}{\gamma}} \left( R e^{-\beta t} W^{-\gamma} \right)^{\frac{\gamma-1}{\gamma}} \\ &= R^{\frac{\gamma-1}{\gamma}} e^{-\beta t} W^{1-\gamma} = (1-\gamma) R^{-\frac{1}{\gamma}} J \end{aligned} \tag{16.45}$$

One can check whether the above solution is the solution of our DP problem, by inserting (16.43), (16.44), (16.46) in (16.42) we obtain

$$\begin{aligned} -\beta J + r(1-\gamma)J + \gamma R^{-\frac{1}{\gamma}} J = \\ J(\gamma R^{-\frac{1}{\gamma}} + r(1-\gamma) - \beta) = 0. \end{aligned} \tag{16.46}$$

If $R$ satisfies $(\cdot) = 0$ in (16.46) we have

$$R = \left( \frac{\beta}{\gamma} + \frac{r(\gamma-1)}{\gamma} \right)^{-\gamma}.$$

We get the solution for our DP problem

$$J(t, W) = \left( \frac{\beta}{\gamma} + \frac{r(\gamma-1)}{\gamma} \right)^{-\gamma} e^{-\beta t} \frac{W^{1-\gamma}}{1-\gamma}. \tag{16.47}$$

Using (16.41) to get the optimal control

$$C^* = (J_W e^{\beta t})^{-\frac{1}{\gamma}} = R^{-\frac{1}{\gamma}} W.$$

Thus,

$$\frac{C^*}{W} = \frac{\beta}{\gamma} + \frac{r(\gamma-1)}{\gamma}. \tag{16.48}$$

Our results show that first, for this example indeed it holds that the consumption over wealth ratio is constant for a constant $r$, second, the ratio increases in $\beta$ (less patience), third, the ratio increases in $r$ (return effect), and fourth, consumption propensity is affected by $\gamma$ (risk aversion).

A dynamic programming method, as sketched in appendix 3, and as developed by Grüne (1997) and Grüne and Semmler (2004a), can be used to compute the value function and the path of the control variable, $C$, the latter in feedback form from the state variable, $W$. We use $\beta = 0.05, \gamma = 0.5$ and $r = 0.03$.

Figure 16.1 shows the value function and figure 16.2 the path of the control variable, $C$, in feedback form from the state variable, $W$, both computed by our dynamic programming algorithm as presented in Appendix 3.

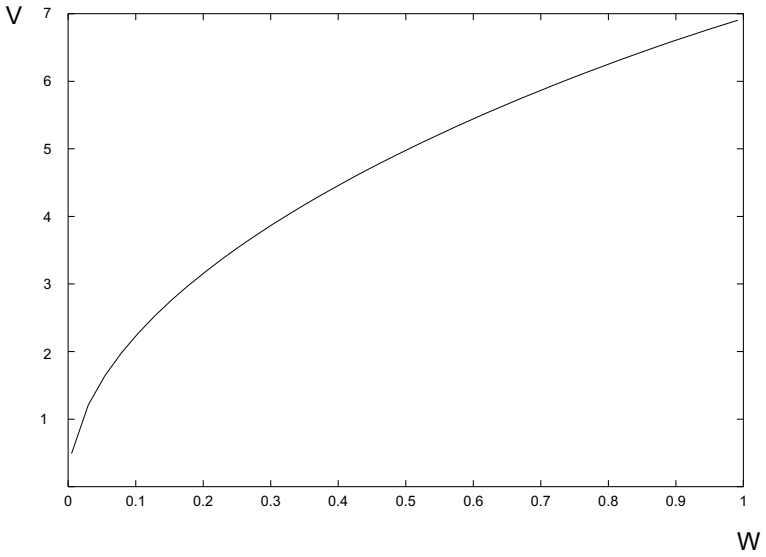



**Fig. 16.1.** Value Function of the Model

### 16.4.2 Time Varying Risk-Free Rate and Equity Return – Two Assets

The next model has two assets. Consumption, $C_t$, and asset allocation, $\alpha$, are the choice variables. Here, too, we first want to study a deterministic model of the following type

$$\max_{C,\alpha} E \int_0^\infty e^{-\beta t} U(C_t) dt \tag{16.49}$$

$$s.t.\ \dot{W} = [\alpha(R_{e,t}(W) - R_{f,t}(W)) + R_{f,t}(W)]W_t - C_t \tag{16.50}$$

We presume some stylized facts on the long run swings in returns of the risk-free asset and equity. In particular we postulate some positive and negative overshooting of the returns for risky assets as compared to the returns of the risk-free asset, see Chap. 5. For reasons of simplicity we presume that the two returns exhibit different amplitudes but also different expected means. In order to keep the number of state variables small we suggest the following wealth dependency of returns for the risk-free asset and equity[182]

$$R_{f,t}(W) = \beta_1 sin(\beta_3 W) \qquad \text{(time varying interest rate)}$$
$$R_{e,t}(W) = \beta_2 sin(\beta_4 W) + \beta_5 \qquad \text{(time varying equity return)}$$

[182] Note that the swings of the above returns might also be derived on the basis of models of interacting heterogeneous agents as in Chiarella and He(2001), LeBaron (2001, 2003), or as discussed in Chaps. 5 and 14.

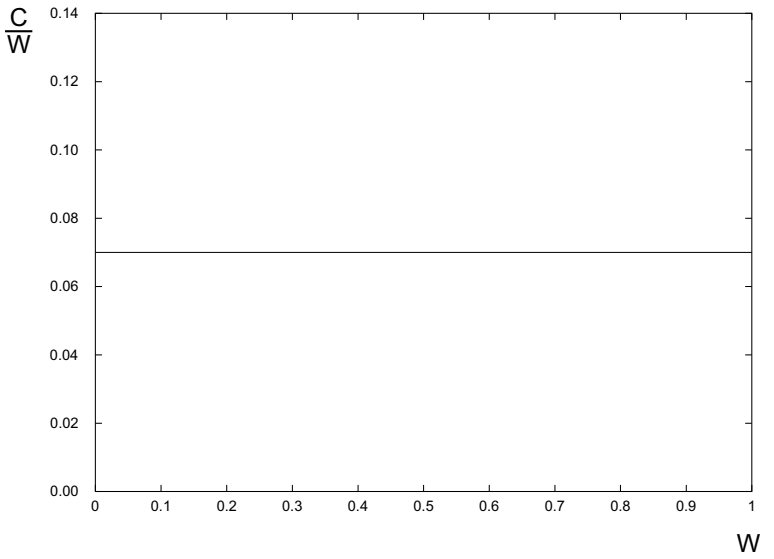



**Fig. 16.2.** Dynamic Solution Path for $C$ of the Model

Specifying our problem of a dynamic portfolio choice with consumption and asset allocation as choice variables by employing a power utility function we can write

$$\max_{C,\alpha} E \int_{t=0}^{\infty} e^{-\beta t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \tag{16.51}$$

$$s.t.\ \dot{W} = [\alpha_t(R_{e,t} - R_{f,t}) + R_{f,t}]W_t - C_t \tag{16.52}$$

with $R_{e,t}$ the equity premium and $R_{f,t}$ the risk-free interest rate, $W_t$ the total wealth, $C_t$ and $\alpha_t$ the choice variables consumption and the fraction of wealth allocated to equity. The model is again solved through a dynamic programming algorithm as sketched in appendix 3. This example shows that the ratio of consumption over wealth is not constant.

In the numerical study applying numerical DP we take $\beta_1 = 0.1, \beta_2 = 0.2, \beta_3 = 0.2, \beta_4 = 0.2, \beta_5 = 0.005$ and $\beta = 0.05, \gamma = 0.5$. Moreover, we use upper and lower bounds for our choice variables $-3 \leq \alpha_t \leq 4.5$ and $0 \leq C_t \leq 70$.

Figure 16.3 and 16.4 show the results of the numerical solution of the above dynamic portfolio decision model. Figure 16.3 shows the value function of the above model. Since the equity return and the risk-free rate, and thus the equity premium, shown by the difference of the solid and dashed line in figure 16.5 move cyclically, the value function also moves cyclically. Moreover, since there is an expected positive equity premium the wealth, $W_t$, growth over time. Note, however, that wealth is reduced each period through consumption.

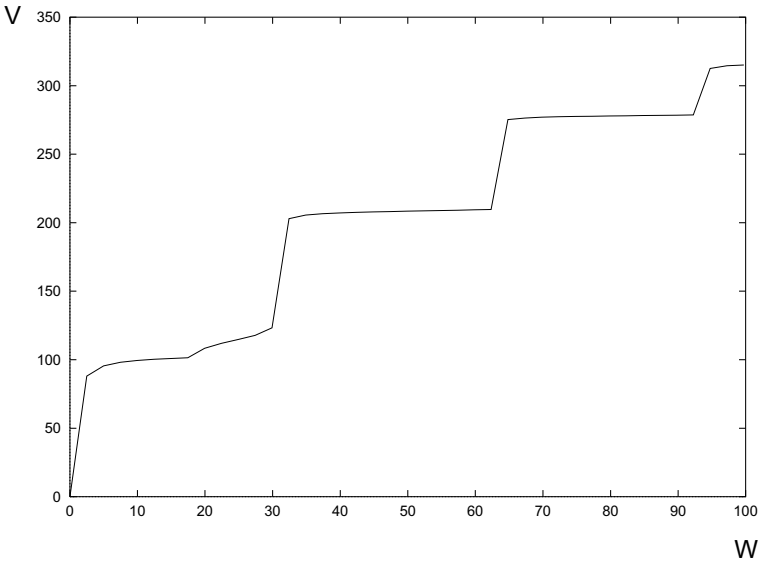



**Fig. 16.3.** Present Value Curve

Figure 16.4 shows the choice variables $C_t$, (dashed line), and $\alpha_t$, (solid line). As can be observed, because of the cyclical nature of the equity premium, consumption is cyclical as well, although on average it grows over time. The ratio of optimal consumption to wealth is not constant over time. The fraction of assets put back into equity also moves cyclically but is bounded by $-3 \leq \alpha_t \leq 4.5$. This can be observed if one compares figure 16.4 and figure 16.5. The large positive (negative) $\alpha_t$ in figure 16.4 appears when there is a positive (negative) excess return in figure 16.5.

Note that in our model, the asset holder is constrained to take a negative position on the risk-free rate not larger than $(1 - 4.5)$, and a negative position on the equity constrained by $\alpha = -3$. Those constraints may be considered reasonable from a practical point of view, where there are some constraints on the allocation choice, for details on such a constrained choice behavior, see Chiarella et al (2002). In practice there are trading rules, for example, margin requirements and adjustment costs, that place limits on short positions in the risk-free asset.

Our solution of the optimal portfolio and consumption choice problem shows that, given softer constraints, the fluctuations of the fraction of wealth allocated to equity would be much larger if positive or negative excess returns occur.

In fact, a similar large fluctuation of the asset allocation is observed in Campbell and Viceira (2002: 78). There, however, $\alpha$, the fraction allocated to equity is only solved as a comparative static solution contingent on certain risk parameters $\gamma$, ranging from $0.75 \leq \gamma \leq 5000$. In contrast to the solution by Campbell and Viceira our solution path of $\alpha_t$ represents a dynamic solution, contingent on the stock of wealth, $W$, for a fixed value of the parameter of relative risk aversion $\gamma$.

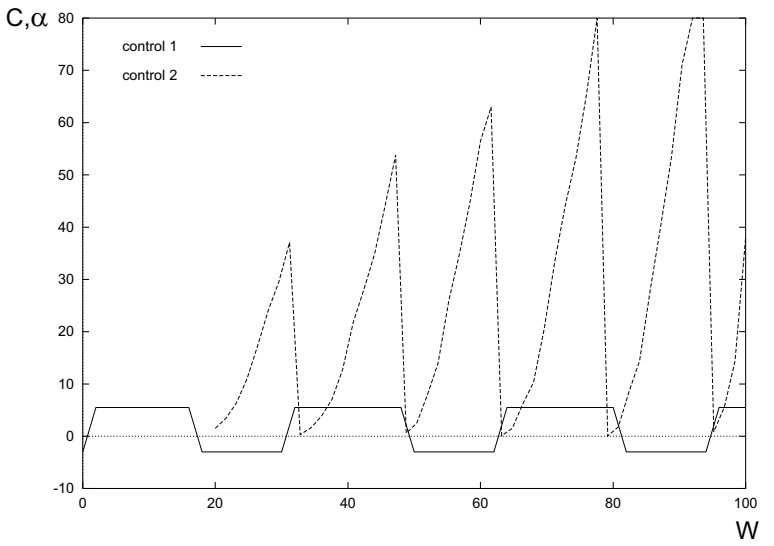



**Fig. 16.4.** Controls $C_t$ (Dashed) and $\alpha_t$ (solid)

## 16.5 Continuous Time Stochastic Dynamic Portfolio Choice

### 16.5.1 A Simple Model

Again, a simple form of a stochastic dynamic choice problem with one state and one control variable can be written following Kamien and Schwartz (2001, sect. 22). This is autonomous and has an infinite time horizon. The optimal expected return can then be expressed in current value terms independently of $t$. Let

$$V(x_0) = \max E \int_0^\infty e^{-\beta t} f(x,u)dt \tag{16.53}$$

$$\text{s.t. } dx = g(x,u)dt + \sigma(x,u)dz, \;\; x(t_0) = x_0, \tag{16.54}$$

This is a general stochastic decision problem with $x$ as state variable, $u$ as control variable and a Brownian motion equ. (16.54). From (16.53) into (16.54) one can obtain a Bellman equation in stochastic form

$$\beta V(x) = \max_u (f(x,u) + V'(x)g(x,u) + (1/2)\sigma^2(x,u)V''(x)) \tag{16.55}$$

Let us turn the above into a simple example, based on the work by Merton. The example has two controls and one state variable. It represents a problem of allocating wealth among current consumption, investment in a risk-free asset, and investment in a risky asset. We here too exclude transaction costs. Denote $W$, total wealth, $\alpha$, fraction of wealth in the risky asset, $R_f$, return on the risk-free asset, $R_e$, expected

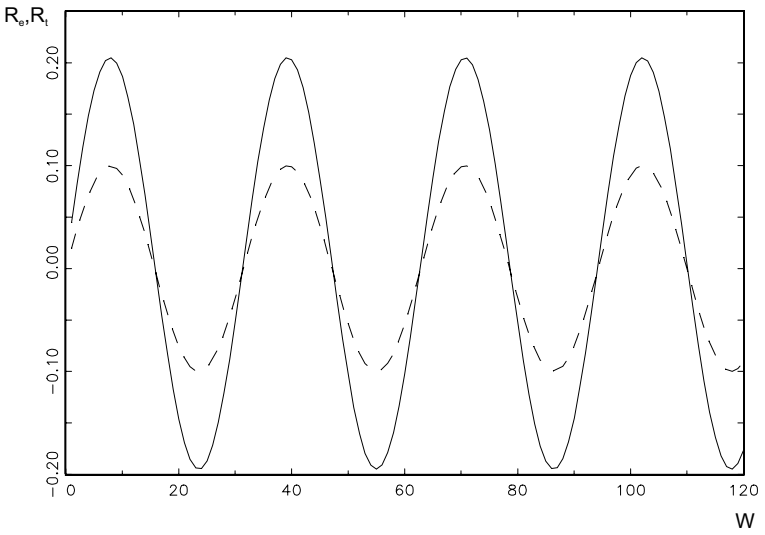



**Fig. 16.5.** Cyclical Equity Return (Solid) and Risk-Free Rate (Dashed)

return on the risky asset, $R_e > R_f$, $\sigma^2$, variance per unit time of the return on the risky asset, and $C$, consumption. Presume preferences $U(C), C^b/b$ with $b = 1 - \gamma$.

The change of wealth can be denoted by

$$dW = [(1-\alpha)R_f W + \alpha R_e W - C]dt + \alpha W \sigma dz. \tag{16.56}$$

There is a deterministic fraction of wealth which is determined by the return on the funds in the risk-free asset, plus the expected return on the funds in the risky asset, minus consumption. The aim of the holder of wealth is the maximization of an expected discounted utility flow. We again assume a model with an infinite horizon:

$$\max_{C,\alpha} E \int_0^\infty (e^{-\beta t} C^b/b) dt \tag{16.57}$$

$$\text{s.t. (16.56) and } W(0) = W_0.$$

This infinite horizon decision problem has indeed one state variable $W$ and two control variables $C$ and $\alpha$. The equation (16.55) was stated for a problem with just one state variable and one choice variable in equs. (16.53)-(16.54). Yet, it can easily be extended. Using the specifications of (16.56) and (16.57), (16.55) is

$$\begin{aligned}\beta V(W) = \max_{C,\alpha} (&C^b/b + V'(W)[(1-\alpha)R_f W + \alpha R_e W - C] + \\ &(1/2)\alpha^2 W^2 \sigma^2 V''(W)).\end{aligned} \tag{16.58}$$

Some calculus[183] provides us with the maximizing values of $C$ and $\alpha$ for the given parameters of the problem, the state variable $W$, and the unknown function $V$:

$$C = [V'(W)]^{b/(b-1)}, \;\; \alpha = V'(W)(R_f - R_e)/\sigma^2 W V''(W). \tag{16.59}$$

It is assumed that the optimal solution involves investment in both assets for all $t$. Using (16.59) and (16.58) and simplifying we obtain

$$\beta V(W) = (V')^{b/b-1}(1-b)/b + R_f W V' - (R_f - R_e)^2 (V')^2 / 2\sigma^2 V''. \tag{16.60}$$

One can try a solution to this nonlinear second order differential equation of the form

$$V(W) = AW^b, \tag{16.61}$$

Hereby $A$ is a positive parameter to be determined. One can compute the required derivatives of (16.61) and use the results in (16.60). With some simplification, one obtains

$$Ab = \{[\beta - R_f b - (R_f - R_e)^2 b / 2\sigma^2 (1-b)]/(1-b)\}^{b-1}. \tag{16.62}$$

Thus, the optimal current value function is (16.61), with $A$ as specified in (16.62). In order to find the optimal choice $C$, use equs. (16.61) and (16.62) in equ. (16.59):

$$C = W(Ab)^{1/(b-1)}, \;\; \alpha = (R_e - R_f)/(1-b)\sigma^2. \tag{16.63}$$

This means that the household consumes a constant fraction of wealth at each instant of time only if the equity premium remains a constant. The optimal choice depends on the parameters. It varies with the discount rate and with the variance of the risky asset. Similarly to our static case, equ. (16.4) and (16.34) the optimal wealth chosen for the two kinds of assets is a constant, independent of total wealth, as long as the equity premium and variance $\sigma^2$ remain constant. As in equ. (16.4) the portion devoted to the risky asset varies with the equity premium. It is related to the variance of that return and the risk aversion parameter, $\gamma$, since $b = 1 - \gamma$.

### 16.5.2 Mean-Reverting Interest Rates and Long Bonds

Next we want to study a portfolio model with short and long bonds. We thus need to discuss the relation of the short term to the long term interest rates for bonds. We assume mean reverting short term interest rates.[184] We take as the price of the bond, $P$, depending on the specification of the interest rate process, $\{r_t\}$, to be defined below, and the time period $N = T - t$. We refer to a bond price $P(r_t, N)$ and consider a zero coupon bond. Let the discount rate for the N-period bond be

$$Y^{(N)} = (1 + r^N)$$

[183] For details, see Kamien and Schwartz (2001, sect. 22).
[184] For further details, see Cochrane (2001, Chap. 19).

and its price

$$P^{(N)} = \frac{1}{(Y^{(N)})^N} \tag{16.64}$$

Let $y^{(N)} = lnY^{(N)}$ and $p^{(N)} = lnP^{(N)}$ then we have for the return of the long bond with maturity $N$

$$y^{(N)} = -\frac{1}{N} p^{(N)} \tag{16.65}$$

This gives us the term structure of interest rates. We presume the return on the long bond to be $r_t^N$ and on the short bond to be $r_t$, the latter generated from the mean reverting interest rate process, introduced below. Then we can specify a dynamic portfolio decision problem with two control variables, $C$ and $\alpha$, with one state variable representing wealth, $W$, and another state variable, representing two assets, the short and long bond, as follows

$$\max_{c,\alpha} E \int_0^\infty e^{-\beta t} \frac{C^{1-\gamma}}{1-\gamma} dt \tag{16.66}$$

s.t.

$$dW = [(\alpha_t (r_t^N - r_t) + r_t) W_t - C_t] dt + \sigma(W_t, \alpha_t) dz_t \tag{16.67}$$

$$dr = \phi(\overline{r} - r) dt + \sigma_r \sqrt{r_t} dz \tag{16.68}$$

Note that the return on the long bond is a function of the return on the short bond, so that we have only one state equation.

Following Cochrane (2001, Chap. 19) let the time series of the discount factor be

$$\frac{d\Lambda}{\Lambda} = -r dt - \sigma_\Lambda(\cdot) dz \tag{16.69}$$

$$dr = \mu_r(\cdot) dt + \sigma_r dz \tag{16.70}$$

Moreover, we specify the interest rate process of equ. (16.70) as mean reverting process such as studied by Vasicek or Cox-Ingersoll-Ross (CIR):

Vasicek:

$$\begin{aligned} \frac{d\Lambda}{\Lambda} &= -r dt - \sigma_\Lambda dz \\ dr &= \phi(\overline{r} - r) dt + \sigma_r dz \end{aligned}$$

CIR:

$$\begin{aligned} \frac{d\Lambda}{\Lambda} &= -r dt - \sigma_\Lambda \sqrt{\Lambda} dz \\ dr &= \phi(\overline{r} - r) dt + \sigma_r \sqrt{r} dz \end{aligned}$$

Bond prices are then

$$P_t^{(N)} = E_t\Big(\frac{\Lambda_{t+N}}{\Lambda_t}\Big) \tag{16.71}$$

A solution of equ. (16.71) can be obtained for solving the discount factor forward. Taking $t = 0$ we have

$$\frac{\Lambda_T}{\Lambda} = e^{-\int_0^T (r_s + \frac{1}{2}\sigma_{\Lambda s}^2)ds - \int_0^T \sigma_{\Lambda s} dz_s}$$

and thus the bond price is

$$P_0^{(T)} = E_0(e^{-\int_0^T (r_s + \frac{1}{2}\sigma_{\Lambda s}^2)ds - \int_0^T \sigma_{\Lambda s} dz})$$

If $\sigma_\Lambda = 0$ we obtain the continuous time present value of a pay-off of one unit

$$P_0^{(T)} = e^{-\int_0^T r_s d_s}$$

and for a constant interest rate we obtain

$$P_0^{(T)} = e^{-rt}$$

Given the estimates of the coefficients of either the Vasicek or the CIR model of mean reverting interest rates one can suggest (see Cochrane, 2001, Chap. 9.5) the bond prices to be a direct solution of

$$P(r, N) = e^{A(N) - B(N)r} \tag{16.72}$$

with $A(N)$ and $B(N)r$ as functions of the underlying[185] estimated coefficients (see Cochrane 2001: 367). Log prices and log yields of log bonds with duration $T$ are the linear functions of the interest rate $r$,

$$p(r, N) = A(N) - B(N)r \tag{16.73}$$

$$y(r, N) = \frac{-A(N)}{N} + \frac{B(N)}{N} r \tag{16.74}$$

Although the $A(N)$ and $B(N)$ in (16.72) are different for the Vasicek and CIR models respectively, the return on the long bond can be directly computed. Empirical results on the estimates of the coefficients of the Vasicek and CIR mean reverting interest rate process are reported and extensively discussed in Chan et al. (1992) and in Chap. 2 of this book.[186] Having obtained the bond returns of long bonds as in (16.74), and given the short rates $r_t$, one can employ then $r_t^N$ from $y^{(N)}$ and the short rate $r_t$, as in the portfolio decision model of equs. (16.67)-(16.68) and solve the model through dynamic programming for the path of the control variables $C_t$ and $\alpha_t$. Next we will pursue a simpler model and solve it through dynamic programming.

---

[185] The functions $A(N)$ and $B(N)r$ in equs. (16.72) and (16.73) can either be obtained by solving a thereby involved partial differential equation or by taking expectations of the discount factor of equ. (16.69), see Cochrane (2001, Chap. 9.5).

[186] Chap. 2 in addition reports empirical results on a mean reverting interest rate process with changing mean.

### 16.5.3 Portfolio Model with Mean Reverting Interest Rate and Equity

A portfolio choice model (see Munk et al., 2004), that also takes into account equity but includes only short term bonds, can be written for power utility as

$$\max_{\alpha,C} \int_0^\infty e^{-\beta t} \frac{C^{1-\gamma}}{1-\gamma} dt \tag{16.75}$$

s.t.

$$dW = \{[\alpha_t(r_t + x_t) + (1-\alpha_t)r_t]W_t - C_t\}dt + \sigma_w dz_t \tag{16.76}$$

$$dx_t = \nu(\overline{x} - x_t)dt + \sigma_x dz_t \tag{16.77}$$

$$dr_t = \kappa(\theta - r_t)dt + \sigma_r dz_t \tag{16.78}$$

Denote, $W_t$, total wealth, $r_t$, the short term interest rate, $\alpha_t$, the fraction of wealth held as equity, $x_t$, the equity premium, $\overline{x}$, the mean equity premium, $\theta$, the mean interest rate and $dz_t$ again, the increment in Brownian motion.

Let us further simplify the model and study a special case which is obtained if we take $x_t = \overline{x}$, presuming hereby that the equity premium is fixed. Following Munk et al. (2004) we assume stylized facts of the U.S. asset market such as $\sigma_x = 0.0069$, $\overline{x} = 0.0648$, $\nu = 0.0608$, $\sigma_r = 0.0195$, $\theta = 0.00369$, $\kappa = 0.0395$. The first and second moment reported here are annualized.

For the control variable $\alpha_t$ we assume $-2 \leq \alpha \leq 2$ and for $C_t$ we presume bounds such as $0 < C_t < 40$. The use of a stochastic dynamic programming algorithm provides us with the following result for the dynamic decision paths for $\alpha_t$ and $C_t$ and the wealth dynamics. Figures 16.6-16.8 show the results of the numerical study using a dynamic programming algorithm similarly to the one sketched in appendix 3.[187]

As observable in figure 16.6 and 16.7 there are two domains of attraction for the wealth dynamics. For low wealth (and not too high interest rates) wealth is contracting and will thus finally be used up. For larger wealth and higher interest rates, given the bounded consumption $0 < C_t < 40$, wealth will persistently increase. This is visible from the value function, figure 16.6, and the vector field, figure 16.7. As the figures 16.6 and 16.7 suggest the two domains of attractions should be separated by a line. In some recent literature (see Grüne and Semmler, 2004a) this line has been called the Skiba-line (shown in figure 16.7 by the line $S - S$). This line should be somewhat blurred because of the stochastic shocks presumed in our model. The optimal response of the decision variable, $C_t$, depending on the state variables, wealth, $W_t$, and interest rate, $r_t$, is shown in figure 16.8. As figure 16.8 indicates for low wealth, roughly for $W \leq 50$, consumption is declining whereas for $W > 50$ consumption is bounded by $C_t \leq 40$. Of course, in this model too, the consumption to wealth ratio will not be a constant.

[187] For the stochastic version of the DP algorithm, see Grüne and Semmler (2004a).

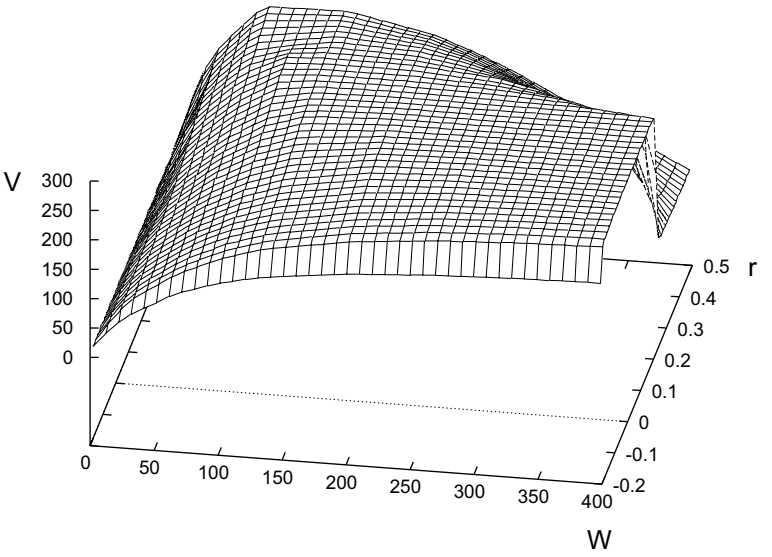



**Fig. 16.6.** Value Function for the Wealth Dynamics

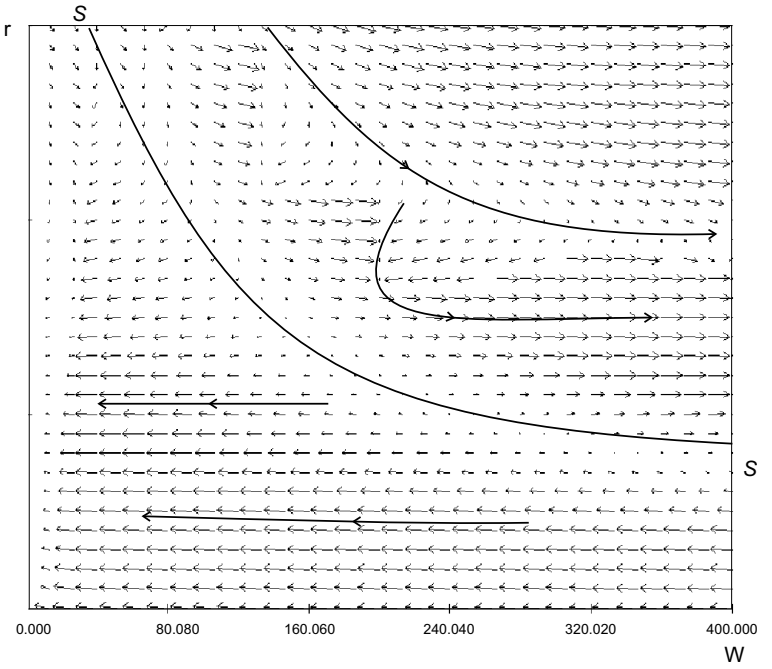



**Fig. 16.7.** Vector Field for the Wealth Dynamics

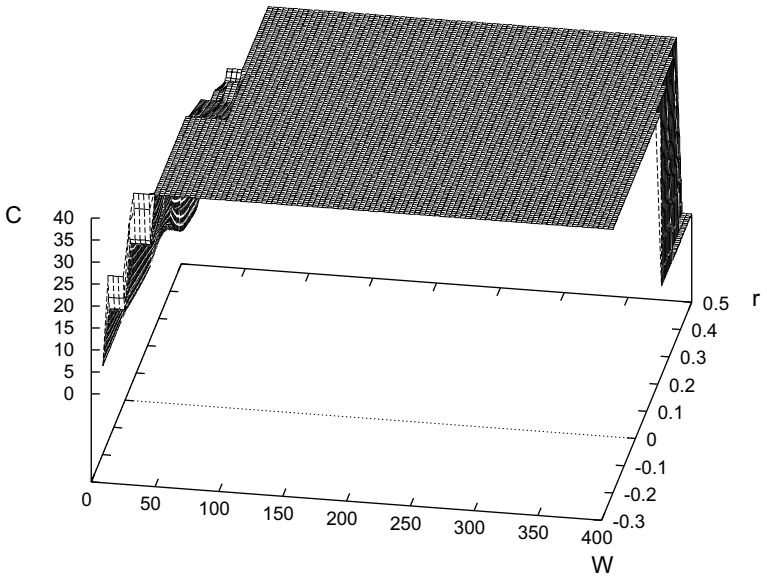



**Fig. 16.8.** Dynamic Consumption Decisions

## 16.6 Conclusions

This chapter, in the line of previous chapters of advanced modeling of the asset market, has employed modern analytical and numerical methods to study the problem of dynamic portfolio choice. Once one goes beyond static portfolio theory by presuming that investors have different risk preferences, care about consumption while they accumulate assets for future consumption and face new investment opportunities as time is evolving, dynamic portfolio choice models are required. We began this chapter by referring to the seminal work by Campbell and Viceira (2002), who recently have made strategic asset allocation a major topic in portfolio theory. In the present chapter we have, using a dynamic programming algorithm, demonstrated the solution for simple deterministic as well as stochastic versions of portfolio choice models. As we have shown, dynamic programming proves to be a powerful tool in solving higher dimensional and more complex portfolio choice models. Thus, it may in fact find fruitful applications in future research on asset pricing and dynamic portfolio theory.

CHAPTER 17

# Some Policy Conclusions

The growth of financial markets has exerted its impact on economic activity. The role of financial markets has grown due to deregulation, liberalization of capital accounts in many countries, financial innovations and development of new financial instruments such as financial derivatives. Moreover, since the 1980s, financial liberalization has been actively advocated by international financial institutions such as the IMF and many governments. For some countries a financial market boom was accompanied by an economic boom. On the other hand, numerous countries have experienced major episodes of financial instability, some times with devastating effects on economic activity. This has happened when a fast liberalization of financial markets has led to a currency crisis, sudden reversal of capital flows followed by financial instability and stock market crashes with consequently declining economic activity and large output losses. Yet, the globalization of real and financial activities have also created new opportunities for the financial market traders and investment firms that invested their funds globally.

This book has dealt with the interaction of financial markets and economic activity. An important part of financial markets are the money and bond markets where short and long-term interest rates are determined. We have presented theories and empirical models on the credit market, credit risk and the term structure of interest rates. We have shown that credit markets, where either commercial papers are traded or where households and firms obtain bank loans, play an important role for economic activity. Credit is still the dominant source for financing of real activity (firms, households and countries). Additionally, financial markets contain the stock market, the credit market and credit risk. We have also studied foreign exchange markets where exchange rate volatility and international capital flows come into play. Economic activity impacted by financial markets was described by the activity of firms, households, banks, governments and countries. In order to study the dynamics of the financial-real interaction we have used micro as well as macro approaches, presumed optimizing and non-optimizing behavior, employed zero horizon and infinite horizon models and have used linear and nonlinear models.

A particularly great concern of ours was the externalities of the financial markets. The experience of financial crises and large output losses in emerging markets in the years 1997-1998 and the large and sudden asset price deflation in advanced market economies during the years 2001-2002, have shown that financial liberalization without proper safety nets, without enforcement of strict accounting standards and government supervision may lead to a failure of financial sectors which may have disastrous effects on real activity. To prevent this, it not only requires regulatory insti-

tutions and public screening and monitoring, but firms and banks need to be required to adhere to strict standards of accounting and publicly reveal information on assets, debt and earnings. Fast liberalization of the financial market entails a greater risk if there is insufficient financial market regulation, inexperienced and loose supervision, no disclosure requirement, no screening and monitoring of financial institutions and no secure safety net for the financial institutions (for example, insurance for bank deposits).[188]

Although implicitly or explicitly discussed throughout the entire book, in Chap. 12 we in particular have demonstrated dynamic mechanisms that help us to explain financial instabilities and financial crises that have occurred in many countries and regions. As we have shown, asset price appreciation through an increased value of collateral, low borrowing costs and wealth effects, can fuel borrowing, lending, and consumption and investment spending and thus economic growth. Asset price deflation on the other hand devalues collaterals, increases borrowing cost and lets consumption and investment spending decrease. Indeed an interesting feature of the monetary and financial environment in industrial countries over the past decade has been that inflation rates remained relatively stable and low, while asset prices, the prices of equities, bonds, and foreign exchanges, experienced a strong appreciation and depreciation as well as short-term volatility with the liberalization of the financial markets.

There have been, of course, certain regulatory measures enacted for reducing asset price volatility and preventing its adverse impact on the macroeconomy. As remarked above, the improvement of the stability of the financial sector through financial market supervision and banking regulation, such as supervision undertaken by the government and monetary authorities (central banks), appears to be the most important means towards this end. Yet, as discussed in Chap. 2, given financial institutions and financial market regulations, an important contribution of central banks might be to not only stabilize output and inflation, but also to stabilize asset prices when they are too volatile.

Stabilizing asset prices, for example, preventing them from depreciating below some level, is not an easy task for monetary authorities. Especially if inflation rates and interest rates are already very low (given the zero bound of the nominal interest rate), it may be impossible. As Japan experienced, in the 1990's, monetary authorities become helpless in stabilizing a further fall of asset prices and output. Central banks, therefore, must, early on, not only respond to forecasted future inflation and output gap but to asset prices as well. Of course estimating asset prices misalignment is at least as difficult as estimating future inflation rates or output gaps, yet one must not forget that future inflation rates or output gaps depend also on future asset prices.

Of course, monetary authorities can and should not target specific levels of asset prices. There are fundamentally justified movements in asset prices as we have shown in Chaps. 2-3 for bond prices and credit cost, Chaps. 5-7 for stock prices and in Chap.

[188] Such weak accounting standards and loose supervision cannot only be found in emerging markets, but also, as the book by MacArby and Millstein (2004) demonstrates, in the U.S. and other advanced market economies.

12 for exchange rates. Although asset price misalignments are difficult to measure, as are potential output, future inflation rates and equilibrium interest rates, this should be no reason to ignore them.[189] Monetary authorities should help to provide stability for the financial market and reduce the likelihood of financial instability not only in the credit market and banking sector, but also instability arising from extreme changes in asset prices.

[189] For a more detailed analysis and for the issues involved see Cecchetti, Genberg, Lipsky and Wadhwani (2000) and Semmler and Zhang (2002).

# Appendices

## Appendix 1: Stochastic Processes

An important example of a stochastic process is Brownian motion[190] where dW is the increment in the Wiener process. This is defined as follows

1. linear:
$$dr_t = \mu dt + \sigma dW_t$$
2. growth rates:
$$dr_t = \mu r_t dt + \sigma r_t dW_t$$
3. square root process:
$$dr_t = \mu r_t dt + \sigma r_t^{\gamma} dW_t$$
In the latter version the larger the $\gamma$ the larger is the state dependent volatility. With $\gamma = 1/2$ one obtains the square root process.
4. mean reverting (fixed mean):
$$dr_t = \lambda(\mu - r_t)dt + \sigma r_t dW_t$$
Note that the mean, $\mu$, could also be time varying, see Chap. 2.
5. stochastic volatility:
$$\begin{aligned} dr_t &= \mu dt + \sigma_t dW_t \\ d\sigma_t &= \lambda(\sigma_0 - \sigma_t)dt + \alpha\sigma_t dW_t \end{aligned}$$
The most suitable process to model interest rates is the mean reverting process. For the stock market one often uses the geometric Brownian motion that with multiplicative noise. This also appears in Black and Scholes (1973).
6. geometric Brownian motion for stock prices:
$$dS_t = \mu S_t dt + \sigma S_t dW_t$$
From 6. we obtain
7. growth rate of stock prices
$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$
or as integral terms
$$\begin{aligned} \int_0^t \frac{dS_u}{S_u} &= \int_0^t \underbrace{\mu du}_{\mu t} + \int_0^t \underbrace{\sigma dW_u}_{+\ \sigma(W_t - W_0)} \\ W_0 &= 0 \end{aligned}$$

[190] For details of the subsequent stochastic processes, see Chan et al. (1992), see also Neftci (1996), Chap. 11.

Using 6.

$$S_t = S_0 + \int_0^t \mu S_u du + \int_0^t \sigma S_u dW_u$$

we get 7.

$$\int_0^t \frac{dS_u}{S_u} = \mu t + \sigma W_t$$
$$(\text{since } W_0 = 0)$$

A candidate for an explicit solution of 6. is using Ito's Lemma, see Kloeden, Platen and Schurz (1991), p.70

$$S_t = S_0 e^{\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right\}}$$

*Proof:*

Consider the stochastic differential $dS_t$ using Ito's Lemma:

$$dS_t = S_0 e^{\left\{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right\}} \left[\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t + \sigma^2 dt\right]$$

The last term on the right corresponds to the second order term in Ito's Lemma. Cancelling similar terms we get

$$dS_t = S_t \left[\mu dt + \sigma dW_t\right]$$

which is 6. If we had not the last term from Ito's Lemma we would have instead from ordinary calculus:

$$dS_t = S_t \left[\left(\mu - \frac{1}{2}\sigma^2\right) dt + \sigma dW_t\right]$$

which is incorrect.

## Appendix 2: Deriving the Euler Equation from Dynamic Programming

Stockey and Lucas (1995) use the dynamic programming approach to derive the Euler equation. Let us write the discrete time Bellman equation as

$$V(k)_{0 \le y \le f(k)} = \max \left\{U\left[f(k) - y\right] + \beta V(y)\right\}$$

where $f(k) - y = C$ and $C$ is consumption.

Using the first-order and envelop conditions

$$U'(f(k_t) - g(k_t)) = \beta V'(g(k_t)) \tag{17.1}$$
$$k_{t+1} = g(k_t);\ k_{t+1} = f(k_t) - C_t;\ k_{t+1} \le f(k_t)$$
$$V'(k_t) = f'(k_t) U'(f(k_t) - g(k_t)). \tag{17.2}$$

Then write:

$$V'(k_{t+1}) = f'(k_{t+1})U'(f(k_{t+1}) - g(k_{t+1})) \tag{17.3}$$

and

$$U'(f(k_{t+1}) - g(k_{t+1})) = U'(C_{t+1}). \tag{17.4}$$

We therefore get from (17.1), (17.3) and (17.4)

$$U'(f(k_t) - g(k_t)) = \beta f'(k_{t+1})U'(C_{t+1}).$$

Thus

$$1 = \beta f'(k_{t+1})\frac{U'(C_{t+1})}{U'(C_t)}.$$

The latter is the Euler equation derived from dynamic programming. For a more detailed treatment of how to solve interemporal dynamic optimization problems using dynamic programming, see Grüne and Semmler (2004a).

## Appendix 3: Numerical Solution of Dynamic Models

We here briefly describe the dynamic programming algorithm as applied in Grüne and Semmler (2004a) that enables us to numerically solve dynamic models as proposed in Chaps. 9-10 and 15-16. The feature of the dynamic programming algorithm is an adaptive discretization of the state space which leads to high numerical accuracy with moderate use of memory.

Such algorithm is applied to discounted infinite horizon optimal control problems of the type introduced in the above mentioned chapters. In our model variants we have to numerically compute $V(x)$ for

$$V(x) = \max_u \int_0^\infty e^{-rt} f(x,u)dt$$

$$\text{s.t. } \dot{x} = g(x,u)$$

where $u$ represents the control variable and $x$ a vector of state variables.

If we have a continuous time problem, such as the one above, in a first step, the continuous time optimal control problem has to be replaced by a first order discrete time approximation given by

$$V_h(x) = \max_j J_h(x,u), \;\; J_h(x,u) = h\sum_{i=0}^{\infty}(1-\theta h)Uf(x_h(i),u_i) \tag{A1}$$

where $x_u$ is defined by the discrete dynamics

$$x_h(0) = x, \; x_h(i+1) = x_h(i) + hg(x_i,u_i) \tag{A2}$$

and $h > 0$ is the discretization time step. Note that $j = (j_i)_{i\in\mathbb{N}_0}$ here denotes a discrete control sequence.

The optimal value function is the unique solution of a discrete Hamilton-Jacobi-Bellman equation such as

$$V_h(x) = \max_j \{hf(x, u_o) + (1 + \theta h)V_h(x_h(1))\} \tag{A3}$$

where $x_h(1)$ denotes the discrete solution corresponding to the control and initial value $x$ after one time step $h$. Abbreviating

$$T_h(V_h)(x) = \max_j \{hf(x, u_o) + (1 - \theta h)V_h(x_h(1))\} \tag{A4}$$

the second step of the algorithm now approximates the solution on grid $\Gamma$ covering a compact subset of the state space, i.e. a compact interval $[0, K]$ in our setup. Denoting the nodes of $\Gamma$ by $x^i, i = 1, ..., P$, we are now looking for an approximation $V_h^\Gamma$ satisfying

$$V_h^\Gamma(X^i) = T_h(V_h^\Gamma)(X^i) \tag{A5}$$

for each node $x^i$ of the grid, where the value of $V_h^\Gamma$ for points $x$ which are not grid points (these are needed for the evaluation of $T_h$) is determined by linear interpolation. We refer to Grüne and Semmler (2004a) for the description of iterative methods for the solution of (A5). Note that an approximately optimal control law (in feedback form for the discrete dynamics) can be obtained from this approximation by taking the value $j^*(x) = j$ for $j$ realizing the maximum in (A3), where $V_h$ is replaced by $V_h^\Gamma$. This procedure in particular allows the numerical computation of approximately optimal trajectories.

In order the distribute the nodes of the grid efficiently, we can make use of a posteriori error estimation. For each cell $C_l$ of the grid $\Gamma$ we compute

$$\eta_l := \max_{k \in c_l} \mid T_h(V_h^\Gamma)(k) - V_h^\Gamma(k) \mid$$

More precisely we approximate this value by evaluating the right hand side in a number of test points. It can be shown that the error estimators $\eta_l$ give upper and lower bounds for the real error (i.e., the difference between $V_j$ and $V_h^\Gamma$) and hence serve as an indicator for a possible local refinement of the grid $\Gamma$. It should be noted that this adaptive refinement of the grid is very effective for computing more complex models for example with steep value functions or with multiple equilibria, see Grüne and Semmler (2004a).

## Exercises[191]

### Exercise 1: Bond Prices and Yields

1. The present value of an n-period cash flow is

$$P_0 = \frac{a_1}{1+r_1} + \frac{a_2}{(1+r_1)(1+r_2)} + \frac{a_3}{(1+r_1)(1+r_2)(1+r_3)...} + \frac{a_n}{(1+r_1)(1+r_2)(1+r_3)..(1+r_n)}$$

A four period debt instrument with promised payment by the borrower: ($M$= Maturity or face; $C$= Coupon payment (=$a_1$))

| year | Interest payment | Principle Payment | Cash Flow (C) |
|---|---|---|---|
| 1 | \$ 100 | 0 | \$ 100 ($a_1$) |
| 2 | \$ 120 | 0 | \$ 120 ($a_2$) |
| 3 | \$ 140 | 0 | \$ 140 ($a_3$) |
| 4 | \$ 150 | (C+M)\$ 1150 | \$ 1150 ($a_4$) |

One discount rates for the next period are $r_1 = 0.07; r_2 = 0.08; r_3 = 0.09; r_4 = 0.1; M = 1000$
Compute the present value of the bond (pay attention to the fact that in the last period there is an interest payment as well as repayment of the principle).

2. The present value of a bond paying an interest income (C) is
   a)
$$P = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + ..... + \frac{C_4+M}{(1+y)^n}$$
   or
   b)
$$\frac{P}{M} = \frac{C}{M}\sum_{t=1}^{n}\frac{1}{(1+y)^t} + \frac{1}{(1+y)^n}$$
   The latter is the present value per \$ of face value. Use the above example of table 17.1 and compute the yield, $y$.

3. Derive for a bond with semi-annual coupon payment

$$\frac{P}{M} = \frac{C}{2M}\left[\frac{1-(1+y/2)^{-2n}}{y/2}\right] + \frac{1}{(1+y/2)^{2n}}$$

(Derive from 2. b), first the above equation for annual coupon payment, and then semiannual payment)

[191] I want to thank Chih-Ying Hsiao for providing most of the exercises presented here.

4. A 20 year bond with maturity or per value of \$100 pays annually 7% as coupon payment, but it has semi-annual coupon payments of 40 six-months payments. Compute for the semi-annual yield $(y/2)$ the present value of the bond $(P/M)$

| interest rate $(y/2)$ | 3.5% | 4.0% | 4.5% | 5.0% | 5.5% | 6.0% | 6.5% |
|---|---|---|---|---|---|---|---|
| present value $(P/M)$ | ? | ? | ? | ? | ? | ? | ? |

**Exercise 2: Money and Interest Rates**

1. Monetary Aggregates
   Give the definition of the monetary aggregates M1, M2, and M3. Describe their current and historical developments, see, for example, "www.ecb.int".

2. Loanable Fund Theory

   2.1 An investor buys a zero-coupon bond with face value \$100 at the end of the third year. What is this bond price if the interest rate is 2% (with the simple interest rate rule)? How does the bond price change if the interest rate goes up to 4%? Describe the relationship between the interest rate and bond demand.

   2.2 Following the "Loanable Fund Theory" in Chap.1, how would the bond demand change if now the expected interest rate goes up? Compare this answer with Exercise 2.1, do they contradict each other?

   2.3 Now, the investor will sell the bond (from Exercise 2.1) at the end of the year. The interest rate remains 2% for these three years. How much is the bond return for this year? If the investor expects that the interest rate goes up to 4% for the next two years, how much is his/her expected bond return by selling his/her bond at the end of the year?

   2.4 Can you explain now this "contradiction"?

3. Simulating Mean-Reverting Processes
   In Chap.2 we assume that the instantaneous interest rate follows the mean-reverting process

$$dr_t = \kappa(\overline{r} - r_t)dt + \sigma dB_t.$$

   The solution of $r_t$ can be expressed as the following autoregressive process of first order (AR1)

$$r_{t+1} = r_t + \tilde{\kappa}(\overline{r} - r_t) + \tilde{\sigma}u_t,$$

   where $u_t$ is white noise with normal distribution $N(0,1)$ and the parameters are

$$\tilde{\kappa} = 1 + e^{-\kappa}$$
$$\tilde{\sigma}^2 = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa}) .$$

   Simulate the mean-reverting process with the parameters $\kappa = 0.5, \sigma = 0.005$. Simulate the process with different $\kappa = 0.1, 1.5, 2$. What is your observation of the simulated paths of different $\kappa$?

**Exercise 3: Credit Market**

1. Markets
   1.1. What are “complete capital markets" and “incomplete capital markets"?
   1.2. Explain the “transversality condition", “debt ceiling" (credit capacity), and the intertemporal budget constraint. How do they hang together?

2. Information
   2.1. Explain the terms “asymmetric information", “adverse selection", and “moral hazard".
   2.2. Explain how asymmetric information affects credit markets.

3. Imperfect Capital Market

   The investment cost for a project is $B$ money unit. The payout of this project is stochastic

   $$\text{Payout} = \begin{matrix} X_a(\text{good})\,, & \text{with probability } p_a \\ X_b(\text{bad})\,, & p_b\,. \end{matrix}$$

   Mark will invest in this project and finance it with credit with constant interest rate $r$. The payout of the project follows $X_b < (1+r)B < X_a$. We assume also “limited liability".[192]

   a) Give the payoff-matrix for the both parties (Mark and the bank) with both situations(good and bad).
   b) Now, Mark can decide to invest in one of the two projects $X, Y$ with the same expected return $X^e = p_a X_a + p_b X_b = p_a Y_a + p_b Y_b = Y^e$ but project $Y$ is more risky $Y_b < X_b < X_a < Y_a$.
   c) Which project would Mark choose? and which project should Mark choose from the view point of the bank? Do Mark and his bank have the same opinion on this investment? Who is more risk-friendly? What is the concept to explain this phenomenon?

4. Credit Rationing

   Assume that there is excess demand on loans on the market.

   a) What is the normal mechanism to bring the credit market back into equilibrium? The debtors on the credit market are heterogeneous as concerning their creditworthiness but the creditors do not have information about the creditworthiness of the debtors.
   b) If the interest rate is raised, how does the fraction of the different kinds of the debtors change? Explain the credit supply in Figure 3.7 in the book. What is the concept behind this phenomenon? Would the credit market go back to the equilibrium?

[192] If the project has a bad outcome, Mark only needs to return $X_b$ instead of $(1+r)B$.

**Exercise 4: Hamiltonian in Finance**

1. The Ramsey-Problem, see Blanchard and Fischer (1989).

   The representative consumer maximizes a utility function

$$\max_{c_t} \int_0^\infty e^{-\delta t} \frac{c^{1-\gamma}}{1-\gamma} dt \,,$$

   where $0 < \gamma < 1$. The budget constraint is given by

$$c_t = f(k) - \frac{dk(t)}{dt},$$

   where $k$ is the capital and $f(k) = k^{1-\alpha}$ is the production function with $0 < \alpha < 1$.

   The Hamiltonian solution for this intertemporal optimization problem is known as

$$\begin{aligned} \frac{\partial}{\partial c} H(c(t), k(t), \lambda(t)) &= 0 \\ \frac{\partial}{\partial k} H(c(t), k(t), \lambda(t)) &= \delta\lambda(t) - \frac{d\lambda(t)}{dt} \,, \end{aligned}$$

   where the Hamiltonian is defined as

$$\begin{aligned} H(c(t), k(t), \lambda(t)) &= u(c(t)) - \lambda(t) \frac{dk(t)}{dt} \\ &= u(c(t)) - \lambda(t)(f(k(t) - c(t)). \end{aligned}$$

   Give the condition for the optimal consumption path and explain how the optimal consumption behavior is affected by the subjective discount factor $\delta$, the parameter of risk aversion $\gamma$, and by the marginal product of capital $f'(k)$.

**Exercise 5: Credit Rating and Transition Matrix**

Presume that there are four different rating levels for credit: $A$ good rating, $B$ bad rating, $D$ default this time, $E$ already in default last time. The transition matrix is defined by

$$\begin{pmatrix} \pi_{AA} & \pi_{AB} & \pi_{AD} & 0 \\ \pi_{BA} & \pi_{BB} & \pi_{BD} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where, for example, $\pi_{AB}$ denote the die transition probability to move from level $A$ this year to level $B$ in next year.

Assume the following number of bonds at different rating levels:

| $A$ | $B$ | $D$ | $E$ |
|---|---|---|---|
| 80 | 170 | 20 | 30 |

and the transition matrix

$$\begin{pmatrix} 0.99 & 0.01 & 0 & 0 \\ 0.03 & 0.96 & 0.01 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

1. Explain why all elements in a row in the transition matrix sum up to 1. Give the rating of different credit levels in the next year and the third year. The annual bond return is $6\%$ and the collateral is $40\%$. The collateral goes to the creditors if bonds are in default at the first time. (For the computation of risk adjusted returns, see Benninga, 1998).

   a) What is the expected return of the bond with rating $A$ for one year?

   b) What is the expected return of the bond with rating $B$ for two years?

**Exercise 6: Detrending Stock Prices**

Compare the stock index in Fig.5.1 and the empirical S&P 500 stock index.[193] You will find that the S&P index in Fig. 5.1 is "stationary" while the empirical S&P index has an increasing trend. In order to detrend the data we should undertake the following steps:

1. Eliminate the price effect and obtain the real stock index $\overline{S}$

$$\overline{S}_t := \frac{S_t}{P_t},$$

   where $S_t$ denotes the S&P index and $P_t$ denotes the consumer price index (CPI).
2. Eliminate the long-term growth trend. Let $g$ be defined as the constant long-term growth rate[194]

$$g = \frac{\ln S_T(real) - \ln S_0(real)}{T}.$$

[193] Date source for S&P index: Robert Shiller's homepage: http://www.econ.yale.edu/∼shiller/data.html, then click on "Annual Stock Market Data 1871-2003".

[194] Recall the growth rate for one year is $\ln S_t - \ln S_{t-1}$. So, the average growth rate from $t = 0$ until $t = T$ is

$$g = \frac{1}{T}\sum_{t=1}^{T}(\ln \overline{S}_t - \ln \overline{S}_{t-1}) = \frac{\ln S_t - \ln S_0}{T}.$$

The detrended index is now defined as [195]

$$S_t(real, detrend) = e^{-gt}\overline{S}_t.$$

Implement these two detrending process and compare it with the original index.

**Exercise 7: Blanchard Model with Perfect Foresight**

The interaction between the output and the stock price under "prefect foresight"[196] is described by the following dynamic system

$$\dot{y} = \kappa_y(aq - by + g) \tag{17.5}$$

$$\dot{q} = q(cy - h(m - p)) - \alpha_0 - \alpha_1 y, \tag{17.6}$$

where $a > 0, 0 < b < 1, c < 0, \alpha_1 > 0$. Comparing the equations (17.5) and (17.6) with the equations (6.16) and (6.20) in the book, you find the dynamic system (17.5) and (17.6) is a special case of the three dimensional dynamic system in Chap. 6.

a) Determine the steady state $\overline{q}$ and $\overline{y}$ of this system.

b) Discuss the stability of the steady state.

c) Show that the steady state is a saddle point when

$$c\overline{q} - \alpha_1 > 0\,.$$

**Exercise 8: Market Price of Risk**

The market price of risk is defined by the Sharpe Ratio

$$\frac{\mu - r_f}{\sigma},$$

where $\mu$ is the expected return and $\sigma$ is the volatility (standard deviation).

a) Evaluate the Sharpe Ratios for S&P500 index and Nasdaq for 2000, 2001 and 2002.[197]

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[195] Recall the relationship

$$\overline{S}_T == e^{gT}\overline{S}_0.$$

[196] See Blanchard (1981).

[197] Date source: finance.yahoo.com. For the risk-free interest rate you might take the "average treasury bill rates in US": 5,84% (2000), 3,45%(2001) und 1,61%(2002). The date source is International Financial Statistics (IMF).

b) Evaluate the Sharpe Ratios for both indices for 2002 at different frequencies: for daily, weekly, and monthly data. Observe how the frequency affects the Sharpe ratio and explain it, see also Lo (2002)

c) Another possibility to measure the price of risk is

$$\frac{\mu - r_f}{\sigma^2} .$$

Evaluate this new risk measure also for different frequencies. Is this measure more robust with respect to different frequencies?

**Exercise 9: Beta Pricing**

Explain and evaluate the Beta-Pricing equation (8.10) in Chap. 8, use an example, see Benninga (1998), why can ot be used to compute the cost of capital of a firm, Chaps. 2 and 8.

**Exercise 10: Asset Pricing**

Answer the following questions:

1. Use power utility: $u(C) = \frac{C^{1-\gamma}}{1-\gamma}$ and derive:
   a) absolute risk aversion:= $-\frac{U''(C)}{U'(C)}$
   b) relative risk aversion:= $-\frac{CU''(C)}{U'(C)}$
2. Given a two period model (see Chap. 9):

$$\begin{aligned} \max_{\varepsilon} U(C_t, C_{t+1} &= U(C_t) + \beta E_t(U(C_{t+1})) \\ \text{s.t. } C_t &= e_t - p_t\varepsilon \\ C_{t+1} &= e_{t+1} + x_{t+1}\varepsilon \end{aligned}$$

$e$= endowment; $\varepsilon$= amount of assets; $x_{t+1}$= pay off next period; $U(C) =$ power utility

Derive: 1. Euler equation, 2. stochastic discount factor, 3. risk-free rate, 4. equity premium, and 5. Sharpe-ratio.

**Exercise 11: Advanced Asset Pricing**

Answer the following questions:

1. Given the fact that for log-normality holds:

$$(*) log E_t X_{t+1} = E_t log X_{t+1} + \frac{1}{2} Var\ log X_{t+1}$$
$$= E_t x_{t+1} + \frac{1}{2}\sigma_{xt}^2$$

   Hereby $x_{t+1} = log X_{t+1}$. Derive for the growth rate of consumption (if log normally distributed):

$$r^f = log(1-\delta) + \gamma E_t(\Delta c_{t+1}) - \frac{1}{2}\gamma^2 \sigma_t^2(\Delta c_{t+1})$$

   Start with:

$$R^f = \frac{1}{E(m)} = \frac{1}{E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}$$

   $r^f = lnR^f$; $\beta = \frac{1}{1+\delta}$; $\gamma =$ parameter of relative risk aversion

2. Why do preferences with habit formation lead to a higher volatility of the discount factor than for power utility and time varying aversion, see Cochrane (2001, Chap. 21), Cochrane and Campbell (2000), Jerman (1998).

3. Why do preferences with loss aversion lead to a time varying risk aversion and higher equity premium than for power utility, see Grüne and Semmler (2005b).

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